General Mathematics 2019 v1.2

Units 1 and 2 sample marking scheme

April 2019

Examination

This sample has been compiled by the QCAA to model one possible approach to allocating marks in an examination. It matches the examination mark allocations as specified in the syllabus (~ 60% simple familiar, ~ 20% complex familiar and ~ 20% complex unfamiliar) and ensures that all assessment objectives are assessed.

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 1 and 2
2. comprehend mathematical concepts and techniques drawn from Units 1 and 2
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 1 and 2.

Queensland Government
Queensland Curriculum & Assessment Authority
For all Queensland schools
190439

Task

See the sample assessment instrument for Units 1 and 2: Examination (available on the QCAA Portal).

Sample marking scheme

The annotations are written descriptions of the expected response for each question and are related to the assessment objectives.

Note: ✓ = $\frac{1}{2}$ mark

Marking scheme

Paper 1 (simple familiar)

Question 1 (5 marks)

1. recall and use facts
3. use mathematical terminology, symbols and conventions

Wage = 220 x 2 ✓ + 6.5% ✓ x (2500 ✓ + 4275 ✓)

= \$440 ✓ + \$440.375 ✓ *(does not round before final answer)* ✓

= \$ ✓880.38 ✓ *(correct rounding)* ✓

Question 2 (3 marks)

1. select rules
2. identify and symbolise relevant concepts
3. present information in symbolic form

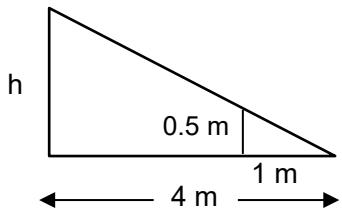


diagram ✓ (not to scale)

Using similar triangles ✓

$\frac{0.5}{1} = \frac{h}{4}$ ✓

$\therefore h = 4 \times 0.5$ ✓

$= 2\,m$

The height of the tree is 2 metres. ✓✓

Question 3 (3 marks)

1. use definitions
2. identify relevant techniques

$\cos\theta = \frac{a}{h}$ ✓

$\cos 28° = \frac{104}{h}$ ✓

$h = \frac{104}{\cos 28°}$ ✓✓

Hypotenuse = $117.79\ cm$ ✓✓

Question 4 (2,3 marks)

1. select rules and put them into effect
2. comprehend mathematical concepts and techniques
3. communicate using mathematical conventions

a) Using the cosine rule: (*select appropriate rule*) ✓

$(AC)^2 = 22^2 + 26^2 - 2 \times 22 \times 26 \cos 130°$ ✓

$= 1895.349025$ ✓

$AC = \sqrt{1895.349025} = 43.5356064\text{cm}$ ✓ (*don't round until the final answer*)

The length of AC is 43.54 cm ✓

b) Area $= \frac{1}{2} ac \sin B$ (*select appropriate rule)* ✓

$= \frac{1}{2} \times 26 \times 22 \times \sin 130°$ ✓

Area $= 219.0887107$ ✓

$= 219.09$ ✓ cm^2 ✓

Question 5 (4 marks)

1. recognise particular features and consider relevance; perform calculations using technology
2. symbolise critical elements
3. organise and present information in symbolic form; use mathematical symbols and conventions

✓

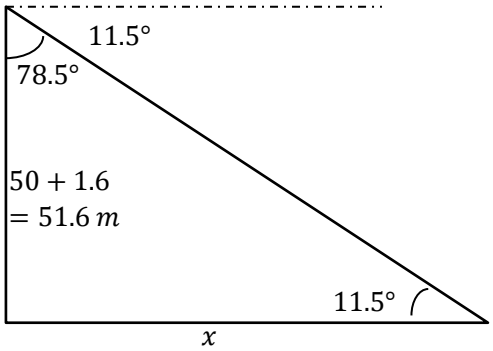


diagram ✓✓ (not to scale)

$\tan 78.5° = \frac{x}{51.6}$ ✓

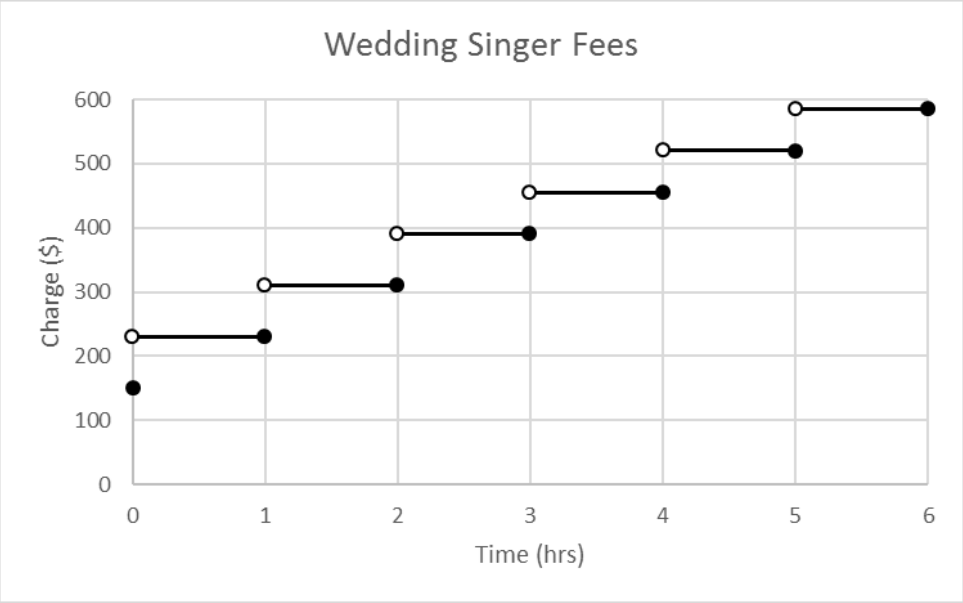
$x = 51.6 \times \tan(78.5°)$ ✓✓

$x = 253.6221028\ m$ ✓

The boat is 253.62 metres from the cliff. (*rounded to two decimal places)* ✓

Question 6 (4 marks)

2. understand the meaning, nature and purpose of the mathematics concepts and techniques
3. organise and present information in graphical form



✓ *graph heading*

✓✓ *axes labels and units*

✓ *correct type of graph*

✓ *step graph lines, open and closed circles ✓ and correct values ✓✓*

Question 7 (2 marks)

1. use facts; present procedures without technology
2. identify relevant techniques
3. use mathematical conventions

$v^2 = u^2 + 2as$

$v^2 - u^2 = 2as$ ✓✓

$\frac{v^2-u^2}{2a} = s$ *$(v^2 - u^2$ as numerator)* ✓ *$(2a$ as denominator)* ✓

Question 8 (2 marks)

1. use procedures
3. organise and present information

GM Geog.

Marks (M) = $\begin{matrix} \boldsymbol{T1} \\ \boldsymbol{T2} \\ \boldsymbol{T3} \end{matrix} \begin{bmatrix} 15 & 20 \\ 9 & 16 \\ 12 & 18 \end{bmatrix}$ ✓✓✓✓

Question 9 (2 marks)

1. use procedures
2. identify relevant techniques
5. justify decisions by explaining mathematical reasoning

Calculate unit cost of each bag and then compare the prices.

$389/1000 = 0.389\ c/g$ ✓

$195/500 = 0.39\ c/g$ ✓

$145/375 = 0.38\dot{6}\ c/g$ ✓

The best value for money is 375 g for $1.45 as it is the cheapest. ✓

Question 10 (4 marks)

1. select and use rules and procedures
2. understand the meaning and purpose of mathematics concepts and techniques
3. use mathematical conventions and present information in graphical form

Percentage of 14- to 16- year-old city high school students having less than 8 hours sleep/night at each school. ✓

Stem	Leaf
4	8
5	6
6	5 8
7	4 5 6 8 9 9 9
8	2 3 4 5 8 9 9
9	0 1

table headings ✓✓

data in columns ✓✓✓✓

Key: 5|6 means 56% ✓

Question 11 (2 marks)

1. perform calculations with technology

Using the statistical capabilities on a scientific calculator:

$\bar{x} = 366.5$ mL ✓✓ $s_x\ (sample\ standard\ deviation) = 8.72$ mL ✓✓

Question 12 (4 marks)

2. identify techniques

30 75 95 120 125 170 **180** 180 190 200 220 240 300

$Q_1 = \frac{95+120}{2} = 107.5 \quad Q_3 = \frac{200+220}{2} = 210$ ✓✓

$IQR = Q_3 - Q_1$

1. recall procedures and recognise features

$= 210 - 107.5$ ✓

$= 102.5$ min/day ✓

2. identify and

articulate relevant concepts and techniques

4. interpret mathematical results in the context of the situation

$Q_1 - 1.5 \times IQR \leq x \leq Q_3 + 1.5 \times IQR$

$107.5 - 1.5 \times 102.5 \leq x \leq 210 + 1.5 \times 102.5$ ✓

$-46.25 \leq x \leq 363.75$ ✓✓

As all values lie between -46.25 and 363.75 inclusive, there are no outliers. ✓

Paper 2 (simple familiar, complex familiar and complex unfamiliar)

1. select, recall and use definitions and procedures
3. use mathematical terminology, symbols and conventions
5. justify procedures and decisions by explaining mathematical reasoning

Question 1 (4 marks) SF

a. $3A + C$

$= 3\begin{bmatrix}1 & -2\\4 & -1\end{bmatrix} + \begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}$

$= \begin{bmatrix}3 & -6\\12 & -3\end{bmatrix} + \begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}$ ✓✓

$= \begin{bmatrix}5 & -3\\16 & 2\end{bmatrix}$ ✓

b. AB

$= \begin{bmatrix}1 & -2\\4 & -1\end{bmatrix}\begin{bmatrix}3 & 0 & 2\\-1 & 4 & 0\end{bmatrix}$

$= \begin{bmatrix}(1\times 3)+(-2\times -1) & (1\times 0)+(-2\times 4) & (1\times 2)+(-2\times 0)\\(4\times 3)+(-1\times -1) & (4\times 0)+(-1\times 4) & (4\times 2)+(-1\times 0)\end{bmatrix}$
✓✓✓✓

$= \begin{bmatrix}5 & -8 & 2\\13 & -4 & 8\end{bmatrix}$ ✓

1. recall facts
2. comprehend mathematical concepts

Question 2 (4 marks) SF

a. continuous ✓✓

b. categorical ✓✓

c. numerical ✓✓ discrete ✓✓

1. recognise features of recalled information
2. make connections between topics

Question 3 (5 marks) CF

$3\begin{bmatrix}y & 4\\6 & x-1\end{bmatrix} + \begin{bmatrix}3 & -7\\-14 & y\end{bmatrix} = \begin{bmatrix}6x & 5\\4 & 6\end{bmatrix}$

$\begin{bmatrix}3y & 12\\18 & 3x-3\end{bmatrix} + \begin{bmatrix}3 & -7\\-14 & y\end{bmatrix} = \begin{bmatrix}6x & 5\\4 & 6\end{bmatrix}$ ✓

$\begin{bmatrix}3y+3 & 5\\4 & 3x-3+y\end{bmatrix} = \begin{bmatrix}6x & 5\\4 & 6\end{bmatrix}$ ✓

6. make decisions about techniques used to develop a solution

$3y + 3 = 6x$ ✓ (eq. 1)

$y + 1 = 2x$ ✓

$\therefore y = 2x - 1$ ✓

$3x - 3 + y = 6$ (eq. 2)

Substitute $y = 2x - 1$ into eq. 2✓

$3x - 3 + 2x - 1 = 6$

$5x - 4 = 6$ ✓

$5x = 10$

$\therefore \boldsymbol{x = 2}$ ✓

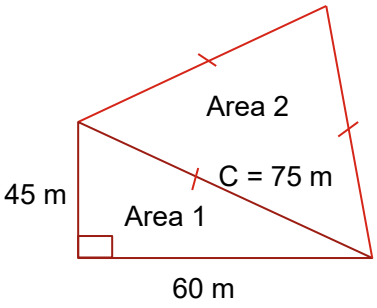
Substitute $\boldsymbol{x = 2}$ into $y = 2x - 1$✓

$y = 3$ ✓

Question 4 (6 marks) CF

3. present information in symbolic form
4. check calculations using relevant facts and procedures
6. translate information into a mathematically workable format

Area 1 = $\frac{45 \times 60}{2} = 1350\ m^2$ ✓✓



(diagram) ✓✓

Length C

$C = \sqrt{45^2 + 60^2}$

$= 75$ m ✓✓

Area 2

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$where \quad s = \frac{75 \times 3}{2} = 112.5$ ✓

$A = \sqrt{112.5(112.5 - 75)(112.5 - 75)(112.5 - 75)}$ ✓✓

$= \sqrt{112.5(112.5 - 75)^3}$

$= 2435.70\ m^2$ ✓✓

The area of the block is $1350m^2 + 2435.7m^2 = \boldsymbol{3785.70\ m^2}$ ✓

4. evaluate the reasonableness of solutions; interpret results in the context of the solution
5. construct mathematical arguments and provide reasons for choices made and conclusions reached
6. analyse the context of the problem and make decisions about techniques and technology used to develop a solution

Question 5 (6 marks) CU

(Logically we know that the cube has a greater surface area but the student must justify their response by showing calculations.)

No marks awarded for an answer only.

$\boldsymbol{V_{cube}} = s^3$

$10 = s^3$ ✓

$\therefore s = \sqrt[3]{10}$ ✓

$s \approx 2.15\ m$ *(2 d.p.)* ✓

$\boldsymbol{SA_{cube}} = 2.15443469 \times 2.15443469 \times 6$ ✓

$= 27.849533\ m^2 \approx 27.85\ m^2$ ✓

$\boldsymbol{V_{sphere}} = \frac{4\pi r^3}{3}$

$10 = \frac{4\pi r^3}{3}$ ✓

$\therefore r^3 = \frac{10 \times 3}{4\pi} = 2.387324146$ ✓

$\therefore r = \sqrt[3]{2.387324146} = 1.34\ m$ *(2 d.p.)* ✓

$\boldsymbol{SA_{sphere}} = 4\pi r^2$

$= 4 \times \pi \times 1.336504618^2$ ✓

$= 22.44661156\ cm^2 \approx 22.45\ m^2$ ✓

To paint both solids one would consider the surface areas. The surface area of the cube is $27.85\ m^2$ and the surface area of the sphere is $22.45\ m^2$, a difference of approximately $5.40\ m^2$. ✓

Therefore, the sphere would be cheaper to paint, since $22.45\ m^2 < 27.85\ m^2$ ✓

2. comprehend mathematical techniques
5. justify procedures and decisions by explaining mathematical reasoning; describe mathematical thinking
6. analyse the context of the problem

Question 6 (5 marks) CF

The photographer is buying Australian currency as they need to convert all prices to Australian dollars: ✓

London: $\text{GBP}\ £345.85 \div 0.6835 = \text{AUD}\ \$506.00$ ✓✓

New York: $\text{USD}\ \$588 \div 1.1002 = \text{AUD}\ \$534.45$ ✓✓

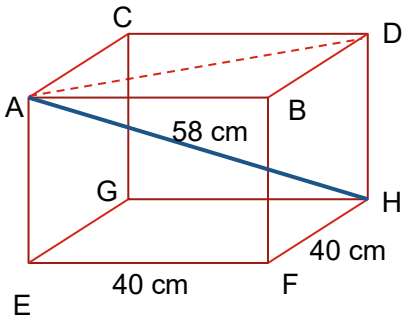
Australia: $\text{AUD}\ \$620$ ✓

It is observed that postage to or within Australia is included in all prices.✓
Assume there are no other factors such as a warranty or exchange of goods. ✓
Therefore, the photographer should purchase the camera from Great Britain.

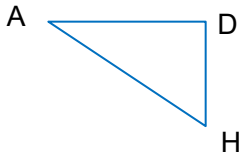
1. select and use procedures
3. communicate using everyday language to present information in symbolic form
5. justify procedures using mathematical reasoning that is rigorous and requires clarity and precision
6. solve problems by applying mathematical concepts and techniques

✓✓

Question 7 (5 marks) CU



Side view



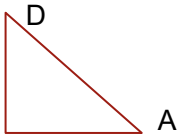
✓✓ *diagram*

We need to calculate the angle between the web $\overline{AH}$ and $\overline{AD}$. First, find the length of $\overline{AD}$ using Pythagoras' theorem.

$\overline{AD} = \sqrt{40^2 + 40^2}$

$= 56.56854249\ cm$ ✓

Top view



Then, calculate the height $\overline{DH}$.

$\overline{DH} = \sqrt{58^2 - 56.56854249^2}$ $= 12.80624847\ cm$ ✓✓

Use three known sides to calculate the angle: ✓

$c^2 = a^2 + b^2 - 2ab\ cos\ C$ ✓

$12.80624847^2 = 58^2 + 56.56854249^2 - 2 \times 58 \times 56.56854249\ cos\ C$

$-12.80624847^2 + 58^2 + 56.56854249^2 = 2 \times 58 \times 56.56854249\ cos\ C$

$\frac{-12.81^2+58^2+56.57^2}{2\times58\times56.57} = cos\ C$ (only round at the end) ✓

$\cos C = 0.975319698$ ✓

$C = cos^{-1}\ 0.9753 = 12.7558717° = 12°45'$

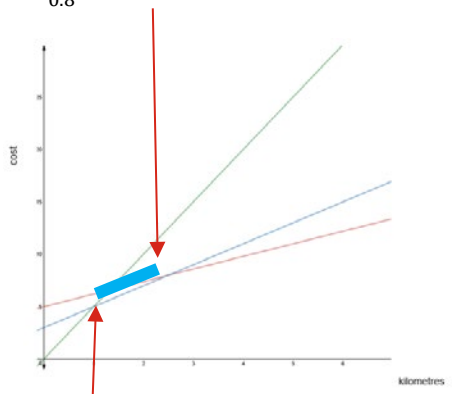
The angle of depression the web makes with the top of the box is $12°45'$. Therefore, the web will hold. ✓

Question 8 (5 marks) CU

2. identify and articulate critical elements of relevant techniques
4. reflect on whether the

Company 1: red line, Company 2: blue line and Company 3: green line.

Company 2 is cheaper than Company 1 at first. Once the lines meet, Company 1 becomes cheaper. Company 2 is more expensive than Company 3 until the lines meet and then it becomes the cheaper option.

problem has been solved 5. construct mathematical arguments and providing reasons for choices made and conclusions reached 6. solve problems by applying mathematical concepts and techniques, make decisions about the technology used to develop a solution	To calculate the exact time that the lines meet I will solve using substitution into simultaneous equations. Company 1: $C = 1.2k + 5$ ✓ Company 2: $C = 2k + 3$ ✓ Company 3: $C = 5k$ ✓ Companies 1 and 2 are the same cost when $C = 1.2k + 5$ and $C = 2k + 3$ meet. So, when $1.2k + 5 = 2k + 3$ ✓ $2k - 1.2k = 5 - 3$ $0.8k = 2$ $k = \frac{2}{0.8} = 2.5km$ ✓  The time that Company 2 is the cheapest option is indicated by the thick blue line on the graph: ✓ The lines representing companies 2 and 3 meet when $C = 2k + 3$ and $C = 5k$ meet. So when $2k + 3 = 5k$ ✓ $5k - 2k = 3$ $3k = 3$ $k = 1$ km ✓ Therefore, Company 2 is the cheapest option for pets being walked between 1 km and 2.5 km. ✓✓