General Mathematics SEE marking guide

External assessment

SEE 1 — Short response (50 marks)

SEE 2 — Short response (100 marks)

SEE 1 Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Unit 3 Topics 1, 2 and 3
2. comprehend mathematical concepts and techniques drawn from Unit 3 Topics 1, 2 and 3
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Unit 3 Topics 1, 2 and 3.

SEE 2 Assessment objectives

This assessment instrument is used to determine student achievement in the following objectives:

1. select, recall and use facts, rules, definitions and procedures drawn from Units 3 and 4
2. comprehend mathematical concepts and techniques drawn from Units 3 and 4
3. communicate using mathematical, statistical and everyday language and conventions
4. evaluate the reasonableness of solutions
5. justify procedures and decisions by explaining mathematical reasoning
6. solve problems by applying mathematical concepts and techniques drawn from Units 3 and 4.

Purpose

This document is an External assessment marking guide (EAMG).

The EAMG:

- Provides a tool for calibrating external assessment markers to ensure reliability of results
- Indicates the correlation, for each question, between mark allocation and qualities at each level of the mark range
- Informs schools and students about how marks are matched to qualities in student responses.

Mark allocation

Where a response does not meet any of the descriptors for a question or a criterion, a mark of ‘0’ will be recorded. Where no response to a question has been made, a mark of ‘N’ will be recorded.

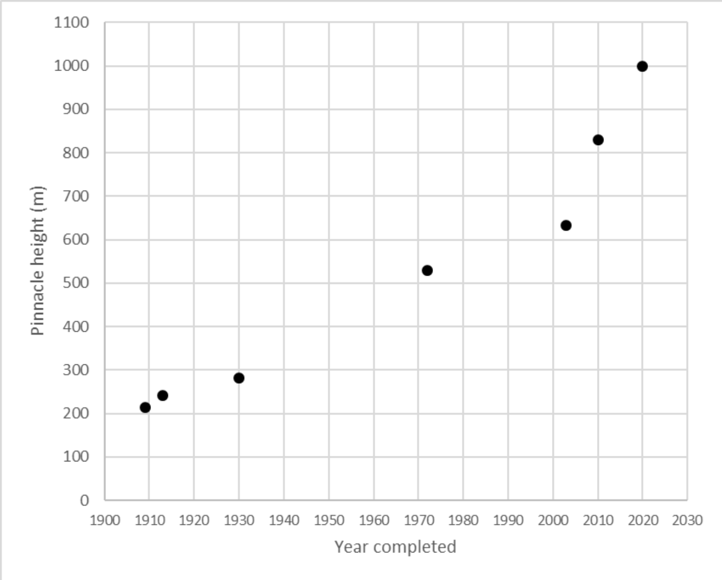
Allow FT mark(s) – refers to ‘follow through’, where an error in the prior section of working is used later in the response, a mark (or marks) for the rest of the response can still be awarded so long as it still demonstrates the correct conceptual understanding or skill in the rest of the response.

This mark may be implied by subsequent working – the full mathematical reasoning and/or working, as outlined in the sample response and associated mark, is not evident in the student response, but by virtue of subsequent working there is sufficient evidence to award mark(s).

SEE 1

External assessment marking guide

Short response (50 marks)

Q	Sample response	The response:
1		correctly labels axes and scales for the scatterplot **[1 mark]** accurately plots the given data points **[1 mark]**

Q	Sample response	The response:
2	Identify variables $x =$ years $y =$ height	correctly identifies the explanatory and response variables **[1 mark]**
	Define variables Let $x =$ the number of years since 1900 (i.e. $x = 9$ in 1909) $y =$ the height of the building in metres	correctly defines the explanatory and response variables **[1 mark]**
	Create model Using a scientific calculator and the data provided in Stimulus 1 $y = 6.229x + 126.214$	determines the parameters for a linear model using all the data values **[1 mark]** determines a linear model **[1 mark]**

Q	Sample response	The response:
3	Define variables Let $n =$ the number of years since 1900 (i.e. $n = 9$ in 1909) $t_n =$ the height of the building in metres Create model @ $n = 13,\ \ t_n = 241$ $t_n = t_1 r^{(n-1)}$ $241 = t_1 r^{12}$… equation 1 @ $n = 110,\ \ t_n = 830$ $t_n = t_1 r^{(n-1)}$ $830 = t_1 r^{109}$… equation 2 Find r equation 2 ÷ equation 1 $\frac{830}{241} = \frac{t_1 r^{109}}{t_1 r^{12}}$ $3.444 = r^{97}$ $r = 1.0128$ Find t_1 sub r into equation 1 $241 = t_1 \times 1.0128^{12}$ $t_1 = 206.81$ $t_n = 206.81 \times 1.0128^{(n-1)}$	correctly defines the response and explanatory variables **[1 mark]** correctly identifies n and t_n values for one point **[1 mark]** determines one of the geometric equations **[1 mark]** identifies n and t_n values for one point **[1 mark]** determines the second geometric equation **[1 mark]** provides evidence of solving simultaneous equations **[1 mark]** determines the r value **[1 mark]** provides evidence of substituting r into one of the equations **[1 mark]** determines the t_1 value **[1 mark]** determines a geometric model **[1 mark]** shows logical organisation, communicating key steps **[1 mark]**

Q	Sample response	The response:

4

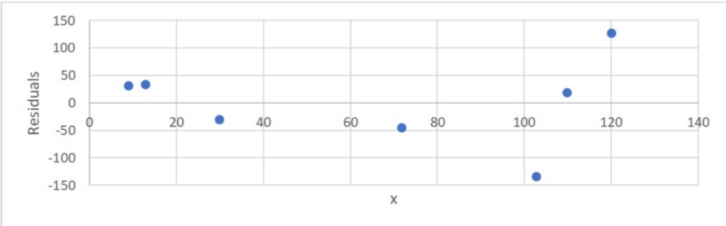
Residuals for linear model

Year	*x*	Height (A)	Linear (P)	Residual (A–P)
1909	9	213	182.272	30.7284
1913	13	241	207.186	33.8137
1930	30	282	313.074	-31.0738
1972	72	530	574.678	-44.6783
2003	103	634	767.767	-133.767
2010	110	830	811.368	18.632
2020	120	1000	873.655	126.345

The response:

- provides an appropriately organised table **[1 mark]**
- correctly calculates the predicted values for each year using the linear model from Question 2 **[1 mark]**
- calculates the residuals for all data values with the linear model **[1 mark]**

Residual plot for linear model



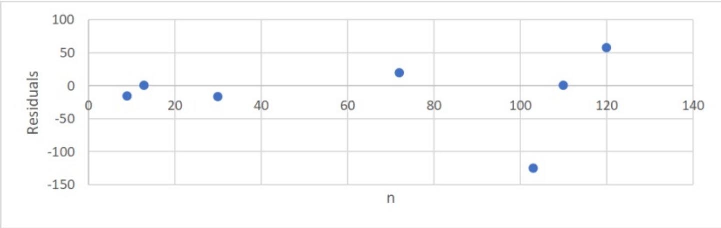
- correctly labels the axes and chooses appropriate scales on at least one residual plot **[1 mark]**
- accurately plots the points on the residual plot for the linear model **[1 mark]**

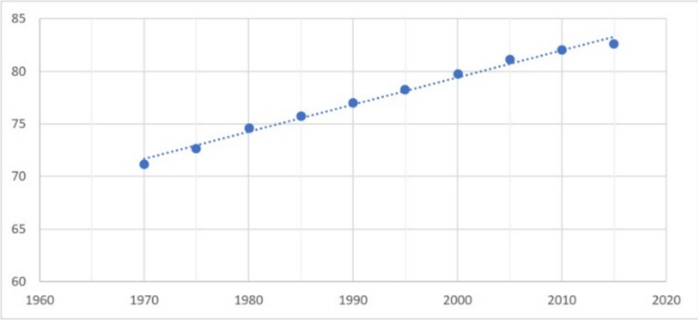
Residuals for geometric model

Year	*n*	Height (A)	Geometric (P)	Residual (A–P)
1909	9	213	229.018	-16.0183
1913	13	241	241	0
1930	30	282	299.324	-17.3237
1972	72	530	511.309	18.6915
2003	103	634	759.139	-125.139
2010	110	830	830	0
2020	120	1000	942.856	57.1443

- provides an appropriately organised table **[1 mark]**
- correctly calculates the predicted values for each year using the geometric model from Question 3 **[1 mark]**
- calculates the residuals for all data values with the geometric model **[1 mark]**

Residual plot for geometric model

Q	Sample response	The response:
		accurately plots the points on the residual plot for the geometric model **[1 mark]**
	Evaluation of models The geometric model is better because the magnitude of each corresponding residual is smaller and the original scatterplot clearly shows that the data is more geometric than linear.	determines the better model **[1 mark]** evaluates the reasonableness of the chosen model by considering the results of the residual plot **[1 mark]** evaluates the reasonableness of the chosen model by considering assumptions and/or observations **[1 mark]** shows logical organisation, communicating key steps **[1 mark]**

Q	Sample response	The response:
5	Model life expectancy Define variables: Let x = the birth year y = life expectancy of newborns	correctly identifies the response and explanatory variables **[1 mark]** correctly defines the response and explanatory variables **[1 mark]**
		correctly labels the axes and chooses appropriate scales for a scatterplot of the life expectancy data **[1 mark]** accurately plots the points on the scatterplot **[1 mark]**
	The data looks linear, so the model is	justifies the reason for the chosen model for life expectancy **[1 mark]**
	$y = 0.259x - 438.145$	determines a model for life expectancy **[1 mark]**
	Life expectancy for someone born in 2021.	provides evidence of substitution **[1 mark]**
	$y = 0.259(2021) - 438.145$ $= 84.9$ Therefore, someone born in Australia in 2021 would be expected to live until 2106.	determines life expectancy for someone born in 2021 **[1 mark]** explains the expected year that the average person born in 2021 will live until **[1 mark]**

Q	Sample response	The response:
	Next, calculate the year of completed construction. Ultima Tower is approximately 3250 m tall.	correctly identifies the height of the tower **[1 mark]**
	Using an iterative process Year / n / Height: 2100 / 200 / 2614.46 2120 / 220 / 3373.78 2119 / 219 / 3331.04 2118 / 218 / 3288.84 2117 / 217 / 3247.18 2116 / 216 / 3206.04	identifies a valid procedure to determine the completion year **[1 mark]**
	Therefore, the model suggests that Ultima Tower will be completed in 2117.	correctly uses the model to determine the completion year **[1 mark]**
	Therefore, someone born in 2021 would not be expected to see the construction of the Ultima Tower completed.	explains whether a newborn is expected to still be alive when the tower is constructed **[1 mark]** shows logical organisation, communicating key steps **[1 mark]**

Year	n	Height
2100	200	2614.46
2120	220	3373.78
2119	219	3331.04
2118	218	3288.84
2117	217	3247.18
2116	216	3206.04

Q	Sample response	The response:
6	Strengths of life expectancy model • A linear model fits the data well because the scatterplot looks very linear. • The line of best fit fits the data closely and is therefore a good model for years close to the domain. Limitations of life expectancy model • Data is for a relatively short domain (from 1970 to 2015), which may limit its usefulness as a long-term model. • Extrapolating the model backwards would show that the average life expectancy was 0 in 1691, and negative before that, which makes no logical sense.	states two relevant strengths and two relevant limitations of using the life expectancy model **[1 mark]** justifies a stated strength and a limitation of using the life expectancy model **[1 mark]**
	Strengths of the building height model • The geometric model fits the shape of the scatterplot better than a linear model, which can be seen in Question 1. • The residual analysis in Question 4 shows that the geometric model was the better of the two models Limitations of the building height model • The model may be a poor predictor as it does not account for technological advances, all the buildings listed are skyscrapers comprised of concrete and steel technology; there is nothing earlier based on timber or large blocks, and nothing later based on future technologies such as nanotubes or carbon fibres • The model is based on data from over 100 years but the prediction is more than 100 years after the final data value. This may not be useful for long-term extrapolation because the domain is relatively small.	states two relevant strengths and two relevant limitations of using the building height model **[1 mark]** justifies a stated strength and a limitation of using the building height model **[1 mark]**

Q	Sample response	The response:
	Evaluation of models • The fact that the life expectancy model is very close to the domain of the data used and that the model predicts values that are very close to the data points suggest that this is probably a reasonable predictor for a child born in 2021. • Because the building height model is projecting so far into the future, it is likely that the technology will improve, the height could potentially be reached earlier, and more people born in 2021 would be able to see it completed.	evaluates the reasonableness of using the life expectancy model to make predictions **[1 mark]** evaluates the reasonableness of using the building height model to make predictions **[1 mark]**

SEE 2

External assessment marking guide

Paper 1: Multiple choice

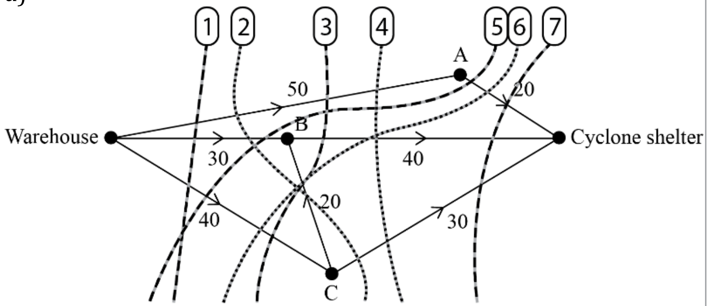
Question	Response
1	C
2	A
3	C
4	D
5	B
6	B
7	C
8	B
9	D
10	D
11	B
12	A
13	A
14	C
15	A

Short response

Q	Sample response	The response:
16	a) AFCFB starts and ends at a different vertex edge repeats: FC = CF ∴ Open walk b) AFCEFBA starts and ends at the same vertex no edges are repeated vertex F is repeated ∴ Closed trail c) ABCDEFA starts and ends at the same vertex no edges are repeated no vertices are repeated ∴ Cycle	• correctly identifies an open walk **[1 mark]** • correctly identifies a closed trail **[1 mark]** • correctly identifies a cycle **[1 mark]**
17	Distance is east / west $\therefore \text{distance} = 111.2 \times \cos\theta \times \text{angular dist.}$ $\text{angular dist.} = 147°\,37' - 140°\,47'$ $= 6°\,50'$ $\text{distance} = 111.2 \times \cos\theta \times \text{angular dist.}$ $= 111.2 \times \cos(37°\,50') \times (6°\,50')$ $= 600.14$ It is approximately 600 km between Mount Gambier and Bairnsdale.	• correctly calculates the angular distance **[1 mark]** • provides evidence of substituting into the appropriate distance rule **[1 mark]** • calculates distance to the nearest km **[1 mark]**

Q	Sample response	The response:
18	a) Arithmetic sequence $t_1 = 353$ $t_3 = 439$ Find d $t_3 = t_1 + 2d$ $439 = 353 + 2d$ $86 = 2d$ $43 = d$ b) Find t_6 $t_6 = t_1 + 5d$ $= 353 + 5 \times 43$ $= 568$ They would expect 568 people to attend the sixth day.	• correctly provides mathematical reasoning to support the answer **[1 mark]** • correctly determines the common difference **[1 mark]** • substitutes into an appropriate rule **[1 mark]** • determines value **[1 mark]**
19	a) $x = 12$ $\therefore y = 2.3(12) + 31.4$ $= 59$ b) A correlation coefficient of 0.688 suggests a moderate association, which means that as the hours spent fishing increase so do the number of fish caught. A coefficient of determination of 0.473 means that 47% of the variation in results can be explained by the variation of hours spent fishing. Therefore the prediction of catching 59 fish after fishing for 12 hours may be valid, however other factors will also come into play.	• correctly calculates 59 **[1 mark]** • correctly describes the strength as either moderate or strong **[1 mark]** • correctly describes the meaning of the coefficient of determination **[1 mark]** • evaluates the reasonableness of the solution **[1 mark]**

Q	Sample response	The response:
20	$A = 350\ 000$ $M = ?$ $i = \frac{0.065}{12}$ $= 0.005416\ldots$ $n = 25 \times 12$ $= 300$ $A = M\left(\frac{1-(1+i)^{-n}}{i}\right)$ $A = M\left(\frac{1-(1+0.005416\ldots)^{-300}}{0.005416\ldots}\right)$ $350\ 000 = M \times 148.102\ldots$ $M = \frac{350\ 000}{148.102\ldots}$ $M = 2363.225\ldots$ The monthly repayment will be $2363.23 each month for 25 years.	• correctly determines the i and n values **[1 mark]** • substitutes into appropriate annuity rule **[1 mark]** • determines monthly repayment **[1 mark]** • states solution with correct units and appropriate rounding **[1 mark]**

Q	Sample response	The response:
21	a)  Cut 1 → 50+30+40 = 120 Cut 2 → 50+30+20+30 = 130 Cut 3 → 50+40+40 = 130 Cut 4 → 50+40+30 = 120 Cut 5 → 20+30+40 = 90 Cut 6 → 20+40+40 = 100 Cut 7 → 20+40+30 = 90 The capacity of the minimum cut is 90, so the maximum flow of this network is 90. b) Cut 1, 2, 3, 4, 5, 6 and 7 change to 70, 80, 80, 70, 70, 80 and 70. The maximum flow is now 70.	• correctly identifies all the possible cuts **[1 mark]** • correctly calculates the flow across all the cuts **[1 mark]** • states maximum flow across minimum cut **[1 mark]** • recalculates flow across minimum cut **[1 mark]** • states new maximum flow **[1 mark]**

<table>
<tr><th>Q</th><th>Sample response</th><th>The response:</th></tr>
<tr><td>22</td><td>a) Total # change uniform = 115
Total # do not change = 95

<table><tr><th></th><th>Change uniform</th><th>Do not change uniform</th></tr><tr><td>Junior staff</td><td>80%</td><td>29.5%</td></tr><tr><td>Senior staff</td><td>20%</td><td>70.5%</td></tr><tr><td></td><td>100%</td><td>100%</td></tr></table>
b) There does appear to be an association between the staff groups and wanting to change the uniform.
The data suggests that junior staff want to change the uniform (80% as opposed to 20% of senior staff) and senior staff do not want to change (70.5% compared with 29.5% of junior staff).</td><td>• correctly determines column totals [1 mark]
• correctly represents the data in a percentaged two-way table [1 mark]
• suggests the presence of an association [1 mark]
• provides reasons to support conclusion [1 mark]</td></tr>
<tr><td>23</td><td>Depart Brisbane 22:45 Monday Brisbane time
Travel <u>+14:35</u>
Arrive Dubai 37:20 Monday
next day – 24:00
13:20 Tuesday

UTC correction <u>– 6:00</u>

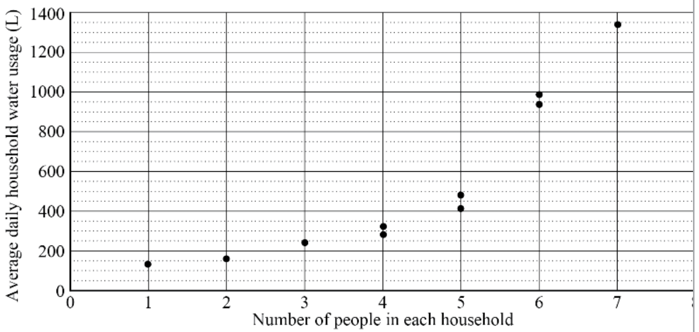
7:20 am on Tuesday in Dubai</td><td>• correctly adds travel time [1 mark]
• calculates arrival time from Brisbane’s perspective [1 mark]
• correctly subtracts time difference [1 mark]
• calculates arrival time and day from Dubai’s perspective [1 mark]</td></tr>
</table>

Q	Sample response	The response:
24	a) 11 cm b) Interpolation c) The least-squares line provided does suggest that at 29 days, the seedling will be 32 cm high. However, the data values are levelling off at about 25 cm, so extrapolation is unwise.	• provides the correct value including units **[1 mark]** • correctly classifies the prediction as interpolation **[1 mark]** • identifies that the least-squares line supports the statement **[1 mark]** • identifies potential dangers of extrapolation **[1 mark]**
25	Option 1 $i_{e1} = (1 + \frac{i}{n})^n - 1$ $= (1 + \frac{0.07}{4})^4 - 1$ ≈ 0.07186 Option 2 $i_{e2} = (1 + \frac{i}{n})^n - 1$ $= (1 + \frac{0.068}{12})^{12} - 1$ ≈ 0.07016 Option 1 is better because it has a slightly higher effective interest rate.	• correctly substitutes into appropriate rule **[1 mark]** • calculates effective interest rate for Option 1 **[1 mark]** • correctly substitutes into appropriate rule **[1 mark]** • calculates the effective interest rate for Option 2 **[1 mark]** • states better option **[1 mark]**

Q	Sample response	The response:
26	a) Determine the common ratio $r = 1 - 0.17$ $= 0.83$ Determine the model let n = the number of years since 2015 and t_n = the number of birds $t_n = t_1 r^{(n-1)}$ $= 483 \times 0.83^{n-1}$ b) $n = 6$ $t_6 = 483 \times 0.83^5$ $= 190.255 \ldots$ Expect 190 birds remaining.	• correctly determines the common ratio **[1 mark]** • determines geometric model **[1 mark]** • correctly determines the n value **[1 mark]** • determines t_6 **[1 mark]** • states a reasonable answer **[1 mark]**

Paper 2

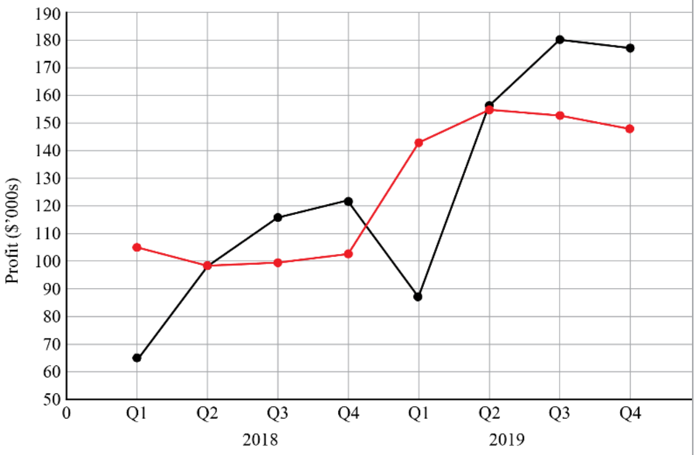
Q	Sample response	The response:
1	**Option 1: Arithmetic sequence** $n =$ the number of minutes starting at 1 $t_n =$ the amount of water in the tank $t_1 = 12\,500$ $d = -135$ $t_n = 5000$ $n = ?$ Find n $t_n = t_1 + (n-1)d$ $\therefore \quad 5000 = 12\,500 - 135(n-1)$ $\therefore 135(n-1) = 7500$ $\therefore \quad n - 1 = 55.5556$ $\therefore \quad n \quad = 56.5556$ The tap was left on until the 57th term. The tap was left on for about 56 minutes.	• correctly defines the variables **[1 mark]** • correctly identifies the parameters t_1, d and t_n **[1 mark]** • substitutes values into appropriate model **[1 mark]** • determines n value **[1 mark]** • states a reasonable answer rounded to the nearest minute **[1 mark]**
	Option 2: Linear function $x =$ the time that the tap has been on $y =$ the amount of water in the tank $c = 12\,500$ $m = -135$	• correctly defines the variables **[1 mark]**
	$y = mx + c$ $\therefore y = -135x + 12\,500$ Find x when $y = 5000$	• correctly identifies the parameters y, m and c **[1 mark]**

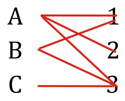
Q	Sample response	The response:
	$\therefore\ 5000 = -135x + 12500$ $\therefore\ 135x = 7500$ $\therefore\quad x = 55.5556$ The tap was left on for about 56 minutes.	• substitutes values into appropriate model **[1 mark]** • determines x value **[1 mark]** • states a reasonable answer rounded to the nearest minute **[1 mark]**
2	 From the calculator $r = 0.886$ A correlation coefficient of 0.886 indicates that the relationship is a very strong positive relationship. However, this relationship as shown in the scatterplot does not appear to be linear, therefore the correlation coefficient should not be used.	• correctly constructs a scatterplot **[1 mark]** • correctly determines the correlation coefficient **[1 mark]** • interprets the value of the correlation coefficient **[1 mark]** • correctly identifies that the scatterplot is not linear **[1 mark]** • correctly identifies that the correlation coefficient should not be used **[1 mark]**
3	$y = 2.1875x + 0.0625$ $\therefore\ b = 2.1875$ $a = 0.0625$ From the table of values $\bar{x} = 5$	• correctly identifies the a and b values **[1 mark]** • correctly determines $\bar{x}$ **[1 mark]**

Q	Sample response	The response:
	Using a $a = \bar{y} - b\bar{x}$ $0.0625 = \bar{y} - 2.1875 \times 5$ $\therefore \bar{y} = 11$ From the table $\bar{y} = \frac{\sum y}{n}$ $\therefore \quad 11 = \frac{4+8+p+q+16}{5}$ $\therefore \quad 55 = 28 + p + q$ $\therefore p + q = 27$ If $q = p + 3$ then $p + p + 3 = 27$ $\therefore \quad 2p = 24$ $\therefore \quad p = 12$ $\therefore \quad q = 15$	• determines $\bar{y}$ **[1 mark]** • determines sum of missing values **[1 mark]** • determines values for p and q **[1 mark]** • shows logical organisation, communicating key steps **[1 mark]**

Q	Sample response	The response:
4	(see table and graph below)	• correctly determines the yearly averages **[1 mark]** • determines profit/yearly average values **[1 mark]** • determines seasonal indices **[1 mark]** • determines deseasonalised values **[1 mark]** • accurately plots deseasonalised data on provided graph **[1 mark]**

Year	Quarter	Profit (in $1000s)	Yearly average	Profit/ yearly average	Seasonal indices	Deseasonalised
2018	1	64	100	0.64	0.61	104.92
	2	98		0.98	1.01	97.03
	3	116		1.16	1.18	98.31
	4	122		1.22	1.2	101.67
2019	1	87	150	0.58	0.61	142.62
	2	156		1.04	1.01	154.46
	3	180		1.2	1.18	152.54
	4	177		1.18	1.2	147.5

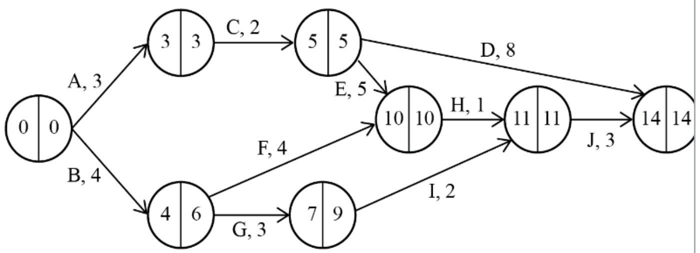


Q	Sample response	The response:
5	Matrix form 3 3 1 4 7 2 4 4 1 row reduction: $R_1 - 1, R_2 - 2, R_3 - 1$ 2 2 0 2 5 0 3 3 0 only need 1 line to cover all the 0s, ∴ column reduction: $C_1 - 2, C_2 - 2$ 0 0 0 0 3 0 1 1 0 need 3 lines to cover all the 0s, ∴ bipartite graph:  contractor task cost A 2 3 B 1 4 C 3 1 Total 8 ∴ Minimum cost is \$8000.	• correctly reduces each row **[1 mark]** • correctly reduces each column **[1 mark]** • allocates each task to one contractor **[1 mark]** • determines minimum cost **[1 mark]** • shows logical organisation, communicating key steps **[1 mark]**

Q	Sample response	The response:

6

Network



Determine minimum completion time.

Shortest path is 14, so with a large enough workforce the job could be completed on the 14th day.

Find how many employees required.

At the start of the project only tasks A and B can be done, so employing more than 2 people at the start would be wasteful.

If the company employed 3 people as suggested, the following job allocation could be used.

Worker 1 follows the critical path to complete the job on day 14.

Worker 2 works on non-critical jobs that are available.

Day 5 is the first day where having 3 employees would be useful.

		Day													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
worker	1	A	A	A	C	C	E	E	E	E	E	H	J	J	J
	2	B	B	B	B	F	F	F	F						
	3					G	G	G	I	I					
	4							D	D	D	D	D	D	D	D

But with 3 workers, activity D could not be completed by day 14.

The owner's belief is incorrect: at least 4 workers must be employed.

The response:

- correctly translates the information into a network **[1 mark]**
- determines LST for each activity **[1 mark]**
- determines EST for each activity **[1 mark]**
- determines minimum completion time **[1 mark]**
- determines whether three workers are sufficient **[1 mark]**
- evaluates reasonableness of the claim **[1 mark]**
- shows logical organisation, communicating key steps **[1 mark]**

Q	Sample response	The response:
7	Perpetuity — find the size of the savings M = 3600 $i = \frac{0.0576}{12}$ $= 0.0048$ A = ? $A = \frac{M}{i}$ $= \frac{3600}{0.0048}$ $= 750\ 000$ Use the total savings to find the size of the monthly payment $A = 750\ 000$ M = ? $i = \frac{0.042}{12}$ $= 0.0035$ $n = 20 \times 12$ $= 240$ $A = M\left(\frac{(1+i)^n - 1}{i}\right)$ $750\ 000 = M \times 375.13 \ldots$ $M = 1999.281$ The monthly savings were $1999.29.	• correctly determines the i value **[1 mark]** • correctly recalls the perpetuity rule **[1 mark]** • determines purchase price of perpetuity **[1 mark]** • correctly determines the i and n values **[1 mark]** • correctly selects the appropriate annuity rule **[1 mark]** • determines payment **[1 mark]** • shows logical organisation, communicating key steps **[1 mark]**