Queensland Students' Understanding of Fractions: Evidence from the NAPLAN test results

Abstract

Fractions are a complex concept and have been recognised as an important foundation for the understanding of our number system. Fractions are also used extensively in everyday life — beyond the context of school — and are therefore an important component of numeracy. There is consensus in the literature that almost all students find this concept challenging and many struggle with it throughout their education. This concept is difficult not only for students to learn, but also for teachers to teach and has been identified by assessment programs as a consistent and recurring area of difficulty. This problem is universal and not a recent occurrence. Historical data from the Queensland Years 3, 5 and 7 Aspects of Literacy and Numeracy Tests (3, 5, 7 Tests) also shows similar evidence.

The National Assessment Program — Literacy and Numeracy (NAPLAN) tests provide data and the opportunity to analyse and compare student performance on specific mathematical concepts. Items that appear on the tests for two year levels — link items — allow us to ascertain the development of students' understanding of these concepts across these year levels. Data from the NAPLAN National Report reveals that Queensland students' performances in questions relating to fractions are below the national average. The analysis of selected link items that assess fractional concepts identifies key areas that require attention. Teachers can perform similar analyses and assist students to develop skills that strengthen their understanding and application of concepts.

Introduction

The NAPLAN tests, introduced in 2008, are designed to test the literacy and numeracy standards of students in Years 3, 5, 7 and 9. These tests are based on the *National Statements of Learning,* reflecting common aspects of curriculum taught in all states and territories.

Students' test performance informs teachers, school administrators, the community and parents, whether educational standards are being met. The tests provide a common measure of student learning and achievement across Australia. Outcomes from the tests are used by governments to assist with future directions for policy development, intervention, resource allocation and systemic practices. The class reports, provided to schools by the Queensland Studies Authority (QSA), are intended to assist educators to examine the cohort's performance, identify strengths and weaknesses, and focus on areas of the curriculum that require more attention. The QSA has developed a data analysis tool (SunLANDA) to perform this function. Teachers can review students' performances on particular questions or within specific strands and make comparisons between their class and school, state and national averages.

This study examines students' performances in the NAPLAN tests on three important concepts relating to fractions within the *Number* sub-strand: part–whole relations, equivalence and percentages. While several ideas have been put forward regarding students' and teachers' interactions with the concept of fractions, this paper investigates the link between sound basic conceptual understanding of fractions and how this facilitates the development of multiplicative and proportional reasoning. The data published in the National Assessment Program Summary Report reveals that students in Year 7 and Year 9 in Queensland performed less well than students in some other states (see Appendix 1). Also important is the rate of improvement between Years 7 and 9.

There are a number of questions in the NAPLAN test that assess students' fractional knowledge. However, the test items that are the focus of this paper are **link** items that relate to specific fractional concepts. Link items allow a comparison of students' performances on the same test item across two test years. This analysis of students' performances in selected link items in Years 7 and Year 9 presents teachers with an overview of students' understanding of, and difficulties with, fractional concepts. Because the data from each year is comparable, it is possible to make general inferences about students' performances over time. Teachers are also able to make similar comparisons through their own analysis.

The literature supports the analysis and contention of this paper that more attention to the development of basic understanding is needed to ensure the majority of students grasp these fundamental concepts. Research has also identified common trends and error patterns to support these conclusions.

From the data in the NAPLAN National Report, the issues that arise with the teaching of fractional concepts appear to be common and not restricted to Queensland. Students in other states also appear to have difficulty with the concept of fractions (see Appendix 2).

Results from the Queensland conducted 3, 5, 7 Tests reveal that some students had problems with these concepts prior to the NAPLAN tests. Performance on the 2008, 2011 and 2012 NAPLAN tests provides similar evidence and that students continue to find these concepts challenging (data from selected items is presented in Appendix 3).

Data

This paper uses data from the 2009 and 2010 NAPLAN tests. The sample was the whole-cohort of Queensland Year 7 and Year 9 students.

Table 1 shows the total number of students who sat the tests in each year in approximately 1700 schools (state, independent and Catholic) across Queensland. Students' backgrounds have not been considered in the analysis of their performances on the tests.

Table 1: Year level distribution of students completing tests in Queensland

Year level	Total number of students completing the tests				
	2008	2009	2010	2011	2012
Year 7	56043	56170	56236	56215	57179
Year 9	56719	56957	55428	55183	55032

Appendix 2 provides a comparison between Queensland and national data and achievement in the Numeracy test by state and territory for 2008–11. It is important to note that the national data is not indicative of the true spread of facility rates (the percentage of students who answered a question correctly) obtained across the states. The data has been deflated by the lower performances of some states (including Queensland). Therefore, care should be taken when drawing inferences regarding Queensland's performance against the national average. However, Queensland's performance appears to be below the national average and the trend shows no significant improvement in achievement.

Appendix 3 presents Queensland students' performance on selected items from tests conducted in 2002, 2004, 2007 (3, 5, 7 Tests), and 2008, 2011 and 2012 (NAPLAN).

Review of literature

Researchers have demonstrated that children engage with mathematical thinking from an early age (before school), although there are considerable differences in their levels of understanding (Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge & Gould 2005). Evidence of different levels of mathematical skills and conceptual diversity within the same year level has also been documented (Steffe & Olive 1991; Norton & Wilkins 2009 and Eriksson 2011). Often, these gaps increase as children move into higher year levels (Wright 2000 & Young-Loveridge 1991).

These gaps in knowledge are particularly evident in the area of fractions. This is because success in fractions depends on the mastery of the basic part–whole relationship which underpins the thinking and reasoning required for other fractional concepts. Because each part of mathematics builds on prior knowledge, students who have not developed competence in fundamental concepts are unable to keep up with students who have been successful in developing the required understanding. Many researchers have agreed that the concept of fractions is particularly complex (Behr, Wachsmuth, Post & Lesh 1984, Lamon 1999; Moss & Case 1999, Streefland 1991). Studies show that, initially, almost all students find the concept challenging (Behr, Harel, Post & Lesh 1993).

There are many reasons why students struggle with fractions. In the early years, it is due to the difficulty that students experience when making the transition from the concept of whole numbers to the concept of parts of numbers where they can no longer rely on their fingers to compute (Wu 2008) and because they have difficulty understanding the written notation used to represent fractions (Hiebert 1988 cited in Saxe, Taylor, McIntosh and Gearhart 2005). Although students are introduced to fractions in primary school, the use of this knowledge transcends the classroom. In everyday life there are many situations that require fractional understanding and representation (multiplicative and proportional thinking). In school, because of the conceptual continuities between the two areas, fractional knowledge forms the foundation for learning of algebra — a fundamental part of secondary mathematics (Wu 2008). Therefore, it is essential that students develop a reasonable level of competency in the area of fractions while in primary school.

The concept of part–whole relationships is a common and recurring hurdle in primary mathematics. It may be helpful for teachers to assess their students' knowledge of fractions before moving to the next step in fractional understanding. To check the level of students' abilities in part–whole relationships, teachers could use the unit fraction scheme which requires reasoning to solve both part-to-whole and whole-to-part problems (Nabors 2003). Poor comprehension of part–whole relationships can often hinder students' understanding of the concept of equivalence. Lesh, Post & Behr (1988) argue that understanding 'equivalence' or the ability to recognise structural similarity is a central component of proportional reasoning. Even after learning fractions for a few years, many students still struggle with the concept of equivalence (Behr, Waschmuth, Post & Lesh 1984; Streefland 1991; Kamii & Clark 1995). Brousseau, Brousseau & Wakerfield (2004) argue that the development of fractional concepts (i.e. part–whole relationship, equivalence, etc.) in isolation does not guarantee understanding of the other concepts. However, competence in these two basic concepts is a prerequisite to success in multiplicative thinking.

Multiplicative relationships underpin most number-related concepts, such as rate of change, ratios and percentages. Saxe *et al.* (2005) summarise the findings of longitudinal studies revealing that students' poor performances can be attributed to their use of additive rather than multiplicative reasoning. That is, they attempt to use their knowledge of whole numbers to interpret fractions. Difficulty in making the transition from additive to multiplicative thinking is common. The basic difference is that the additive approach focuses on the difference between quantities but in multiplicative thinking students need to understand the rate of change. Therefore, in these situations, additive thinking is often inadequate.

Because of the confusion between additive and multiplicative thinking, students in the middle years of schooling may find the concept of percentages bewildering and one of the most difficult to learn (Parker & Leinhardt 1995). Other studies have shown that students who are exposed to a variety of problems involving multiplicative thinking show higher levels of performance. This is because these students have developed different types of problem-solving skills and a certain degree of confidence in their ability to apply them as opposed to students who have not had such a range of experiences and therefore resort to simply applying rote-learned algorithms.

Miller & Fey (2000) and Saxe *et al*. (2005) found that students who learn through investigation and exploration are also less likely to display common errors and misconceptions and to perform better than students who are taught in the traditional instruction method. Another important concern raised about development of problem-solving skills is the inability of students to correctly interpret word problems, identify the relevant quantities and understand the relationship of these quantities to the question (Behr, Harel, Post & Lesh 1993; Wu 1999). Often, students attempt to solve the problem using only the numbers they see without reading the entire text (Bell, Fischbein & Greer 1984).

Success with fractions also requires conceptual and procedural knowledge. Highly achieving students are able to make connections between mathematical concepts and to check the accuracy of their responses (Clark, Berenson & Cavey 2003). For students to be able to perform across conceptual boundaries, a proficiency in the fundamentals is essential.

Teacher effectiveness has been identified as being central to students' success in mathematics (Stepek *et al.*1997, Wu 1999, Gearhart *et al*.1999). Teachers may be able to raise the level of proficiency of selected students by revisiting concepts that need further attention. Intervention in the early years has produced results in bridging the gap between poor and successful performers (Wright, Martland & Stafford 2000) and there is an undeniable link between student learning and the instruction they receive (Wu 2008). The approach selected should be informed by students' existing knowledge and their ability to integrate new information to develop conceptual understanding. The literature strongly advocates that teachers use a student-centred approach to teaching, especially when introducing new concepts. Students have different levels of understanding and although they can be at similar conceptual levels, their learning requirements may vary (Steffe 2004). To allow students to extend their conceptual understanding and develop higher level fractional thinking skills such as proportional reasoning, teachers could identify appropriate strategies for each student rather than employing a standard approach.

Research also reveals that on ongoing professional development plays a crucial role in developing teachers' pedagogical content knowledge. Wu makes a strong case that 'in mathematics, content guides pedagogy' (2002, p. 42). He argues that prospective teachers are not sufficiently equipped by universities to manage the challenges arising from classroom mathematics and recommends more emphasis on mathematics education for teachers. Content knowledge alone is insufficient for pedagogical competence but, once there is mastery of the relevant content, the focus can shift to pedagogy. Wu also makes an important point about teachers understanding the implications of the interconnections of concepts and the 'longitudinal coherence of the curriculum' (2002, p. 19). Strategies for improving mathematics teaching include a support network to provide time for teachers to engage in discussion of instructional effectiveness, assessment of student learning, and opportunities to improve the validity of their assessments (Stepek *et al.* 1997).

Gearhart *et al.* (1999) have summarised their findings on the effect of professional development programs on teachers' frameworks and classroom practices that are critical to support effective instruction. These are:

- gain a deep understanding of the concepts being taught
- gain an understanding of the way children learn mathematics
- support pedagogies that build on students' thinking
- engage in an analytic reflection of their classroom practices.

Many studies reveal that effective teaching requires teachers to understand the mathematical concepts, as well as the way students interpret problems and build knowledge (Stepek *et al.* 1997).

Some studies raise the concern that research is often focused on an adult's view of the concept rather than on children's construction of fractional knowledge (Olive & Lobato 2008). Moss & Case (1999) summarise the proposed explanations contributing to students' difficulties in learning rational numbers, suggesting that these difficulties are partly due to the emphasis of teaching on (i) syntactic over semantic knowledge (focus on the four operations rather than on conceptual meaning), (ii) adult- versus child-centred approach (resulting in the rote application of rules), (iii) representations of rational and whole numbers (students are confused), and (iv) problems with notations. Therefore, teachers face the challenge of providing support that is informed by individual student's conceptual understanding.

When used in conjunction with school-based assessment, the NAPLAN test data provides an excellent opportunity for teachers to reflect on their instruction informed by performance on particular test items. Nabors (2003) and Gearhart *et al*. (1999) and Moss & Case (1999) suggest that teachers identify the specific schemes of understanding with which each student operates, place them in contexts with which they are familiar and gradually build up their skills to perform more sophisticated operations. According to Nabors (2003), a strategy that has often proved successful in developing the skills of students who are unable to understand what is required of a current problem is to allow them to revisit familiar contexts, and work progressively towards more abstract contexts.

It is evident in the literature that mastery of fundamental concepts is necessary for the development of, and success in, higher-order skills, and that teachers play an important role in facilitating this development. Observations from many researchers suggest a stronger focus on developing competency in foundation concepts before students move to performing complicated operations in an area of mathematics that is widely regarded as complex.

Discussion of the data

In Queensland, the Years P–2 Numeracy indicators* show that students are introduced to fractions from Prep. According to the *Statements of Learning for Mathematics* (SOL)† in Year 5, students are able to work with simple common fractions, recognise equivalence and solve practical problems involving fractions. SOL excerpts are provided for Years 7 and 9 in Appendix 1.

Research has established that conceptual understanding, especially in the area of fractions, develops with experience and age. However, even in Year 7 and 9, the facility rates for problems involving fractional concepts are below the national average, indicating that some students still struggle with the basic concepts of part–whole relationship and equivalence. While there is improvement in performance from Year 7 to Year 9, the National Report data reveals that the required skills are not developing as expected when comparing Queensland's NAPLAN performance to most other states' (see Appendix 2). For some students this lack of conceptual understanding of the basic fundamental of part–whole relationships may have compounded their difficulties in developing higher-order thinking such as proportional reasoning.

It is important to remember that each student's ability to unpack word problems may affect their interpretation of a question. It is therefore possible that students who do have some knowledge of fractional concepts may not have been able to demonstrate this due to the multiple choice and open ended question format. Research indicates that providing opportunities such as posing non-routine questions and encouraging the use of multiple representations of problems can broaden students' thinking and often facilitates their abilities to apply critical thinking to interpret word questions and apply conceptual knowledge.

The NAPLAN Test Item Analysis in SunLANDA provides ideas, strategies and activities such as linear models, number lines, folding paper strips and area models to assist teachers in the classroom. (More strategies can be found by accessing the NAPLAN Test Item Analysis at http://www.qsa.qld.edu.au/8101.html).

Year 7 and Year 9 students struggle with the concept of parts of a whole

Confusion between parts and a whole is common among students. Data shows that even in Year 7 and Year 9, students' conceptual understanding of fractions and parts of a whole is incomplete, with fundamental systematic errors across both year levels (see Appendix 2).

Students appear to be unable to identify the whole in a problem, especially in problems that are contextualised, and so they attempt to solve it with fragmented understanding. As a result, some students choose a number from the question without recognising that they must identify the parts to calculate the whole. Although there are various reasons why students are unable to perform this first step, it is most likely due to one of the following:

- not reading the entire question
- misinterpreting the question
- still developing their understanding of a whole.

Because they could not correctly identify the whole, these students perform successive calculations that lead to the incorrect response.

---

* http://www.qsa.qld.edu.au/downloads/p_10/numeracy_indicators_p-2.pdf

† http://www.curriculum.edu.au/verve/_resources/SOL_Mathematics_2006.pdf

The gaps in students' conceptual understanding suggest that they have underdeveloped procedural knowledge, which hinders their application of mathematical content knowledge to check the reasonableness of their answers.

The following link item provides an example of how students' underdeveloped understanding of the part–whole concept affects their performance. In this item, Queensland students achieved below the national average in Year 7 (around 4.4%) and Year 9 (around 5.6%) (Table 5, Appendix 2).

Link item 1: Part–whole thinking

Students were required to interpret a word problem to recognise that they needed to express part of a collection (green apples) as a fraction of the whole collection (red plus green apples).

Figure 1: Link item 1 (2009)

Helen has 24 red apples and 12 green apples.

What fraction of the apples are green?

$\frac{1}{2}$ ○ $\frac{1}{3}$ ○ $\frac{1}{4}$ ○ $\frac{1}{12}$ ○

Table 2 compares response rates between Year 7 and Year 9 for *Link item 1*. The facility rate for this item was 47.5% in Year 7. The data shows an increase in the facility rate in Year 9 to 57.3%. Although this shows an improvement of almost 10%, the improvement in the national average was also similar to this rate, indicating that the gap has not closed between Queensland and the states where students have performed better.

Table 2: Comparison of facility rates (%) for *Link item 1*

Year level	Option A	Option B (answer)	Option C	Option D	Other
Year 7	27.3	47.5	9.9	14.7	2.2
Year 9	23.7	57.3	8.7	9.7	2.9

The first step to solve this problem was to identify the whole (24 + 12 = 36). The most common error was *Option A*. Students who selected this option as their response have not identified this step and have interpreted 24 — the larger of the two given numbers — as the "whole". Therefore they have calculated ${}^{12}/_{24} = {}^{1}/_{2}$ and incorrectly marked *Option A* as the answer.

Option D was the second most common incorrect response. In Year 7, 14.7% of students and 9.7% of students in Year 9 selected this option. According to the data, these are the lower performing students across the entire test. Their conceptual understanding of fractions is poor. They may have chosen this option because this is the only fraction that contains a number (12) that is given in the question.

With a response rate of 9.9% in Year 7 and 8.7% in Year 9, the students who have chosen *Option C* as their response demonstrate little knowledge of fractions. They have interpreted the question incorrectly and may have selected this option for a couple of reasons: (i) ¼ is a known unit fraction or (ii) the numbers in the question (12 and 24) are multiples of 4.

By Year 7, students should have developed the ability to read and interpret word problems and to eliminate obvious distractors such as *Option D,* recognising that it mirrors one of the numbers in the problem.

A sound understanding of the concept of parts of a whole is necessary to support the development of other fractional concepts. The research advises teachers to revisit concepts that students have not mastered to ensure that the understanding of the next level of conceptual understanding can be developed.

Data from the 3, 5, 7 Tests shows that students' difficulties with the simpler concept of identifying fractions from concrete examples such as diagrams have been evident for some time. For example, in 2004, in Year 3, fewer than 55% of students were able to answer a question relating to a graphical representation of the common fraction three-quarters (see Appendix 3, Figure 5, Table 10). In 2002, 85% of Year 5 students correctly identified $^{3}/_{5}$ from a graphical representation (see Appendix 3, Figure 4, Table 9). While this is significantly better, there were still a substantial number of students showing they did not understand the basic concept of a fraction being part of a whole when given a diagram divided into equal parts. In NAPLAN 2011, a similar Year 9 item had a facility rate of 91%. Although this may seem high, for an item testing such basic understanding at this year level, the 9% of students who are unable to answer this should be cause for concern (see Appendix 3, Figure 9, Table 14).

In an item from the 2007, 3, 5, 7 Tests, Year 7 students found it difficult to correctly identify a diagrammatic representation of a fraction of a whole. The item required students to decode the diagram, correctly partition the rectangle into **equal** parts and count the parts to determine the fraction (see Appendix 3, Figure 6, Table 11). This required a higher level of understanding than simply counting parts of equal size in an area model. A facility rate of approximately 30.8% shows that students found the fundamental concept of partitioning into equal parts challenging. The 2007, Year 7 cohort is the same group shown as Year 9 in Table 2.

Concept of equivalence underdeveloped in Year 7 and 9

Students whose understanding of parts of a whole is incomplete, or who are still developing their understanding, are more likely to find the concept of equivalence challenging. The 2009 test data indicates that approximately 44% of students in Year 7 and 36% of students in Year 9 had difficulty with the concept of equivalence. Based on the demands of the curriculum, the percentage decrease reflects expected chronological improvement rather than improvement achieved by a cohort of students who have developed a richer conceptual understanding.

It is evident that students have trouble calculating with unit fractions which have related denominators. They may find it difficult to interpret the word problem correctly and either use the fractions in the order they appear in the question or miss key steps — adding parts or subtracting parts from the whole — while attempting to solve the problem. Also, because there may be gaps in their knowledge, students are not able to apply logical reasoning to their conceptual understanding to check the reasonableness of their answers.

Students were required to integrate their knowledge of the concept of a whole with their understanding of equivalent fractions to solve the problem shown in Figure 2. Of the three link items analysed in this paper, it is in this item that Queensland students' performance was closest to the national average with a gap of around 1% in Year 7 increasing to 2.4% in Year 9. Error patterns for this item again show how students' lack of part–whole conceptual understanding can affect their performance.

Unpacking the question was the first step required to solve this multistep problem.

Figure 2: *Link item 2*: Equivalent fractions (2010)

A set of traffic lights is red for half the time, orange for $\frac{1}{10}$ of the time and green for the rest of the time.

For what fraction of the time is the set of traffic lights green?

$\frac{1}{3}$ $\frac{2}{5}$ $\frac{6}{10}$ $\frac{10}{12}$

The correct answer is $^{2}/_{5.}$ The students who selected *Option B* as their response have demonstrated sound conceptual understanding of fractions. They have correctly interpreted the question and identified the sequence of steps needed to solve it. They have applied the concept of a whole and, using the correct operation (subtraction), found the equivalent fraction with 10 as the common denominator to arrive at the response.

Time for orange light: $^{1}/_{10}$
Time for red light: $^{5}/_{10}$
Total red and orange time: $^{5}/_{10} + {}^{1}/_{10} = {}^{6}/_{10}$
Time for the green: $^{10}/_{10} - {}^{6}/_{10} = {}^{4}/_{10}$
Equivalent fraction: $^{4}/_{10} = {}^{2}/_{5}$

In Year 7, 56.2% of the students were able to solve this problem and the facility rate for Year 9 was 63.9%.

Option C — the most popular incorrect response — was selected by 25.5% and 19.4% of students in Year 7 and Year 9 respectively. Students who selected this option have recognised that half is equivalent to $^{5}/_{10}$ (the total time for red light) and have added $^{1}/_{10}$ (time for orange light) to this but have failed to subtract $^{6}/_{10}$ from 1 (the whole) to calculate the time for the green light.

Students who selected *Option A*, appear to have ignored the fractions in the question ($^{1}/_{2}$ and $^{1}/_{10}$) or did not know how to respond to the question. Three different colours are listed in the question, one of which is green (1 of 3). This may have led them to choose $^{1}/_{3}$ as the answer. In Year 7, 10.6% of students and 11.1% students in Year 9 may have selected this familiar fraction because it seems relevant to the question.

Students who selected *Option D*, have added the numerators and denominators of the given fractions — $^{1}/_{2}$ and $^{1}/_{10}$ — to get $^{2}/_{12}$ and subtracted this from 1 to arrive at their answer, demonstrating that they do not know how to calculate with fractions.

Therefore students in both year levels require more opportunities to extend their understanding of fractions and how to work with equivalent fractions.

Table 3 compares response rates for *Link item 2*. In Year 7, this item had a facility rate of 56.2%. The facility rate for Year 9 increased to 63.9%.

Table 3: Comparison of facility rates (%) for *Link item 2*

Year level	Option A	Option B (answer)	Option C	Option D	Other
Year 7	10.6	56.2	25.5	6.9	1.6
Year 9	11.1	63.9	19.4	4.1	1.6

A Year 7–9 link item from NAPLAN 2011, which required students to match equivalent representations of a common fraction (graphical and symbolic), had facility rates of 17.2% and 24.7% respectively (see Appendix 3, Figure 8, Table 13). These results indicate that students lack a sound understanding of equivalence which is necessary to enable them to solve practical problems.

Proportional reasoning challenges students' understanding

The data shows that students have difficulty solving problems involving percentages. About 59.1% of Year 7 students and 50.5% of Year 9 students could not demonstrate their knowledge of the concept of percentages in a multistep problem with friendly numbers and a common fraction.

Many students, even in Years 7 and 9, have not fully understood the concepts of part–whole and equivalence. Students who have not fully developed their understanding of these concepts may struggle to comprehend problems involving multiplicative reasoning and to interpret equivalent representations of fractions. Therefore, even if these students have the computational abilities, their responses are incorrect because they have not calculated with the correct quantities.

There are a number of reasons why students may have been unable to solve this problem. These include:

- misinterpretation of the question
- not understanding the concept being tested
- an inability to identify the parts of the whole
- an inability to understand the relationships between the values in the problem
- having inadequate skills to complete higher-level multistep problems.

Research suggests that students find the concept of percentages challenging and, because it is built on prior knowledge, teachers should provide a variety of problems using familiar contexts to activate this knowledge. Such contextualised problems have often proven successful in further developing students' skills and understanding.

In Queensland, the performance of the Year 7 students on *Link item 3* was below the national average by 3 %, increasing to 4.4% in Year 9.

Table 4 compares response rates for Years 7 and 9 for *Link item 3*. It is possible that the Year 9 students have performed better (an improvement of 9.2% in Queensland and 10.4% in the national average) because, according to research, conceptual understanding is a skill that can develop with age.

Table 4: Comparison of facility rates (%) for *Link item 3*

Year level	Option A (answer)	Option B	Option C	Option D	Other
Year 7	41.5	36.0	17.2	4.5	2.3
Year 9	50.7	32.0	13.2	3.5	3.0

Figure 3: *Link item 3* (2009)

A copier prints 1200 leaflets.
One-third of the leaflets are on yellow paper and the rest are on blue paper.
There are smudges on 5% of the blue leaflets.

How many blue leaflets have smudges?

40	60	400	800

To solve this problem, students were required to interpret the question and complete the correct sequence of steps. The first step was to recognise that 1200 was the whole and that they had to find the number of blue leaflets out of 1200. The question stated that one-third of the leaflets were yellow, so the next step was to find this value. By computing that $^{1}/_{3} \times 1200 = 400$ (yellow leaflets), they could then calculate the number of blue leaflets. This could be worked out in different ways: $1200 – 400 = 800$; or $^{2}/_{3} \times 1200 = 800$. The final step in solving this problem was to find 5% of the blue leaflets, $800 \times {}^{5}/_{100} = 40$.

In Year 7, 41.5% of students were able to solve this problem. The increase in the Year 9 facility rate to 50.5% may indicate that the conceptual understanding of some students is improving with age, as the literature suggests. However, the rate itself is a cause for concern. Almost 50% of Year 9 students could not solve this problem. The solution may be to revisit some meaningful contexts and then encourage students to apply those strategies to solve abstract problems. Once conceptual understanding is developed, students should be able to make the transition from concrete to abstract contexts.

The reason that 36% of students in Year 7 and 32% of students in Year 9 selected *Option B* as their response could be because they have merely skimmed the question instead of reading it carefully. They have worked with the given numbers (1200 and 5) and have missed the fraction written in words (one-third). They have simply computed 5% of 1200 and therefore selected 60 as their response.

The students who have chosen *Option C* as their response have completed the first two steps by identifying that 1200 is the whole and finding one-third of this total (400). They have not done any further computation to calculate the number of blue leaflets or find 5% of any number. This option was selected by 17.2% of Year 7 students and 13.2% of Year 9 students. These students have demonstrated some understanding of fractions, but only a limited ability to unpack the question.

It could be argued that the low response rates for *Option D* at 4.5% and 3.5% in Year 7 and 9 respectively are the result of these students completing the initial steps of the sequence (identifying 1200 as the whole, calculating $^{1}/_{3}$ of 1200, computing that the number of blue leaflets is twice the number of the yellow leaflets — $400 \times 2 = 800$ — but failing to complete the last step; that is, finding 5% of 800). Alternatively, they may have selected this option for no logical reason.

Interpreting the word problem correctly is the first step in solving this problem. The data demonstrates that a significant number of students may have been unable to unpack the question to identify what was required of them. Teachers should focus on strengthening students' ability to unpack contextualised problems to improve their performance by teaching them specific problem-solving strategies.

Data from other items also suggests that students do not have a good understanding of proportional reasoning. In NAPLAN 2008, an item assessing a simple estimate of a common fraction from a given proportion involving whole numbers showed a facility rate of 54% (see Appendix 3, Figure 7, Table 12). Similar test items in NAPLAN 2012 reported facility rates of approximately 32% in Year 7 and 37% in Year 9 suggesting that there has been no significant improvement in the understanding of this concept (see Appendix 3, Figure10, Table 15 and Figure 11, Table 16).

Learning and teaching implications

The learning and teaching of fractions has been acknowledged as complex. The data reveals gaps in students' fractional knowledge and that a large number of students in Year 7 and 9 still have difficulty with fundamental concepts (e.g. part–whole relationships) that should have been mastered in earlier years.

It is essential for teachers to be aware of the different levels of conceptual understanding in their classrooms and to provide opportunities for all students. This could be achieved by looking at ways of differentiating the learning to match the conceptual understanding of individual students. It is clear from the data that continuing to teach students fractional ideas such as percentages or proportional reasoning when they are still struggling with part–whole understandings is not helpful. Unless the basic understanding is fully grasped, students will continue to rely on misguided or even incorrect methods to solve problems and likely become further confused.

One way teachers can determine the understanding their students have around fractions is to conduct some diagnostic testing and gather formative assessment information. Teachers could use some of the test items from NAPLAN to gauge students' level of understanding. Having whole cohort data to compare performances will give teachers a sense of difficulty for the items. Alternatively there are many diagnostic tests available to determine a baseline understanding.

Another way to address the teaching of fractional understanding is to review the current strategies and pedagogy used and how that may be interpreted or misinterpreted by students. For example, if using multi attribute base materials (MAB blocks) in the delivery of whole number understanding, then consider how the student may be confused if the same material is used to demonstrate fractional understanding. For more advanced students, who may not need as many concrete representations, this leap from whole number representation to fractions will be easily understood, however for the very students who need more concrete representation it may be bewildering. The use of paper strips and fraction walls could be a better choice for these students.

Teaching percentage is problematic and often it is taught in isolation from fractions. Although adults may be able to link the representation of the same number as a percentage, decimal, fraction or even ratio, students need this explicitly demonstrated. The demystifying of the representation and language is important. Mathematics is a language and uses symbols and words to represent itself. Students need to be taught this language so they can represent their thinking. Knowing that the same fraction can be represented in a variety of ways is often a pivotal moment for a student. It helps them to build a fractional schema and link the learning that they have together. When faced with a problem involving percentages, they may use an equivalent representation to help them with the calculation. But they can only do this if they have the knowledge and experience to do so.

Conclusion

It is clear that managing the teaching and learning of fractions can be difficult for teachers. Gaps in learning may be overlooked, leading to students being introduced to the more difficult concepts before they understand earlier ones. This can hinder their understanding and limit their ability to solve problems relating to these concepts. Therefore, to ensure that students gain a robust understanding of fraction concepts, teachers need to develop logical schemas to ensure that students are able to connect new concepts to their existing knowledge and understanding. According to the literature, revisiting fundamental concepts to develop understanding has often proved successful. It also strongly recommends the adoption of a student-centred approach to teaching and learning. As the role of teachers is critical to the success of their students, it is important that at each year level, they understand the continuing relevance of foundation mathematical skills. Teachers' pedagogical competence should be supported and strengthened through ongoing professional development and mathematics education.

Appendix 1

Statements of Learning for Mathematics and Professional Elaborations — Opportunities to Learn for Mathematics	
Year 7	Students extend their use of mathematical inquiry and employ a range of investigative, modelling and problem solving strategies and processes…They develop models, investigate and test propositions, hypotheses and conjectures, and identify key assumptions and conditions that apply to working mathematically in different contexts. • Students form estimates for calculations involving whole numbers, decimal fractions and common fractions using their knowledge of number systems • Students identify and represent integers and decimal fractions, and compare and order them using a variety of methods and models. They calculate with the four operations... • Students represent and describe common fractions, including simplest form, and find their equivalent representations as decimals and percentages • Students apply a range of strategies and approaches to calculate simple proportion, percentages and simple rates in practical situations
Year 9	Students apply a broad range of mathematical and logical skills, processes and strategies as they make deductions, and verify and generalise their reasoning. They identify and describe key features of a context or situation…They compare different models for a given context, make predictions, solve problems and reflect on solution methods, carry out mathematical investigations… • Students work with fractions, decimal numbers and percentages. They are familiar with different representations of numbers… • They apply the relevant operations, with attention to the meaning and order of the operations involved, in practical and theoretical situations • Students are familiar with rational numbers in different forms and use these to formulate and solve ratio, proportion, percentage and rate problems, using mental, written and technology-assisted methods • They carry out investigations, develop, compare and refine models, and solve problems in familiar and unfamiliar contexts • Students use mental, written and technology-assisted methods to carry out computations and solve practical problems with attention to the type of numbers and operations involved, and order of operation • Students solve ratio, proportion, percentage and rate problems using mental, written and technology-based approaches

Appendix 2

Comparison of Queensland facility rates (%) with national average

Table 5: Part–whole relationship — *Link item 1* (2009)

Year level, Question number	Performance	Options selected				
		A	B (answer)	C	D	Other/no answer
7, NCQ14	Queensland	27.3	47.5	9.9	14.7	2.2
7, NCQ14	National	25.94	51.22	8.86	13.10	0.87
9, NCQ14	Queensland	23.7	57.3	8.7	9.7	2.9
9, NCQ14	National	21.73	61.64	7.79	8.13	0.71

Graph 1: Comparison of selected options for *Link item 1* between national and Queensland data for Year 7 and Year 9, testing the concept of part–whole relationship

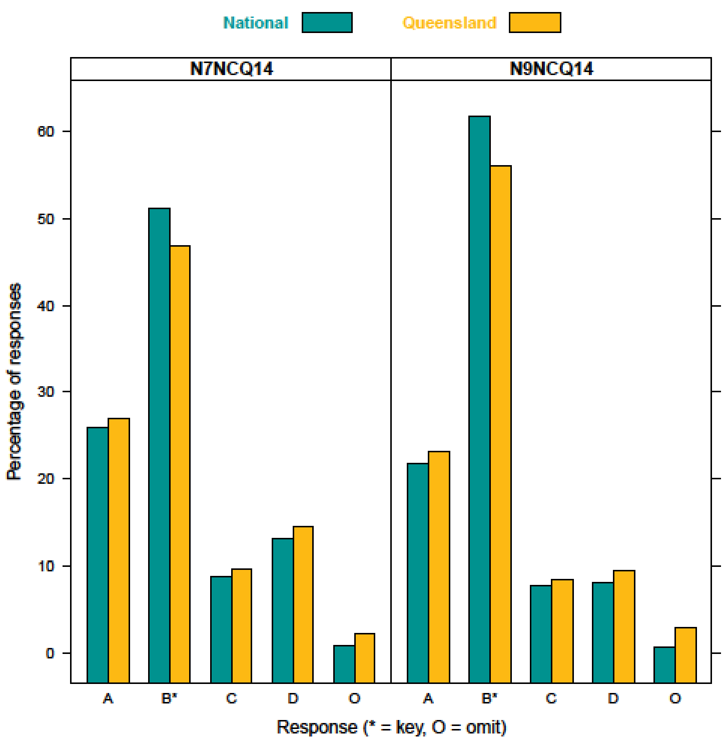


Table 6: Equivalent fractions — *Link item 2* (2010)

Year level, Question number	Performance	Options selected (%)				
		A	B (answer)	C	D	Other/no answer
7, NCQ13	Queensland	10.6	56.2	25.5	6.9	1.6
7, NCQ13	National	10.26	56.85	24.80	6.59	1.50
9, NCQ9	Queensland	11.1	63.9	19.4	4.1	1.6
9, NCQ9	National	10.07	66.30	18.74	4.00	0.89

Graph 2: Comparison of selected options for *Link item 2* between national and Queensland data for Year 7 and Year 9, testing the concept of equivalence

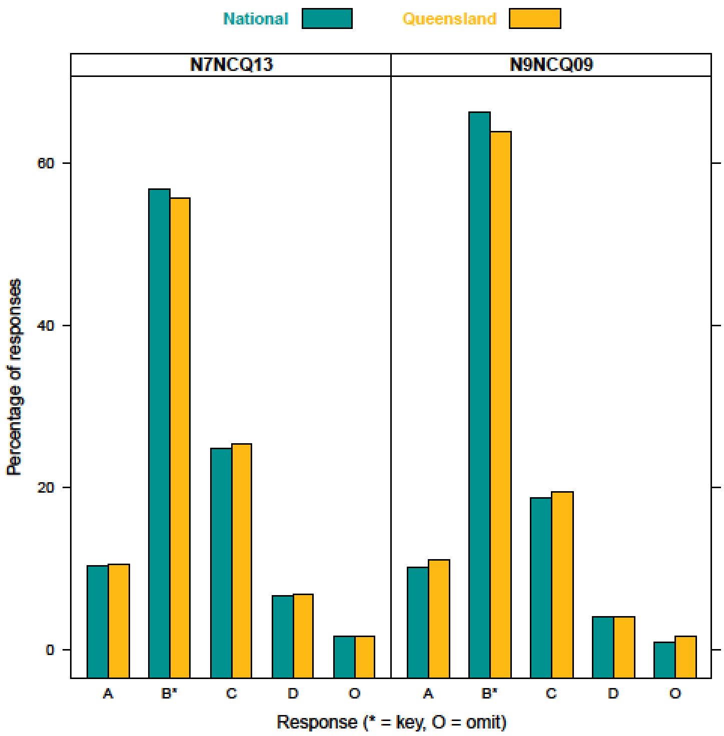
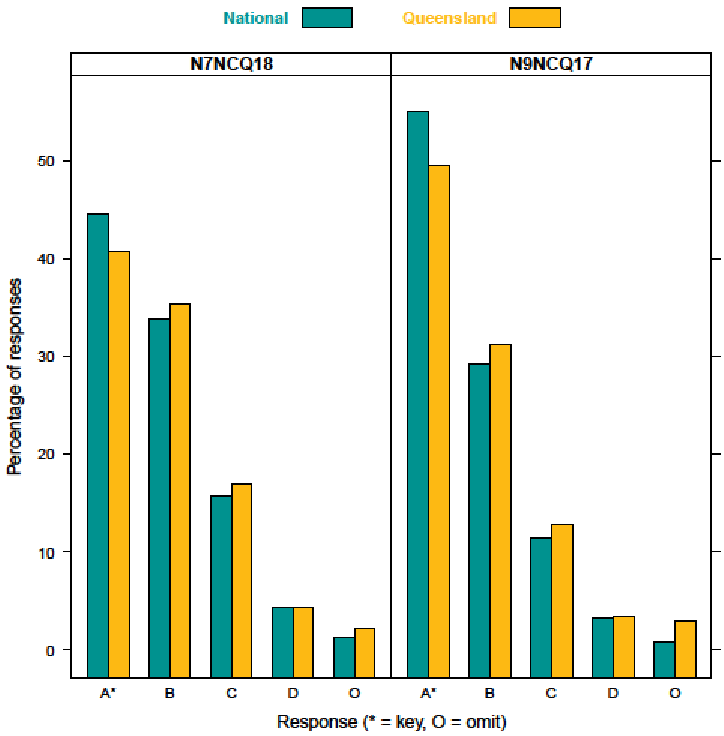


Table 7: Proportional reasoning — *Link item 3* (2009)

Year level, Question number	Performance	Options selected (%)				
		A (answer)	B	C	D	Other/no answer
7, NCQ18	Queensland	41.5	36.0	17.2	4.5	2.3
7, NCQ18	National	44.62	33.96	15.77	4.34	1.30
9, NCQ17	Queensland	50.7	32.0	13.2	3.5	3.0
9, NCQ17	National	55.05	29.29	11.42	3.34	0.89

Graph 3: Comparison of selected options for *Link item 3* between national and Queensland data for Year 7 and Year 9, testing the concept of proportional reasoning



Appendix 3

Table 8: Year level distribution of students completing the 3, 5, 7 Tests

Year level	Total number of students completing Numeracy tests					
	2002	2003	2004	2005	2006	2007
Year 3	51165	51641	51434	51174	51995	52410
Year 5	51982	52077	53050	53099	52987	53043
Year 7	51553	52026	53659	53463	54505	55000

Figure 4: 2002 the 3, 5, 7 Tests — Year 5, Question 25*

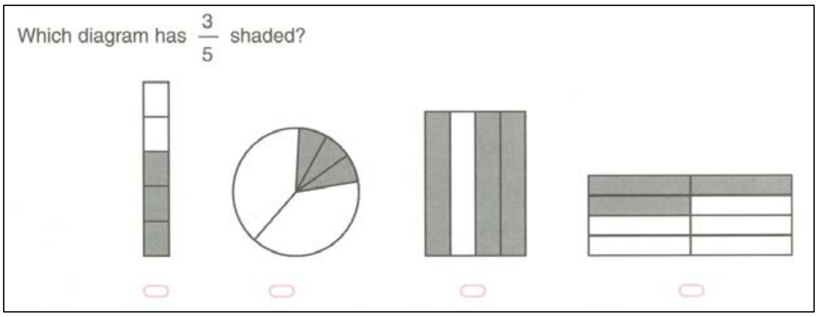


Table 9: Option chosen (%) — 2002, Year 5, Question 25

Performance	Option A (answer)	Option B	Option C	Option D	Other/no answer
Queensland	85.79	4.04	0.96	3.85	5.36

* No National data for 3,5,7 Tests prior to NAPLAN tests.

Figure 5: 2004, 3, 5, 7 Tests — Year 3, Question 28*

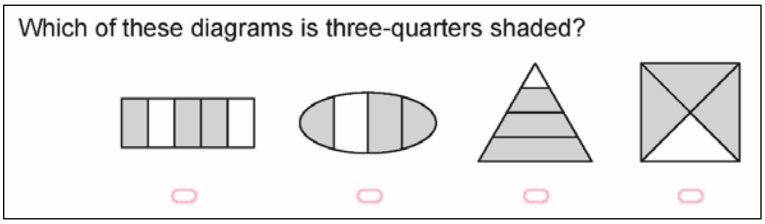


Table 10: Option chosen (%) — 2004, Year 3, Question 28

Performance	Option A	Option B	Option C	Option D (answer)	Other/no answer
Queensland	10.06	5.96	18.22	54.48	11.27

* No National data for 3, 5, 7 Tests prior to NAPLAN tests.

Figure 6: 2007, 3, 5, 7 Tests — Year 7, Question 41*

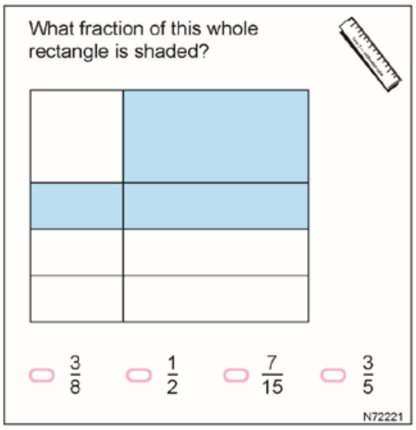


Table 11: Option chosen (%) — 2007, Year 7, Question 41

Performance	Option A	Option B	Option C (answer)	Option D	Other/no answer
Queensland	50.93	5.48	30.72	11.98	0.9

* No National data for 3, 5, 7 Tests prior to NAPLAN tests

Figure 7: NAPLAN 2008 — Year 7, Numeracy Calculator, Question 15

A school has 150 students.
80 of the students each have a book on loan from the library.

The fraction of students who have a book on loan from the library is closest to

one-fifteenth. one-eighth. one-quarter. one-half.

Table 12: Option chosen (%) — 2008, Year 7, Question 15

Performance	Option A	Option B	Option C	Option D (answer)	Other/no answer
Queensland	6.58	25.24	12.84	54.35	0.99
National	Not available				

Figure 8: NAPLAN 2011 — Link item — Year 7 Question 25, Year 9, Question 24

Tam cuts letters from squares of metal.

Which of these letters uses exactly $\frac{5}{6}$ of the metal square?

Table 13: Comparison of facility rates (%) — 2011, Year 7 Question 25, Year 9 Question 24

Year level, Performance	Option A (answer)	Option B	Option C	Option D	Other/no answer
Year 7, Queensland	17.15	15.5	32.6	33.93	1.11
Year 7, National	20.35	14.63	28.35	35.2	1.47
Year 9, Queensland	24.74	14.62	26.9	33.17	0.69
Year 9, National	30.13	13.38	23.11	32.4	0.97

Figure 9: NAPLAN 2011 — Year 9, Numeracy Calculator, Question 2

The picture shows a set of lights. Two of the lights are off.

What fraction of the set of lights is off?

$\frac{1}{2}$ $\frac{1}{7}$ $\frac{2}{7}$ $\frac{2}{9}$

Table 14: Option chosen (%) — 2011, Year 9, Numeracy Calculator, Question 2

Performance	Option A	Option B	Option C	Option D (answer)	Other/no answer
Queensland	0.24	0.55	7.66	91.57	0.1
National	0.29	0.52	6.21	92.85	0.13

Figure 10: NAPLAN 2012 — Year 7, Numeracy Non-calculator, Question 23

There are 600 vehicles in a car park.
One-third of them are trucks. The rest are cars. One-quarter of the cars are white.

How many white cars are in the car park?

Table 15: Responses (%) — 2012, Year 7, Numeracy non-calculator, Question 23

Performance	Answer	4 Most common incorrect responses			
	100	**150**	**50**	**200**	**250**
Queensland	31.7	17.0	6.0	6.0	3.0
National	37.05	Incorrect/no answer = 62.96			

Figure 11: NAPLAN 2012 — Year 9, Numeracy Calculator, Question 17

Sarah is painting a wall using three colours.

She paints $\frac{1}{3}$ of the wall blue, $\frac{1}{4}$ of the wall green and the rest yellow.

What fraction of the wall does she paint yellow?

$\frac{10}{12}$ $\frac{5}{12}$ $\frac{5}{7}$ $\frac{7}{12}$ $\frac{2}{7}$

Table 16: Option chosen (%) — 2012, Year 9, Numeracy Calculator, Question 17

Performance	Option A	Option B (answer)	Option C	Option D	Option E	Other/no answer
Queensland	4.81	37.29	22.42	14.42	19.99	1.11
National	4.31	44.53	18.18	15.24	16.48	1.27

References

Behr, MJ, Waschmuth, I, Post, TR & Lesh, R 1984, 'Order and equivalence of rational numbers: A clinical teaching experiment', *Journal for Research in Mathematics Education*, vol. 15, pp. 323–341.

Behr, M, Harel, G, Post, T, & Lesh, R 1993, 'Rational numbers: Toward a semantic analysis – Emphasis on the operator construct' in Carpenter, T, Fennema, E & Romberg, T (eds), *Rational Numbers: An Integration of Research*, Lawrence Erlbaum Associates , Hillsdale, NJ, pp. 13–47.

Bobis, J, Clarke, B, Clarke, D, Thomas, G, Wright, R, Young-Loveridge, J & Gould, P 2005, 'Supporting teachers in the development of young children's mathematical thinking: Three large scale cases', *Mathematics Education Research Journal*, vol. 16 no. 3, pp. 27–57.

Brousseau, G, Brousseau, N & Warfield, V 2004, 'Rationals and decimals as required in the school curriculum. Part 1: Rationals as measurements', *Journal of Mathematical Behavior* vol. 23, pp. 1–20.

Clark, MR, Berenson, SB & Cavey, LO 2003, 'A comparison of ratios and fractions and their roles as tools in proportional reasoning', *Journal of Mathematical Behaviour*, vol. 22, pp. 297–317.

Eriksson, G 2011, 'Toward a student-centred process of teaching arithmetic', *The Journal of Mathematical Behavior*, vol. 30, pp. 62–79.

Gearhart, M, Geoffrey, B, Seltzer, M & Schlackman, J 1999, 'Opportunities to learn fractions in elementary mathematics classrooms', *Journal for Research in Mathematics Education*, vol. 30, no. 3, pp. 286–316.

Kamii, C & Clark, F 1995, 'Equivalent Fractions: Their Difficulty and Educational Implications', *Journal of Mathematical Behaviour*, vol. 14, pp. 365–378.

Lamon, S 1999, *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*, Lawrence Erlbaum, Mahwah, NJ.

Lesh, R, Post, T & Behr, M 1988, 'Proportional Reasoning' in Hiebert, J & Behr, M (eds.) *Number Concepts and Operations in the Middle Grades,* Lawrence Erlbaum & National Council of Teachers of Mathematics, Reston, VA, pp. 93–118.

Miller, JL & Fey, JT 2000, 'Proportional reasoning', *Mathematics Teaching in the Middle School*, vol. 5, no. 5, pp. 310–313.

Mitchell, A & Horne, M 2008, 'Fractional number line tasks and the additivity concept of length measurement', in Goos, M, Brown, R & Makar, K (eds) *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australia*.

Moss, J & Case, R 1999, 'Developing children's understanding of the rational numbers: A new model and an experimental curriculum', *Journal for Research in Mathematics Education*, vol. 30, no. 2, pp. 122–147.

Nabors, W 2003, 'From fractions to proportional reasoning: a cognitive schemes of operation approach', *Journal of Mathematical Behaviour*, vol. 22, pp. 133–179.

Norton, A & Wilkins, JLM 2009, 'A quantitative analysis of children's splitting operations and fractions schemes', *The Journal of Mathematical Behavior*, vol. 28, nos. 2–3, pp. 150–161.

Olive, J & Lobato J 2008, 'The learning of rational number concepts using technology', in Heid, K and Blume, G (eds), *Research on technology in the teaching and learning of mathematics: syntheses and perspectives*, Information Age Publishing, Greenwich.

Parker, M & Leinhardt, G 1995, ‘A Privileged Proportion’, *Review of Educational Research*, vol. 65, no. 4, pp. 421–481.

Saxe, GB, Taylor, EV, McIntosh, C & Gearhart, M 2005, ‘Representing Fractions with Standard Notation: A Developmental Analyses’, *Journal for Research in Mathematics Education*, vol. 36, no. 2, pp. 137–157.

Streefland, L 1991, *Fractions in realistic mathematics education: A paradigm of developmental research,* Kluwer Academic Publishers, The Netherlands http://books.google.com.au/books?hl=en&lr=&id=Y5Skj-EA2_AC&oi=fnd&pg=PA1&dq=Streefland+-+Fractions+-+a+realistic+approach&ots=aacP5ajrsu&sig=Yn4thv2CtH3PoBX-t8LBY-jfc9k#v=onepage&q=Streefland%20-%20Fractions%20-%20a%20realistic%20approach&f=false

Steffe, LP 2004, ‘On the construction of learning trajectories of children: The case of commensurate fractions’, *Mathematical Thinking and Learning*, vol. 6, no. 2, pp. 129–162.

Steffe, LP & Olive, J 1991, ‘The Problem of Fractions in the Elementary School’, *The Arithmetic Teacher*, vol. 38, no. 9, pp. 22–24.

Stepek, D, Gearhart, M, Denham, W 1997, ‘Mathematics instruction: What works?’, *Thrust for Educational Leadership*, vol. 26, no. 6, pp. 22–25.

Wright, RJ 2000, ‘Professional development in recovery education’ in Steffe, LP & Thompson, PW (eds), *Radical constructivism in action. Building on the pioneering work of Ernst von Glasserfeld*, Routledge, London, pp. 134–151, http://books.google.com.au/books?hl=en&lr=&id=iM18PfvWr54C&oi=fnd&pg=PA134&dq=professional+development+in+recovery+education&ots=p0cZpARKul&sig=eXzd3YvpOfNK7AsuAIHQVyke4to#v=onepage&q=professional%20development%20in%20recovery%20education&f=false.

Wright, RJ, Martland, J & Stafford, A 2000, *Early numeracy in Assessment for teaching and intervention*, Paul Chapman Publications/Sage, London.

Wu, H 1999, *Basic skills versus conceptual understanding: A bogus dichotomy in Mathematics Education*, accessed 1 March 2012, http://enumeracy.com/files/basic_skills_versus_concepts.pdf.

Wu, H 2002, *What is so difficult about the preparation of Mathematics teachers?*, accessed 1 March 2012, http://www.mathnet.or.kr/mathnet/paper_file/UCB/Wu/pspd3d.pdf.

Wu, H 2008, *Fractions, decimals, and rational numbers*, accessed 1 March 2012, http://math.berkeley.edu/~wu/NMPfractions4.pdf.

Young-Loveridge, J 1991, ‘The development of children’s number concepts from ages five to nine. Early mathematics learning project: Phase II, Vol. 1’, *Report of Findings*, University of Waikato, New Zealand.