General Mathematics 2019 v1.2

IA3 sample marking scheme

January 2020

Examination (15%)

This sample has been compiled by the QCAA to model one possible approach to allocating marks in an examination. It matches the examination mark allocations as specified in the syllabus (~ 60% simple familiar, ~ 20% complex familiar and ~ 20% complex unfamiliar) and ensures that all assessment objectives are assessed.

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.

Queensland Government
Queensland Curriculum & Assessment Authority
191481
For all Queensland schools

Instrument-specific marking guide (ISMG)

Criterion: Foundational knowledge and problem-solving

Assessment objectives

1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 4 topics
2. comprehend mathematical concepts and techniques drawn from all Unit 4 topics
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from all Unit 4 topics.

The student work has the following characteristics:	Cut-off	Marks
• consistently correct selection, recall and use of facts, rules, definitions and procedures; authoritative and accurate command of mathematical concepts and techniques; astute evaluation of the reasonableness of solutions and use of mathematical reasoning to correctly justify procedures and decisions; and fluent application of mathematical concepts and techniques to solve problems in a comprehensive range of simple familiar, complex familiar and complex unfamiliar situations.	> 93%	15
	> 87%	14
• correct selection, recall and use of facts, rules, definitions and procedures; comprehension and clear communication of mathematical concepts and techniques; considered evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and proficient application of mathematical concepts and techniques to solve problems in simple familiar, complex familiar and complex unfamiliar situations.	> 80%	13
	> 73%	12
• thorough selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of solutions and use of mathematical reasoning to justify procedures and decisions; and application of mathematical concepts and techniques to solve problems in simple familiar and complex familiar situations.	> 67%	11
	> 60%	10
• selection, recall and use of facts, rules, definitions and procedures; comprehension and communication of mathematical concepts and techniques; evaluation of the reasonableness of some solutions using mathematical reasoning; and application of mathematical concepts and techniques to solve problems in simple familiar situations.	> 53%	9
	> 47%	8
• some selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of mathematical concepts and techniques; inconsistent evaluation of the reasonableness of solutions using mathematical reasoning; and inconsistent application of mathematical concepts and techniques.	> 40%	7
	> 33%	6

The student work has the following characteristics:	Cut-off	Marks
• infrequent selection, recall and use of facts, rules, definitions and procedures; basic comprehension and communication of some mathematical concepts and techniques; some description of the reasonableness of solutions; and infrequent application of mathematical concepts and techniques.	> 27%	5
	> 20%	4
• isolated selection, recall and use of facts, rules, definitions and procedures; partial comprehension and communication of rudimentary mathematical concepts and techniques; superficial description of the reasonableness of solutions; and disjointed application of mathematical concepts and techniques.	> 13%	3
	> 7%	2
• isolated and inaccurate selection, recall and use of facts, rules, definitions and procedures; disjointed and unclear communication of mathematical concepts and techniques; and illogical description of the reasonableness of solutions.	> 0%	1
• does not satisfy any of the descriptors above.		0

Task

See the sample assessment instrument for IA3: Examination (15%) available on the QCAA Portal.

Sample marking scheme

Criterion	Allocated marks	Marks awarded
Foundational knowledge and problem-solving Assessment objectives 1, 2, 3, 4, 5, 6	15	–
Total	**15**	–

The annotations are written descriptions of the expected response for each question and are related to the assessment objectives.

Note: ✓ = $\frac{1}{2}$ mark

1a.

select and use definition for recurrence relation

Marking scheme

Question 1 (6 marks) SF

a. $t_{n+1} = 1.008t_n$ ✓✓

b.

n	t_n
0	12 000
1	12 096
2	12 192.768
3	12 290.31014
4	12 388.63263

✓✓

After one year Anna will have $12 388.63. ✓✓

1b.

recall and use iterative procedure identified in 1a.

communicate findings using appropriate symbols

c. $A = ?$
$P = 12\,000$
$i = \frac{3.2}{400}$ ✓
$= 0.008$
$n = 24$ ✓

1c.

select and use compound interest formula

$A = P(1+i)^n$ ✓
$= 12\,000(1.008)^{24}$ ✓
$= 14\,528.94$

communicate findings using appropriate symbols

The investment would be worth $14 528.94. ✓✓

Question 2 (3 marks) SF

2.

recall and use:

- recurrence relation
- iterative procedure for reducing balance of loan

communicate findings using appropriate symbols

n	t_n
0	$50\,000$
1	$\left(1+\frac{6.6}{1200}\right)\times 50\,000-750=49\,525$
2	$\left(1+\frac{6.6}{1200}\right)\times 49\,525-750=49\,047.3875$
3	$\left(1+\frac{6.6}{1200}\right)\times 49\,047.3875-750$ $=48\,567.14813$

✓✓

✓✓

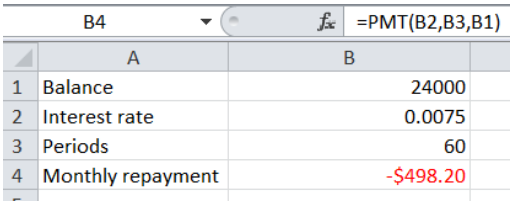
After three months Halim still owes $48 567.15. ✓✓

Question 3 (4 marks) SF

3a.

comprehend elements in the question and transfer this information to the relevant cell

a.



✓✓✓✓

b. =PMT(B2,B3,B1) ✓✓

3c.

use rules

communicate findings using appropriate symbols

c. Total repayment $= 60 \times 498.20$ ✓

$= 29\,892$

Cameron will repay $29 892. ✓

3b.

recall relevant spreadsheet function to calculate the payment for a loan based on constant payments and interest rate

Question 4 (4 marks) CU

4.

select and use rule to determine total amount repaid using monthly and fortnightly repayments

Option 1

$\boldsymbol{A = 350\,000}$

$\boldsymbol{M}$ =?

$\boldsymbol{i = \frac{4.71}{2600}}$

$\boldsymbol{n = 25 \times 26}$ ✓ (for $\boldsymbol{i}$ and $\boldsymbol{n}$)

Fortnightly repayment:

$\boldsymbol{A = M\left(\frac{1-(1+i)^{-n}}{i}\right)}$ ✓

$\boldsymbol{350\,000 = M\left(\frac{1-(1+0.001811)^{-650}}{0.001811}\right)}$

$\boldsymbol{350\,000 = M \times 381.788}$

$\boldsymbol{M = 916.74}$ ✓

Total repayment $= \boldsymbol{25 \times 26 \times 916.74}$ ✓

$\boldsymbol{= 595\,881}$ ✓

Option 2

Total repayment $= \mathbf{18} \times \mathbf{12} \times \mathbf{2500}$
$= \mathbf{540\,000}$ ✓

Difference $= \mathbf{595\,881} - \mathbf{540\,000}$
$= \mathbf{55\,881}$ ✓

The bank manager is incorrect, she only saves $55 881. ✓

justify decisions by providing reasons for choices

Question 5 (1.5 marks) SF

5.

recall and use definitions

a. There are five edges. ✓
b. There is one loop edge. ✓
c. Vertex B has the loop edge. ✓

Question 6 (3 marks) SF

6.

recall and use definitions

justify decisions made

a. Simple
Connected ✓✓

b. Simple
Connected
Complete ✓✓

c. Simple
Bipartite
Connected ✓✓

Question 7 (3 marks) SF

7.

recall and use definitions

a. 3 ✓
b. Boulia and Cloncurry ✓✓
c. Cloncurry and Kynuna ✓✓
Degree of vertices is 3 ✓

Question 8 (4 marks) SF

8a.

recall and use definition of adjacency matrix

a.

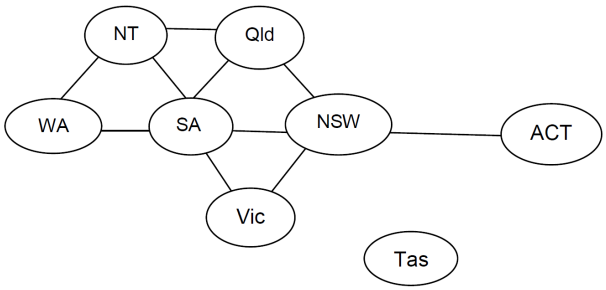
	WA	*NT*	*SA*	*Qld*	*NSW*	*Vic*	*ACT*	*Tas*
WA	0	1	1	0	0	0	0	0
NT	1	0	1	1	0	0	0	0
SA	1	1	0	1	1	1	0	0
Qld	0	1	1	0	1	0	0	0
NSW	0	0	1	1	0	1	1	0
Vic	0	0	1	0	1	0	0	0
ACT	0	0	0	0	1	0	0	0
Tas	0	0	0	0	0	0	0	0

✓✓✓✓

8b.

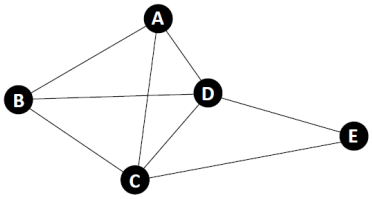
recall and use definition of graph

b.



✓✓✓✓

Question 9 (8, 2.5 marks) SF, CU



9a.

recall and use definition of Hamiltonian graph:

- start and finish at A (Hamiltonian cycle)
- connect every vertex only once (Hamiltonian path)

determine the Hamilton cycle

a. ADECBA ✓✓
(begins and ends at the same vertex ✓✓ and connects every vertex only once ✓, starts and finishes at A)

9b.

recall and use definition of edge

b. AC, BD and CD ✓✓✓

9c.

recall and use definition of a Euler trail

c. For a Euler trail to exist, all vertices must be of an even degree OR there must be exactly two vertices of odd degree. ✓

Degree of each vertex currently:✓
B: 3
A: 3
C: 4
D: 4
E: 2

justify procedures and decisions

find a possible trail

Begin with an odd-numbered vertex and end with the other odd-numbered vertex. ✓
Determine a trail that uses every edge once only (may visit a vertex more than once). ✓
Therefore, beginning with vertex A, ✓✓
one possible trail is:
A – B – C – A – D – C – E – D – B ✓✓

9d.

comprehend total edge distance is required

recall and use rule: $speed = \frac{distance}{time}$

solve for the latest time

d. Total distance to walk is 16 kilometres. ✓
Time to walk $= \frac{16}{5} = 3.2$ hours. ✓
2.5 hours castle visiting time.
Total time $= 2.5 + 3.2 = 5.7$ hours $= 5$ hours 42 minutes. ✓
To finish the walk by 5 pm, the family must leave at 11:18 am.✓✓

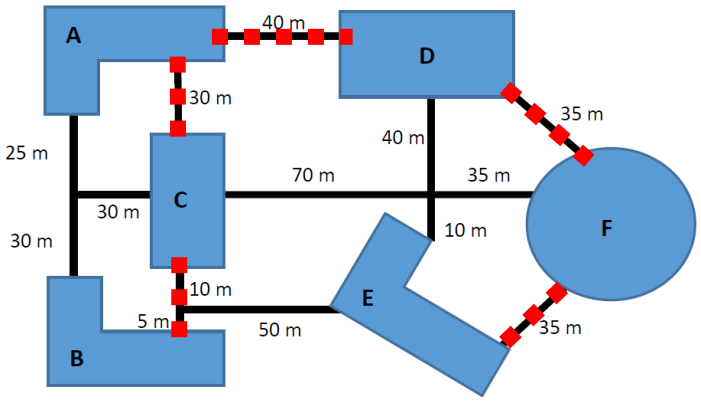
Question 10 (6.5 marks) CU

✓✓

10.

comprehend use of spanning trees

justify decision to use spanning tree



use procedures to:

- determine total length of walkway
- determine the number of connections of a walkway to the stormwater system
- determine the cost (solve the problem)

Using spanning tree will ensure that students can walk to any building without getting wet, but cost of walkways is reduced. ✓✓

Total length of walkway = $5 + 10 + 30 + 40 + 35 + 35$

$= 155$ m ✓✓

Number of stormwater connections = 10 ✓✓

Cost $= 155 \times 180 + 10 \times 1220$

$= 40\ 100$ ✓✓

The total cost to build the walkway will be \$40 100. ✓

✓✓

communicate using

- everyday language
- appropriate symbols
- clear organisation and presentation of information

Question 11 (3 marks) SF

11.

use procedure for finding minimum spanning tree:

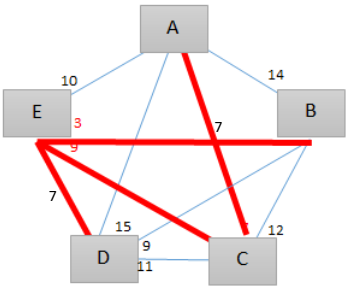
choose smallest edge to begin

select smallest edge from the two vertices included so far

inspect all vertices included so far and select smallest edge

continue procedures until all vertices in the graph are included in the tree

recall the rule for finding minimum length

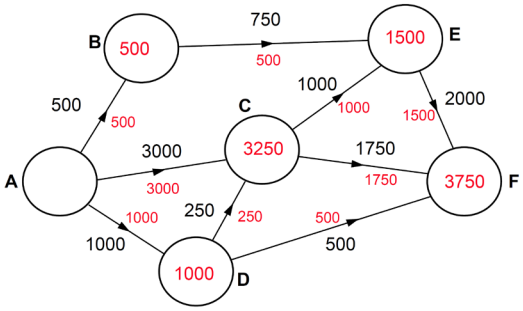


a. AC, BE, CE, DE ✓✓✓✓

b. Minimum spanning tree length $= 7 + 3 + 9 + 7$

$= 26$ m ✓✓

Question 12 (3 marks) CF



✓

✓

✓

Time = 450 000 ✓ ÷ 3 750 ✓
= 120 minutes ✓

It will take two hours to fill the tank.

12.

comprehend use of network flow that allows for flow in one direction only

select and recall:

- outflow from each node
- inflow from each node

use facts to determine the maximum flow for the network

use rule to convert kL to L

recall and use rule to solve for the time

Question 13 (5.5 marks) CF

a. Assuming a team is only allocated one job: ✓

	Job 1	Job 2	Job 3
Team A	12	17	11
Team B	11	20	7
Team C	8	16	5

13a.

comprehend a bipartite graph in matrix/tabular form is required to represent the allocation problem

	Job 1	Job 2	Job 3	
Team A	1	6	0	(–11)
Team B	4	13	0	(–7)
Team C	3	11	0	(–5)

✓

All zeros can be covered with one line (< job numbers); therefore, continue allocation: ✓

recall and use row reduction procedure

justify continued use of procedure by recalling and using procedure of covering all zeros with smallest number of lines

	Job 1	Job 2	Job 3
Team A	0	0	0
Team B	3	7	0
Team C	2	5	0
	(–1)	(–6)	(–0)

✓

perform and use column reduction procedure

comprehend that use of Hungarian algorithm procedure is required

Apply Hungarian algorithm: ✓

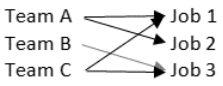
	Job 1	Job 2	Job 3
Team A	2	2	4
Team B	3	7	2
Team C	2	5	2

recall and use procedure for performing Hungarian algorithm

	Job 1	Job 2	Job 3
Team A	0	0	2
Team B	1	5	0
Team C	0	3	0

✓

recall definition to attempt allocation and produce bipartite graph



✓

justify procedures and decisions

Therefore, Team A will do Job 2 (because no other team can), Team B will do Job 3 (because they only have one option), and Team C will do Job 1 (because it is the only option left). ✓

solve for the minimum cost

The minimum cost will be 17 + 7 + 8 = $32 million. ✓

13b.

evaluate the reasonableness of solutions by considering assumptions made

b. If the original assumption was changed to allow a team to complete more than one job, the cost is minimised if Team C completes all jobs (8 + 16 + 5 = $29 million) or the same if Team C completes Job 2 and Job 3, and Team B completes Job 1 (16 + 5 + 11 = $32 million). ✓

Hence, the $32 million cost is reasonable if it is assumed that each team can accept one job only. ✓

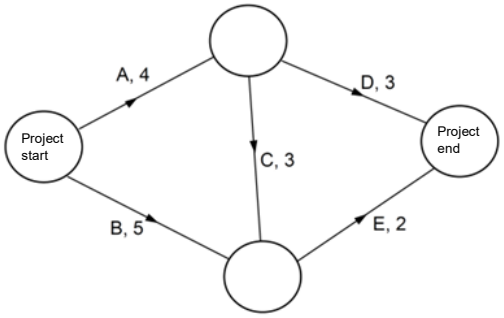
Question 14 (2, 4 marks) SF, CF

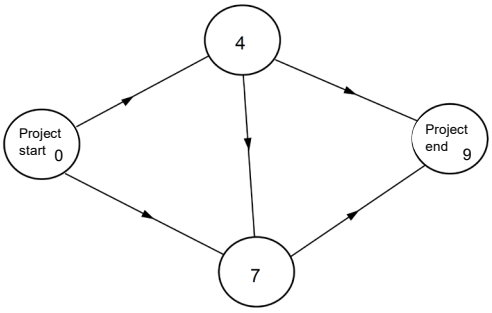
14.

recall and use procedure for constructing a network diagram to represent durations and interdependencies of activities

a.

✓✓✓✓



comprehend that critical path must be determined select and use procedures to find the critical path	b.  ✓✓ Critical path = longest path = 9 weeks ✓✓ Therefore, the project cannot be completed in less than nine weeks. ✓ However, activities B and D ✓✓ could be delayed or extended by up to two weeks ✓ each without affecting the minimum project completion time.	justify procedures and decisions by explaining mathematical reasoning comprehend that float or slack time are required and use procedure to determine activities that may be delayed