



Is it fair?

Sample responses



6

Mathematics

Queensland Comparable
Assessment Tasks
(QCATs) 2011

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A Sample: Response 1

Guide to making judgments — Year 6 Mathematics

Name

Focus: Apply knowledge of chance events, variation and bias to develop arguments and improve game fairness.

Knowledge and understanding	Thinking and reasoning Reflecting	Communicating
<p>Uses possible outcomes and frequency to estimate likelihood as a common fraction.</p> <p>Compares and orders estimates of likelihood.</p> <p>Identifies differences between graphs.</p> <p>Questions 1–5</p>	<p>Reflects on learning and applies new understandings to:</p> <ul style="list-style-type: none"> develop mathematical arguments plan, evaluate and explain changes to game rules. <p>Questions 6–9</p>	<p>Communicates using mathematical language and representations to justify thinking and reasoning.</p> <p>Questions 1–3, 6, 8, 9</p>
<p>Explains differences between graphs using a comprehensive understanding of the factors influencing chance events and variation.</p> <p>Correctly compares and orders estimates of likelihood, and identifies outcomes with equal likelihoods. Considers chance events when explaining differences between graphs.</p> <p>Correctly expresses likelihoods as common fractions.</p> <p>Translates information from graphs into tables. Identifies obvious mathematical differences between graphs. Identifies a most and least likely outcome.</p> <p>Identifies numbers of outcomes from grid.</p>	<p>Justifies arguments using a comprehensive mathematical interpretation of the quantitative data and factors influencing chance events and bias.</p> <p>Makes changes to rules which make the game fair. Accurately judges Player B's chances of winning, and justifies using an understanding of chance events.</p> <p>Makes changes to rules which improve the fairness of the game. Considers chance events in arguments.</p> <p>Makes a relevant statement about why the game is unfair or why the rule changes are fair. Identifies likelihood of Player B winning.</p> <p>Makes a change to the rules.</p>	<p>Communicates and justifies thinking and reasoning using clear mathematical language.</p> <p>Communicates using appropriate mathematical language.</p> <p>Records possible outcomes and frequency in graphs.</p>
Demonstrates all descriptors for this assessable element.	Demonstrates all descriptors for this assessable element. Note that in Q6 there is an inaccuracy in the qualitative description "almost always certain" to describe $\frac{2}{3}$.	Demonstrates all descriptors for this assessable element.

A

B

C

D

E

Overall grade

This response demonstrates a very high level of achievement across all assessable elements. It is judged to be an A.

Collecting and analysing experimental data

Activity

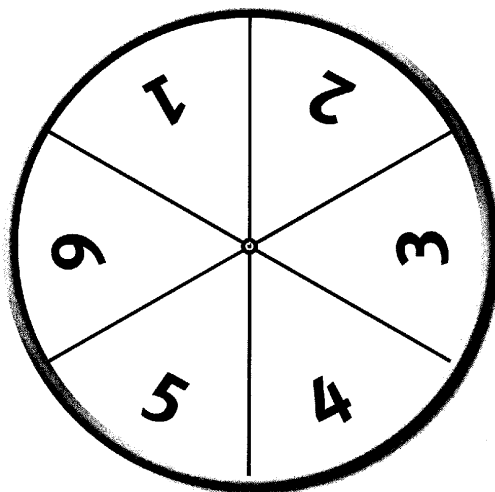
Play "Take a spin" in your pairs again. Complete Question 1 as you play.



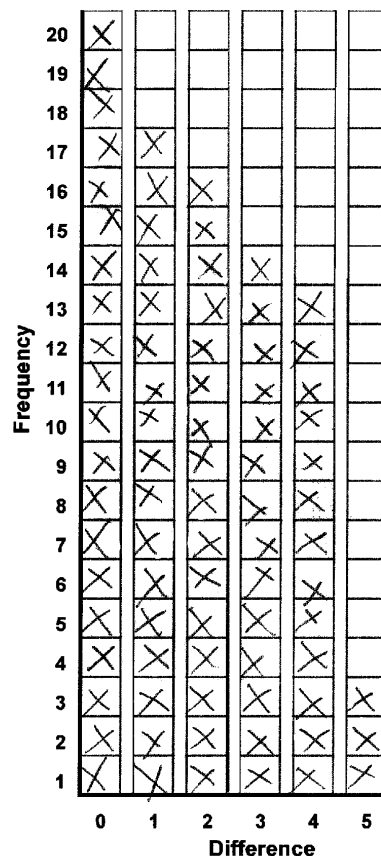
1. Complete Graph 1 by using a cross (**X**) to record the difference for each spin while you play the game.

Stop recording if one of the differences reaches the top of the graph.
Finish the game you are playing if this happens.

Take a spin 



Graph 1: Frequency of each difference



Game	Player A (0, 1 or 2)	Player B (3, 4 or 5)
1		
2		
3		
4		
5		
6		

A Sample: Response 1

Use Graph 1 on page 4 to answer Question 2.

2. a) Complete the Frequency column in Table 1 below.
- b) In the Likelihood column, express the likelihood of spinning each difference as a common fraction.
- c) Order the differences from most likely to least likely in Diagram 1.

Table 1: Likelihood of spinning each difference

Difference	Frequency	Likelihood
0	20	$\frac{20}{83}$
1	17	$\frac{17}{83}$
2	16	$\frac{16}{83}$
3	14	$\frac{14}{83}$
4	13	$\frac{13}{83}$
5	3	$\frac{3}{83}$
Total	83	

Diagram 1: Order of likelihood

Difference	Likelihood
0	most likely
1	
2	
3	
4	
5	least likely



Stop here: Wait for your teacher's directions.

Exploring outcomes and theoretical likelihood

This grid shows all **possible outcomes** when using two spinners and finding the difference. Use the grid to answer the questions below.

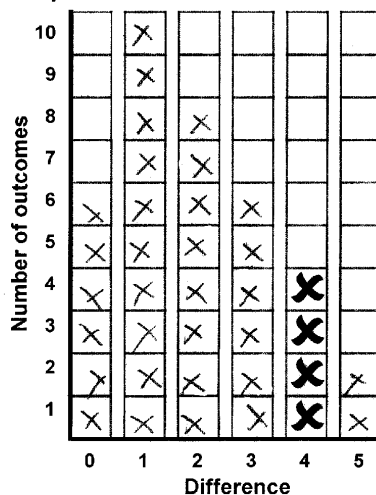
		Player B					
		1	2	3	4	5	6
Player A	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

The difference of 4
has 4 possible outcomes.

These can be written as:
(1, 5) (2, 6) (5, 1) (6, 2)

3. a) How many possible outcomes have a difference of 2? 8
- b) Write all the possible outcomes that have a difference of 3.
(1, 4) (2, 5) (3, 6) (4, 1) (5, 2) (6, 3)
- c) Complete Graph 2 by using a cross (X) to indicate the number of outcomes for each difference.

Graph 2: Number of outcomes for each difference



The possible outcomes for
the difference of 4 have
been completed for you.

A Sample: Response 1

Look at the shape of Graph 1 on page 4 and Graph 2 on page 6, then complete the sentence below.

4. The two graphs may not be the same shape because: 'graph one' relies on chance and every player might have different luck of what actually comes up but in 'graph two' it is only theoretical of what could come up. So these two graphs might be different shapes. It'd take several hundred games to get the 'graph two' shape.

Use Graph 2 to answer the following.

5. a) Complete Table 2 below.
b) Order the differences from most likely to least likely in Diagram 2.

Table 2: Likelihood of spinning each difference

Difference	Number of outcomes	Likelihood
0	6	$\frac{6}{36}$
1	10	$\frac{10}{36}$
2	8	$\frac{8}{36}$
3	6	$\frac{6}{36}$
4	4	$\frac{4}{36}$
5	2	$\frac{2}{36}$
Total	36	

Diagram 2: Order of likelihood

Difference	Likelihood
1	most likely
2	
3, 0	
4	
5	
	least likely



Stop here: Wait for your teacher's directions.

It's not fair!

During the group discussion (page 3), you talked about the question:

Is the game "Take a spin" fair or unfair?

6. Explain why the game is not fair.

Use the information you have collected to support your explanation.

The game 'Take a spin' is not a fair game because player 'a' is almost always certain to win. Player 'a' has a $\frac{34}{36}$ chance of winning the points, while player 'b' has $\frac{17}{36}$ chance. This means the game 'Take a spin' is unfair.

Applying your learning

In a fair game, each player has an equal chance of winning.

7. **Make the game "Take a spin" fair.**
Complete the rules below.

Take a spin

Rules for play

Getting ready

- Both players spin.
- The player who spins the lowest number will be Player A.

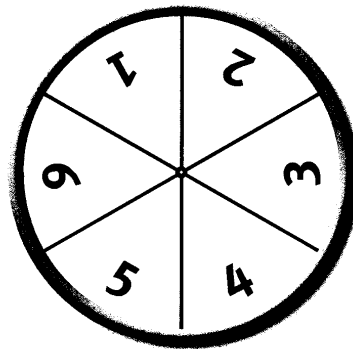
Playing the game

- Both players spin.
- When each spinner stops, find the difference between the numbers.

- If the **difference** is *an even number (0, 2, 4)*.....
then Player A scores *a point (one 1 point)*.....
- If the **difference** is *an odd number (1, 3, 5)*.....
then Player B scores *a point (one 1 point)*.....
- Use a tally mark to record each point.

Winning the game

The first player to ten (10) points wins.....
.....



8. **Explain how your changes to the game make it fair.**

When added, the chances to have a difference of 0, 2, or 4 is 18. When the chances of having a difference of 1, 3, or 5 they also equal 18 chances. So there's an equal amount of differences.

.....

.....

A Sample: Response 1

9. If "Take a spin" is now a fair game and Player A has won 5 out of 5 games, what is the likelihood that Player B will win game 6?

Use a cross to indicate the likelihood on the line below.

impossible unlikely **X** likely certain
equally likely

Explain your answer.

If a game is reliant on chance, and it is fair, both players have an
equal opportunity to win. It is purely luck.

A Sample: Response 2

Guide to making judgments — Year 6 Mathematics

Name

Focus: Apply knowledge of chance events, variation and bias to develop arguments and improve game fairness.

Knowledge and understanding	Thinking and reasoning Reflecting	Communicating
<p>Uses possible outcomes and frequency to estimate likelihood as a common fraction.</p> <p>Compares and orders estimates of likelihood.</p> <p>Identifies differences between graphs.</p> <p>Questions 1–5</p>	<p>Reflects on learning and applies new understandings to:</p> <ul style="list-style-type: none"> develop mathematical arguments plan, evaluate and explain changes to game rules. <p>Questions 6–9</p>	<p>Communicates using mathematical language and representations to justify thinking and reasoning.</p> <p>Questions 1–3, 6, 8, 9</p>
<p>Explains differences between graphs using a comprehensive understanding of the factors influencing chance events and variation.</p> <p>Correctly compares and orders estimates of likelihood, and identifies outcomes with equal likelihoods. Considers chance events when explaining differences between graphs.</p> <p>Correctly expresses likelihoods as common fractions.</p> <p>Translates information from graphs into tables. Identifies obvious mathematical differences between graphs. Identifies a most and least likely outcome.</p> <p>Identifies numbers of outcomes from grid.</p>	<p>Justifies arguments using a comprehensive mathematical interpretation of the quantitative data and factors influencing chance events and bias.</p> <p>Makes changes to rules which make the game fair. Accurately judges Player B's chances of winning, and justifies using an understanding of chance events.</p> <p>Makes changes to rules which improve the fairness of the game. Considers chance events in arguments.</p> <p>Makes a relevant statement about why the game is unfair or why the rule changes are fair. Identifies likelihood of Player B winning.</p> <p>Makes a change to the rules.</p>	<p>Communicates and justifies thinking and reasoning using clear mathematical language.</p> <p>Communicates using appropriate mathematical language.</p> <p>Records possible outcomes and frequency in graphs.</p>
		<p>A</p> <p>B</p> <p>C</p> <p>D</p> <p>E</p>

Demonstrates the descriptors below and aspects of the descriptor above.

The factors influencing chance events and variation are not drawn out enough in Q5 to be considered “comprehensive”.

Demonstrates all descriptors for this assessable element.

Demonstrates all descriptors for this assessable element.

Overall grade

This response demonstrates a high level of achievement in **knowledge and understanding** and a very high level of achievement in **thinking and reasoning, reflecting** and **communicating**.

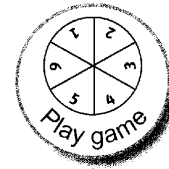
The focus of this QCAT is on **thinking and reasoning** and **reflecting**.

On balance, it is judged to be an A.

Collecting and analysing experimental data

Activity

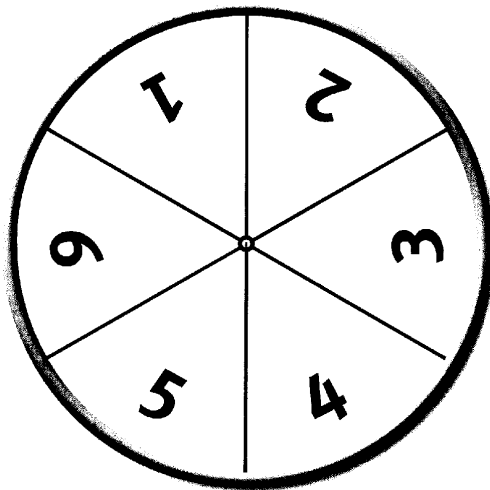
Play "Take a spin" in your pairs again. Complete Question 1 as you play.



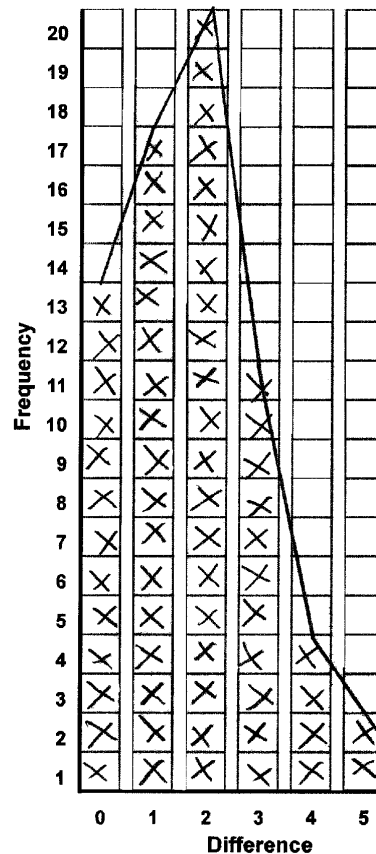
- Complete Graph 1 by using a cross (**X**) to record the difference for each spin while you play the game.

Stop recording if one of the differences reaches the top of the graph. Finish the game you are playing if this happens.

Take a spin



Graph 1: Frequency of each difference



Game	Player A (0, 1 or 2)	Player B (3, 4 or 5)
1		
2		
3		
4		
5		
6		

A Sample: Response 2

Use Graph 1 on page 4 to answer Question 2.

2. a) Complete the Frequency column in Table 1 below.
- b) In the Likelihood column, express the likelihood of spinning each difference as a common fraction.
- c) Order the differences from most likely to least likely in Diagram 1.

Table 1: Likelihood of spinning each difference

Difference	Frequency	Likelihood
0	13	$\frac{13}{68}$
1	18	$\frac{18}{68}$
2	20	$\frac{20}{68}$
3	11	$\frac{11}{68}$
4	4	$\frac{4}{68}$
5	2	$\frac{2}{68}$
Total	68	

Diagram 1: Order of likelihood

Difference	Likelihood
2	most likely
1	
0	
3	
4	
5	
	least likely



Stop here: Wait for your teacher's directions.

Exploring outcomes and theoretical likelihood

This grid shows all **possible outcomes** when using two spinners and finding the difference. Use the grid to answer the questions below.

		Player B					
		1	2	3	4	5	6
Player A	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

The difference of 4 has 4 possible outcomes.

These can be written as:

(1, 5) (2, 6) (5, 1) (6, 2)

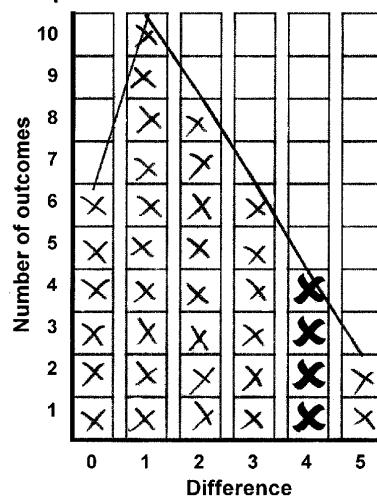
3. a) How many possible outcomes have a difference of 2? 9

b) Write all the possible outcomes that have a difference of 3.

(1, 4) (2, 5) (3, 6) (4, 1) (5, 2) (6, 3)

c) Complete Graph 2 by using a cross (**X**) to indicate the number of outcomes for each difference.

Graph 2: Number of outcomes for each difference



The possible outcomes for the difference of 4 have been completed for you.

A Sample: Response 2

Look at the shape of Graph 1 on page 4 and Graph 2 on page 6, then complete the sentence below.

4. The two graphs may not be the same shape because: *because if it is a game of chance it would be quite hard to get each graph exactly the same because the paper clip spinning is random*

Use Graph 2 to answer the following.

5. a) Complete Table 2 below.
b) Order the differences from most likely to least likely in Diagram 2.

Table 2: Likelihood of spinning each difference

Difference	Number of outcomes	Likelihood
0	<i>6</i>	<i>$\frac{6}{36}$</i>
1	<i>10</i>	<i>$\frac{10}{36}$</i>
2	<i>8</i>	<i>$\frac{8}{36}$</i>
3	<i>6</i>	<i>$\frac{6}{36}$</i>
4	<i>4</i>	<i>$\frac{4}{36}$</i>
5	<i>2</i>	<i>$\frac{2}{36}$</i>
Total	<i>36</i>	

Diagram 2: Order of likelihood

Difference	Likelihood
1	most likely
2	
3, 0	
4	
5	
	least likely



Stop here: Wait for your teacher's directions.

It's not fair!

During the group discussion (page 3), you talked about the question:

Is the game "Take a spin" fair or unfair?

6. Explain why the game is not fair.

Use the information you have collected to support your explanation.

This game is unfair because player A will score a point whenever the difference is 0, 1 and 2. That makes player A most likely to win because the differences have the most possible outcomes in a total of 24 possible outcomes. So that obviously leaves player B with the difference of 3, 4 and 5. That means that player B is unlikely to win with only a total of 12 possible outcomes. It is highly probable that player A will win with a double amount of outcomes than player B.

Applying your learning

In a fair game, each player has an equal chance of winning.

7. **Make the game "Take a spin" fair.**
Complete the rules below.

Take a spin

Rules for play

Getting ready

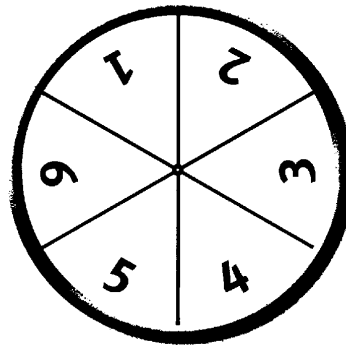
- Both players spin.
- The player who spins the lowest number will be Player A.

Playing the game

- Both players spin.
- When each spinner stops, find the difference between the numbers.
- If the difference is 0, 1 or 5.....
then Player A scores 1 point.....
- If the difference is 2, 3 or 4.....
then Player B scores 1 point.....
- Use a tally mark to record each point.

Winning the game

For players to win the game they must keep on spinning and
recording their score, until they reach the score of 10



8. **Explain how your changes to the game make it fair.**

My changes have made the game fair, because now that I
have swapped the differences around, both player A and
player B have each 18 possible outcomes. That makes
both players chances of winning equally likely

A Sample: Response 2

9. If "Take a spin" is now a fair game and Player A has won 5 out of 5 games, what is the likelihood that Player B will win game 6?

Use a cross to indicate the likelihood on the line below.

impossible unlikely \times equally likely likely certain

Explain your answer.

Because the chances have favoured each player a 50% chance of winning the next game.