Essential Mathematics 2019 v1.1

IA1 high-level annotated sample response

September 2021

Problem-solving and modelling task

This sample has been compiled by the QCAA to assist and support teachers to match evidence in student responses to the characteristics described in the instrument-specific standards.

Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Fundamental topic: Calculations and Unit 3 Topics 1, 2 and/or 3
2. comprehend mathematical concepts and techniques drawn from Fundamental topic: Calculations and Unit 3 Topics 1, 2 and/or
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Fundamental topic: Calculations and Unit 3 Topics 1, 2 and/or 3.

Queensland Government
Queensland Curriculum & Assessment Authority
210972
For all Queensland schools

Instrument-specific standards

Formulate	Solve	Evaluate and verify	Communicate	Grade
The student work has the following characteristics:				
• documentation of appropriate assumptions • accurate documentation of relevant observations • accurate translation of all simple and complex aspects of the problem by identifying mathematical concepts and techniques.	• accurate use of complex procedures to reach a valid solution • discerning application of simple and complex mathematical concepts and techniques relevant to the task • accurate and appropriate use of technology.	• evaluation of the reasonableness of solutions by considering the results, assumptions and observations • documentation of relevant strengths and limitations of the solution and/or model • justification of decisions made using mathematical reasoning.	• correct use of appropriate technical vocabulary, procedural vocabulary and conventions to develop the response • coherent and concise organisation of the response, appropriate to the genre, including a suitable introduction, body and conclusion.	**A**
• statements of appropriate assumptions • statements of relevant observations • translation of simple and complex aspects of the problem by identifying mathematical concepts and techniques.	• use of complex procedures to reach a reasonable solution • application of simple and complex mathematical concepts and techniques relevant to the task • appropriate use of technology.	• statements about the reasonableness of solutions by considering the context of the task • statements about relevant strengths and limitations of the solution and/or model • statements about decisions made relevant to the context of the task.	• use of technical vocabulary, procedural vocabulary and conventions to develop the response • organisation of the response, including a suitable introduction, body and conclusion.	**B**
• statement of assumptions • statement of observations • translation of simple aspects of the problem by identifying mathematical concepts and techniques.	• use of simple procedures to make some progress towards a solution • application of simple mathematical concepts and techniques relevant to the task • use of technology.	• statement about the reasonableness of solutions • statement about strengths and/or limitations of the solution and/or model • statement about decisions made.	• use some of appropriate language and conventions to develop the response • adequate organisation of the response.	**C**
• statement of an assumption or an observation • translation of some simple aspects of the problem by identifying mathematical concepts and techniques.	• application of some simple procedures, mathematical concepts or techniques • superficial use of technology.	• statement about a decision and/or the reasonableness of a solution.	• use of everyday language to develop a response • basic organisation of the response.	**D**
• statement of an assumption, observation or translation of an aspect of the problem.	• inappropriate use of technology or procedures.	• inappropriate statement about a decision or the reasonableness of a solution.	• unclear and disjointed organisation of the response.	**E**

Task

See IA1 sample assessment instrument: Problem-solving and modelling task (available on the QCAA Portal)

Sample response

Criterion	Grade awarded
Formulate Assessment objectives 1, 2, 5	**A**
Solve Assessment objectives 1, 6	
Evaluate and verify Assessment objectives 4, 5	
Communicate Assessment objective 3	

The annotations show the match to the standard descriptors of the instrument-specific standards.

Communicate [A]

coherent and concise organisation of the response

The introduction clearly describes what the task is about and concisely outlines the intent of the task.

Formulate [A]

documentation of appropriate assumptions and accurate documentation of relevant observations

Introduction

Using local information about Tully's rainfall and the variety of rainwater tanks suitable for your house plans, in conjunction with internet and phone apps, our company has provided a recommendation for a rainwater tank to meet the needs of your family of five.

Assumptions and observations

- The number of people in the household will determine the total amount of water usage as the home is used for domestic purposes only (i.e. no additional commercial business).
- There is no loss of rainwater in the tank (e.g. evaporation or leaking) as the tank is insulated, otherwise a larger tank will be required.
- The rainwater tank is positioned at ground level to minimise the installation costs.
- Rainfall data from Tully for the last 10 years was collected and analysed and is deemed to be sufficient to capture long-term rainfall patterns.
- The 'Careful water use' table provides information about a person's typical water use. It is assumed that children and adults both use the same amount of water because they have the same water-consumption needs (e.g. drinking and cooking, bathroom and toilet, clothes washing).

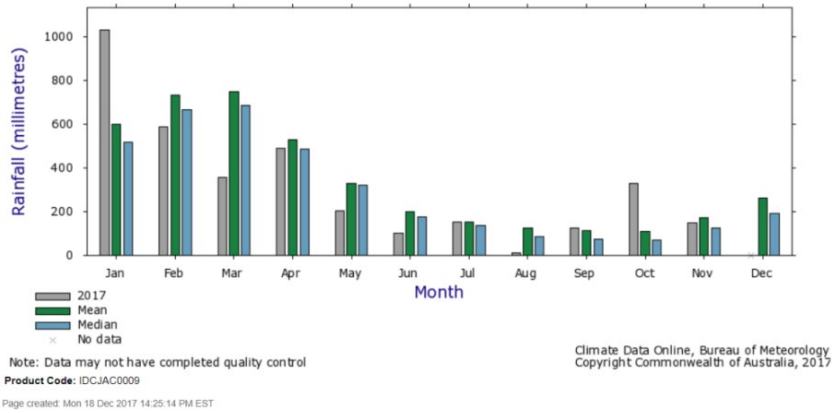
Communicate [A]

correct use of appropriate technical vocabulary, procedural vocabulary and conventions to develop the response

Analysis and discussion

The mean and median rainfall data for Tully was obtained from the Bureau of Meteorology (BOM) website. The past 90 years of data has been used to create the bar graphs below

Comparison of 2017 rainfall to the long-term averages at the Tully Sugar Mill



Source: Bureau of Meteorology, '2017 Rainfall: Tully Sugar Mill', www.bom.gov.au/jsp/ncc/cdio/weatherData/av?p_display_type=dataSGraph&p_stn_num=032042&p_nccObsCode=136&p_month=13&p_startYear=2017.

Formulate [A]

accurate documentation of relevant observations

In all months, the median values are lower than the mean. From the graph, for example, in February the median is less than 700 mm and the mean is more than 700 mm. Examining the data for February over the last 10 years (see table below) shows why this is the case.

February annual rainfall for Tully Sugar Mill — 2007 to 2016

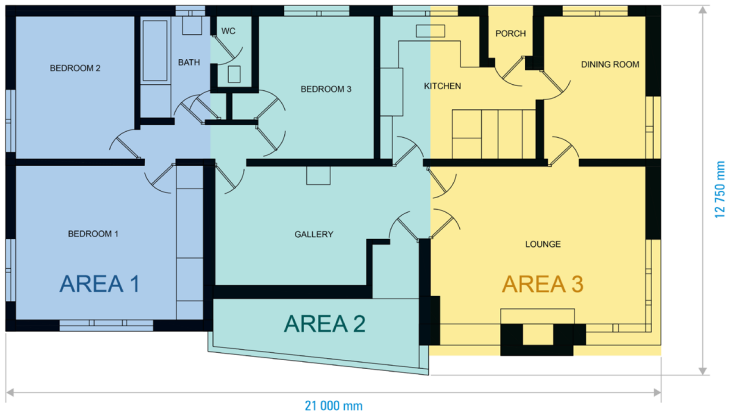
Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Rainfall (mm)	1420	500	106	573	1058	446	281	996	632	181

Solve [A]

discerning application of simple and complex mathematical concepts relevant to the task

There is a significant amount of variation from year to year. The reason the median is lower than the mean is that there are years with significantly higher rainfall in the month, which skews the average higher, e.g. in 2007 and 2011. This may be due to heavy downpours (or floods) that occurred in those years. As such, the median is a better predictor of typical rainfall than the mean.

Calculating the rain collected



Adapted from MichaelScott99 2014, *Ballarat Waller Estate Floor Plan*, https://commons.wikimedia.org/wiki/File:BALLARAT_Waller_Estate_Floor_plan.pdf , CC BY-SA 3.0.

The dimension ratios from the floor plan have been calculated as follows:

Formulate [A]

accurate documentation of relevant observations

documentation of appropriate assumptions

Length: 11.5 cm : 21 000 mm →11.5 cm : 21.00 m → 1 cm : 1.826 m

Width: 6.9 cm : 12 750 mm → 6.9 cm : 12.75 m → 1 cm : 1.848 m

Using a scale factor of 1.8 (correct to 1 d.p.), the above floor plan was divided into sections to approximate the total roof area of the house.

Communicate [A]

coherent and concise organisation of the response

Use of a table to clearly summarise information and data.

Summary of area calculations (rounded to 1 d.p.):

Area (m^2)	Working out
1	$A_1 = 6.0 \times 3.7 \times 1.8^2$ $A_1 \approx 71.9\ m^2$
2	$A_2 = \frac{(6.9 + 6.4)}{2} \times 3.8 \times 1.8^2$ $A_2 \approx 81.9\ m^2$
3	$A_3 = 6.5 \times 4.0 \times 1.8^2$ $A_3 \approx 84.2\ m^2$
Total roof area	Area = $A_1 + A_2 + A_3$ Area ≈ 238.0 m^2

Solve [A]

discerning application of mathematical concepts and techniques relevant to the task

To the nearest 10 m^2 the approximate total roof area is 240 m^2. The volume of water collected is the total roof area × depth of rainfall. For example, in 2013, based on 281 mm (i.e. 0.281 m) of rainfall, the amount of rainwater (rounded to 1 d.p.) collected by the roof is:

$$V = A \times h$$
$$V = 240 \times 0.281$$
$$V \approx 67.4\ \text{m}^3$$

Formulate [A]

accurate translation of all simple and complex aspects of the problem by identifying mathematical concepts and techniques

Evaluate and verify [A]

evaluation of the reasonableness of solutions by considering the results, assumptions and observations

The approximate potential volume of rainwater collected was 67 400 L, i.e. 67.4 × 1000.

However, the rainwater tank can't realistically collect 100% of the rainfall. Given a flow-off rate of 75%, the approximate potential amount of rainwater was 50 550 L, i.e. 67 400 × 0.75.

Typical monthly rain collection

Communicate [A]

coherent and concise organisation of the response

Use of a table to clearly summarise information and data.

A spreadsheet (see Appendix) was used to calculate the expected median amount of rainwater collected each month, as summarised below. The monthly median rainfall data for Tully is rounded to the nearest millimetre.

Month	Median rainfall (mm)	Median rainwater collected (kL)
Jan	517	93
Feb	667	120
Mar	687	124
Apr	487	88
May	323	58
Jun	176	32
Jul	137	24
Aug	87	16
Sep	73	13
Oct	72	13
Nov	121	22
Dec	192	34
Average	**295**	**53**

Here's a snippet of the spreadsheet values:

	A	B	C	N
1	**Monthly rainfall summary statistics for all years**			
2	**Month**	**Jan**	**Feb**	**Average**
3	Median rainfall (mm)	516.5	667.3	**295**
4	Median amount of rainwater collected by roof (kL)	**93**	**120**	**53**
5	Roof area (m^2)			
6	240			
7	Flow rate			
8	0.75			

Solve [A]

accurate and appropriate use of technology

Use of spreadsheet software to produce entries for repeated data manipulations.

Here's a snippet of the spreadsheet entries with the formulas showing:

	A	B	C	N
1	Monthly rainfall summary statistics for all years			
2	Month	Jan	Feb	Average
3	Median rainfall (mm)	516.5	667.3	=AVERAGE(B3:M3)
4	Median amount of rainwater collected by roof (kL)	=$A6*B3*$A8/1000	=$A6*C3*$A8/1000	=AVERAGE(B4:M4)
5	Roof area (m^2)			
6	240			
7	Flow rate			
8	0.75			

Solve [A]

accurate use of complex procedures to reach a valid solution

The solution consists of an involved combination of parts that are interconnected. Monthly median rainfall data and the total roof area are used to calculate the volume of rainwater collected each month, which is then compared to the total amount of water consumed by the family each month to recommend a rainwater tank.

Typical monthly water usage

This report has used the 'Careful water use' information to calculate your family's needs.

Purpose	Amount per person per month (L)	Total amount consumed by the family per month (L)
Drinking and cooking	300	300 × 5 = 1500
Dishwashing	150	150 × 5 = 750
Bathroom and toilet	2500	2500 × 5 = 12 500
Washing clothes	200	200 × 5 = 1000
Watering garden (30 min session)	350 L (per half hour) × 3 (times a week) × 52 (weeks in a year) ÷ 12 (months in a year) = 4550 L	
Total amount of rainwater required (per month) = 20 300 L		

Communicate [A]

correct use of appropriate technical vocabulary, procedural vocabulary and conventions to develop the response

The average for the expected median amount of rainwater collected each month is 53 000 L (see 'Typical monthly rain collection' table). Therefore, the average excess amount of water remaining in the tank per month is 32 700 L, i.e. 53 000 – 20 300.

Recommendation

Evaluate and verify [A]

documentation of relevant strengths and limitations of the solution and/or model

Based on our modelling, it is recommended that you opt for a rainwater tank larger than 20 000 L (your average monthly water consumption). This should be sufficient to cater for most rainfall periods. A larger tank (e.g. 40 000 L) would allow you to store extra rainwater from the heavy rain months (January to May) for use in the lighter rain months (August to October).

Our recommendation considers the gutter height of 2.4 m (the ceiling height of the house) meaning that a tank needs a height less than this to allow for water to drain via the pipes.

Evaluate and verify [A]

justification of decisions made using mathematical reasoning

After considering a number of rainwater tanks, a 22 900 L round tank is recommended, having a diameter of 3.45 m and a height of 2.45 m. With an overall height of 2.45 m it will need to be located a few metres from the side of the house, which is only 2.4 m high.

The tank could be placed immediately beside the house but would need to be dug into the ground, increasing the installation costs, but providing no additional benefits.

Communicate [A]

coherent and concise organisation of the response, appropriate to the genre, including a suitable conclusion

Conclusion

Based on a house roof area of 240 m^2, historical rainfall data and predicted water usage, a rainwater tank of 22 900 L capacity would be ideal. Some strengths and limitations for choosing this sized tank are:

Evaluate and verify [A]

documentation of relevant strengths and limitations of the solution and/or model

- This tank would provide sufficient water for your family's typical needs each month (as 22 900 L > 20 300 L).
- It assumes average (not below average) rainfall.
- It doesn't account for the very dry months, so you may need to supplement your water supply.

Communicate [A]

coherent and concise organisation of the response, appropriate to the genre, including a suitable conclusion

Therefore, it is recommended that you purchase a 22 900 L round tank with a diameter of 3.45 m and a height of 2.45 m.

Appendix

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Monthly rainfall summary statistics for all years													
2	Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Average
3	Median rainfall (mm)	516.5	667.3	686.6	487	322.9	177.5	132.3	89.2	72.8	71.9	120.6	191.6	295
4	Median amount of rainwater collected by roof (kL)	93	120	124	88	58	32	24	16	13	13	22	34	53
5	Roof area (m^2)													
6	240													
7	Flow rate													
8	0.75													

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Monthly rainfall summary statistics for all years													
2	Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Average
3	Median rainfall (mm)	516.5	667.3	686.6	487	322.9	177.5	132.3	89.2	72.8	71.9	120.6	191.6	=AVERAGE(B3:M3)
4	Median amount of rainwater collected by roof (kL)	=$A6*B3*$A8/1000	=$A6*C3*$A8/1000	=$A6*D3*$A8/1000	=$A6*E3*$A8/1000	=$A6*F3*$A8/1000	=$A6*G3*$A8/1000	=$A6*H3*$A8/1000	=$A6*I3*$A8/1000	=$A6*J3*$A8/1000	=$A6*K3*$A8/1000	=$A6*L3*$A8/1000	=$A6*M3*$A8/1000	=AVERAGE(B4:M4)
5	Roof area (m^2)													
6	240													
7	Flow rate													
8	0.75													