D

Is it fair?

Sample responses

6

Mathematics

Queensland Comparable
Assessment Tasks
(QCATs) 2011

Queensland Government
Queensland Studies Authority

Queensland Studies Authority

Contact information

Direct questions about receipt of QCAT materials or QCAT implementation to the Senior Operations Officer.
Phone: (07) 3120 6187 email: qcats.administrator@qsa.qld.edu.au

Queensland Studies Authority PO Box 307 Spring Hill Qld 4004
Phone: (07) 3864 0299 Fax: (07) 3221 2553 Email: office@qsa.qld.edu.au Website: www.qsa.qld.edu.au

Cover image © QSA.

D Sample: Response 1

Guide to making judgments — Year 6 Mathematics

Name .....

Focus: Apply knowledge of chance events, variation and bias to develop arguments and improve game fairness.

Knowledge and understanding	Thinking and reasoning Reflecting	Communicating
Uses possible outcomes and frequency to estimate likelihood as a common fraction. Compares and orders estimates of likelihood. Identifies differences between graphs. Questions 1–5	Reflects on learning and applies new understandings to: • develop mathematical arguments • plan, evaluate and explain changes to game rules. Questions 6–9	Communicates using mathematical language and representations to justify thinking and reasoning. Questions 1–3, 6, 8, 9

◀ Explains differences between graphs using a comprehensive understanding of the factors influencing chance events and variation.	◀ Justifies arguments using a comprehensive mathematical interpretation of the quantitative data and factors influencing chance events and bias.	◀ Communicates and justifies thinking and reasoning using clear mathematical language.	A
	◀ Makes changes to rules which make the game fair. Accurately judges Player B's chances of winning, and justifies using an understanding of chance events.		B
◀ Correctly compares and orders estimates of likelihood, and identifies outcomes with equal likelihoods. Considers chance events when explaining differences between graphs. ◀ Correctly expresses likelihoods as common fractions.	◀ Makes changes to rules which improve the fairness of the game. Considers chance events in arguments.	◀ Communicates using appropriate mathematical language.	C
◀ Translates information from graphs into tables. Identifies obvious mathematical differences between graphs. Identifies a most and least likely outcome.	◀ Makes a relevant statement about why the game is unfair or why the rule changes are fair. Identifies likelihood of Player B winning.		D
◀ Identifies numbers of outcomes from grid.	◀ Makes a change to the rules.	◀ Records possible outcomes and frequency in graphs.	E

Demonstrates all descriptors up to and including this level.

Calculations of theoretical probability demonstrate a lapse in understanding of the probability formula.

Demonstrates most descriptors up to and including this level.

Q7 demonstrates lapses in understanding of game construction and the requirement that someone is able to win.

Demonstrates the descriptors below and aspects of the descriptor above.

Explanations are brief.

Overall grade

This response demonstrates a limited level of achievement across all assessable elements.

It is judged to be a D.

Collecting and analysing experimental data

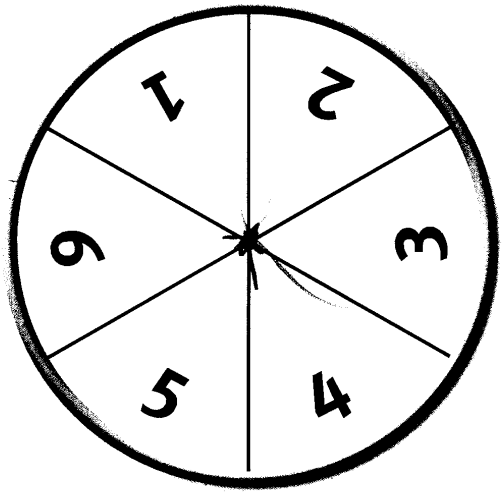
Activity

Play "Take a spin" in your pairs again. Complete Question 1 as you play.

1. Complete Graph 1 by using a cross (✕) to record the difference for each spin while you play the game.

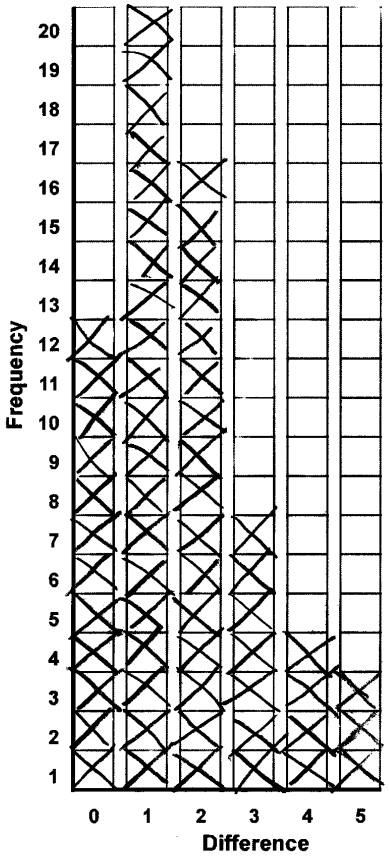
Stop recording if one of the differences reaches the top of the graph.
Finish the game you are playing if this happens.

Take a spin



Game	Player A (0, 1 or 2)	Player B (3, 4 or 5)
1		
2		
3		
4		
5		
6		

Graph 1: Frequency of each difference



D Sample: Response 1

Use Graph 1 on page 4 to answer Question 2.

2. **a) Complete the Frequency column in Table 1 below.**

b) In the Likelihood column, express the likelihood of spinning each difference as a common fraction.

c) Order the differences from most likely to least likely in Diagram 1.

Table 1: Likelihood of spinning each difference

Difference	Frequency	Likelihood
0	12	
1	20	1
2	14	
3	7	
4	2	
5	1	
Total	39	

Diagram 1: Order of likelihood

Difference	Likelihood
1	most likely
5	least likely

Stop here: Wait for your teacher's directions.

Queensland Studies Authority | 5

Exploring outcomes and theoretical likelihood

This grid shows all **possible outcomes** when using two spinners and finding the difference.
Use the grid to answer the questions below.

Player A: 0, 1, 2
Player B 3, 4, 5

Player A \ Player B	1	2	3	4	5	6
1	0 A	1 A	2 A	3 B	4 B	5 B
2	1 A	0 A	1 A	2 A	3 B	4 B
3	2 A	1 A	0 A	1 A	2 A	3 B
4	3 B	2 A	1 A	0 A	1 A	2 A
5	4 B	3 B	2 A	1 A	0 A	1 A
6	5 B	4 B	3 B	2 A	1 A	0 A

The difference of 4 has 4 possible outcomes.

These can be written as:

(1, 5) (2, 6) (5, 1) (6, 2)

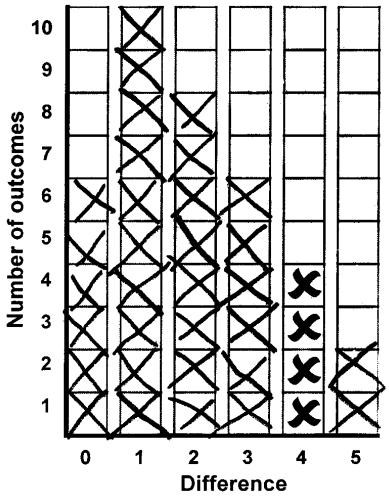
3. a) How many possible outcomes have a difference of 2? ..... 8 .....

b) Write all the possible outcomes that have a difference of 3.

(4, 1) (5, 2) (6, 3) (1, 4) (6, 3) (3, 6) (2, 5) (4, 1)

c) Complete Graph 2 by using a cross (✕) to indicate the number of outcomes for each difference.

Graph 2: Number of outcomes for each difference



The possible outcomes for the difference of 4 have been completed for you.

D Sample: Response 1

Look at the shape of Graph 1 on page 4 and Graph 2 on page 6, then complete the sentence below.

4. **The two graphs may not be the same shape because:** .....

the may have not had as many turns as they got on the first game

.....

.....

.....

Use Graph 2 to answer the following.

5. **a) Complete Table 2 below.**

b) Order the differences from most likely to least likely in Diagram 2.

Table 2: Likelihood of spinning each difference

Difference	Number of outcomes	Likelihood
0	6	60%
1	10	100%
2	8	80%
3	6	60%
4	4	40%
5	2	20%
Total	36 outcomes	

Diagram 2: Order of likelihood

Difference	Likelihood
1	most likely
	least likely

Stop here: Wait for your teacher's directions.

Queensland Studies Authority | 7

It's not fair!

During the group discussion (page 3), you talked about the question:

Is the game "Take a spin" fair or unfair?

6. Explain why the game is not fair.

Use the information you have collected to support your explanation.

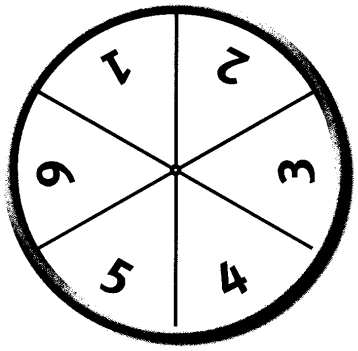
The game is unfair because when we were playing only A came up the most and B does not come much. I was A and the next player was B. And when we started playing I was the one who is winning

Applying your learning

In a fair game, each player has an equal chance of winning.

7. Make the game "Take a spin" fair.
Complete the rules below.

Take a spin



Rules for play

Getting ready

- Both players spin.
- The player who spins the lowest number will be Player A.

Playing the game

- Both players spin.
- When each spinner stops, find the difference between the numbers.
- If the **difference** is 0, 1, 2, 3
  then Player A scores 3
- If the **difference** is 0, 1, 2, 4
  then Player B scores 3
- Use a tally mark to record each point.

Winning the game

first player to 30 wins the game

8. Explain how your changes to the game make it fair.

if they have the same difference the game will be fair

D Sample: Response 1

9. If "Take a spin" is now a fair game and Player A has won 5 out of 5 games, what is the likelihood that Player B will win game 6?

Use a cross to indicate the likelihood on the line below.

impossible unlikely equally likely likely certain

Explain your answer.

because the A got more chance of winning

D Sample: Response 2

Guide to making judgments — Year 6 Mathematics

Name .....

Focus: Apply knowledge of chance events, variation and bias to develop arguments and improve game fairness.

Knowledge and understanding	Thinking and reasoning Reflecting	Communicating
Uses possible outcomes and frequency to estimate likelihood as a common fraction. Compares and orders estimates of likelihood. Identifies differences between graphs. Questions 1–5	Reflects on learning and applies new understandings to: • develop mathematical arguments • plan, evaluate and explain changes to game rules. Questions 6–9	Communicates using mathematical language and representations to justify thinking and reasoning. Questions 1–3, 6, 8, 9
◀ Explains differences between graphs using a comprehensive understanding of the factors influencing chance events and variation.	◀ Justifies arguments using a comprehensive mathematical interpretation of the quantitative data and factors influencing chance events and bias.	◀ Communicates and justifies thinking and reasoning using clear mathematical language.	A
	◀ Makes changes to rules which make the game fair. Accurately judges Player B's chances of winning, and justifies using an understanding of chance events.		B
◀ Correctly compares and orders estimates of likelihood, and identifies outcomes with equal likelihoods. Considers chance events when explaining differences between graphs.	◀ Makes changes to rules which improve the fairness of the game. Considers chance events in arguments.	◀ Communicates using appropriate mathematical language.	C
◀ Correctly expresses likelihoods as common fractions.			
◀ Translates information from graphs into tables. Identifies obvious mathematical differences between graphs. Identifies a most and least likely outcome.	◀ Makes a relevant statement about why the game is unfair or why the rule changes are fair. Identifies likelihood of Player B winning.		D
◀ Identifies numbers of outcomes from grid.	◀ Makes a change to the rules.	◀ Records possible outcomes and frequency in graphs.	E

Demonstrates all descriptors up to and including this level.

Ordering of estimates of likelihood in Q2 demonstrates a lapse in understanding of position and place, and equal likelihood.

Demonstrates most descriptors up to and including this level.

Q7 and Q8 demonstrate lapses in understanding of equal chance.

Demonstrates all of the descriptors up to this level and aspects of the descriptor above.

Explanations are brief.

Overall grade

This response demonstrates a sound level of achievement in knowledge and understanding and a limited level of achievement in thinking and reasoning, reflecting and communicating.

The focus of this QCAT is on thinking and reasoning and reflecting.

On balance, it is judged to be a D.

Collecting and analysing experimental data

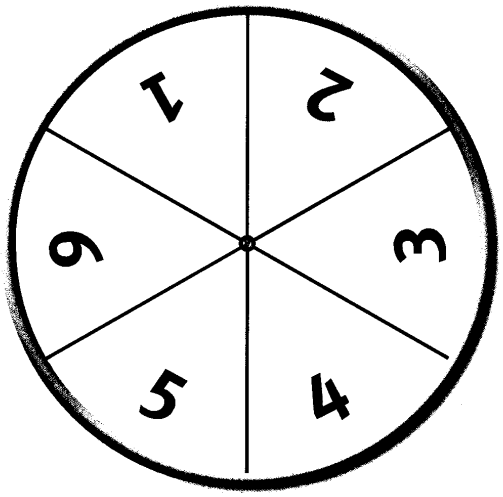
Activity

Play "Take a spin" in your pairs again. Complete Question 1 as you play.

1. Complete Graph 1 by using a cross (✕) to record the difference for each spin while you play the game.

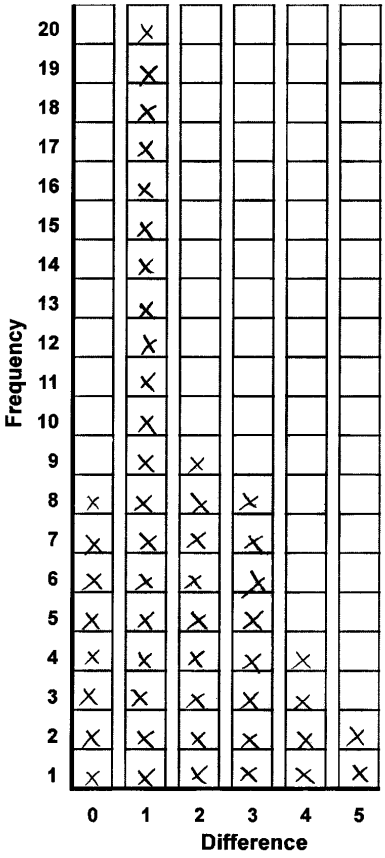
Stop recording if one of the differences reaches the top of the graph.
Finish the game you are playing if this happens.

Take a spin



Game	Player A (0, 1 or 2)	Player B (3, 4 or 5)
1	𝍸 𝍸	𝍸
2	𝍸 𝍸	𝍸
3	𝍸 𝍸	𝍸 II
4	III	II
5		
6		

Graph 1: Frequency of each difference



D Sample: Response 2

Use Graph 1 on page 4 to answer Question 2.

2. **a) Complete the Frequency column in Table 1 below.**

b) In the Likelihood column, express the likelihood of spinning each difference as a common fraction.

c) Order the differences from most likely to least likely in Diagram 1.

Table 1: Likelihood of spinning each difference

Difference	Frequency	Likelihood
0	8	$\frac{8}{51}$
1	20	$\frac{20}{51}$
2	9	$\frac{9}{51}$
3	8	$\frac{8}{51}$
4	4	$\frac{4}{51}$
5	2	$\frac{2}{51}$
Total	51	

Diagram 1: Order of likelihood

Difference	Likelihood
1 0 2	most likely
3 4 5	least likely

Stop here: Wait for your teacher's directions.

Exploring outcomes and theoretical likelihood

This grid shows all **possible outcomes** when using two spinners and finding the difference.
Use the grid to answer the questions below.

Player A \ Player B	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

The difference of 4 has 4 possible outcomes.

These can be written as:

(1, 5) (2, 6) (5, 1) (6, 2)

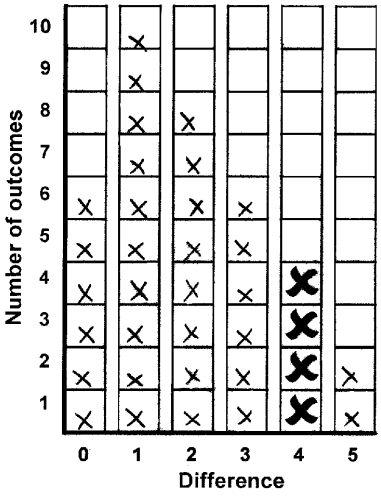
3. a) How many possible outcomes have a difference of 2? ....8....

b) Write all the possible outcomes that have a difference of 3.

(1,4)(2,5)(3,6)(4,1)(5,2)(6,3)

c) Complete Graph 2 by using a cross (✕) to indicate the number of outcomes for each difference.

Graph 2: Number of outcomes for each difference



The possible outcomes for the difference of 4 have been completed for you.

D Sample: Response 2

Look at the shape of Graph 1 on page 4 and Graph 2 on page 6, then complete the sentence below.

4. The two graphs may not be the same shape because: the might not have had the the same amount of numbers on the bigger graph than the smaller graph

Use Graph 2 to answer the following.

5. a) Complete Table 2 below.

b) Order the differences from most likely to least likely in Diagram 2.

Table 2: Likelihood of spinning each difference

Difference	Number of outcomes	Likelihood
0	8	8/51
1	20	20/51
2	9	9/51
3	8	8/51
4	4	4/51
5	2	2/51
Total	51	

Diagram 2: Order of likelihood

Difference	Likelihood
	most likely
	least likely

Stop here: Wait for your teacher's directions.

D Sample: Response 2

It's not fair!

During the group discussion (page 3), you talked about the question:

Is the game "Take a spin" fair or unfair?

6. Explain why the game is not fair.

Use the information you have collected to support your explanation.

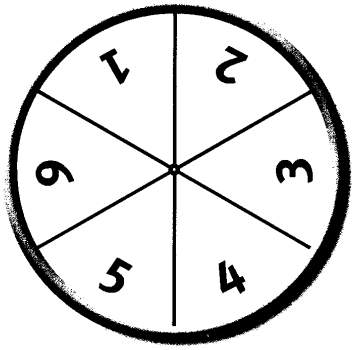
I think the game of take a spin is an unfair game because our class had a vote if the game is fair or unfair and all of the class went to the side that was unfair and the whole class was write

Applying your learning

In a fair game, each player has an equal chance of winning.

7. Make the game "Take a spin" fair.
Complete the rules below.

Take a spin



Rules for play

Getting ready

- Both players spin.
- The player who spins the lowest number will be Player A.

Playing the game

- Both players spin.
- When each spinner stops, find the difference between the numbers.
- If the **difference** is 6, 1, 2, 4
  then Player A scores 1 point
- If the **difference** is 5, 1, 4, 3
  then Player B scores 1 point
- Use a tally mark to record each point.

Winning the game

the first player to 15 points wins the game

8. Explain how your changes to the game make it fair.

I made it so the outcomes for player a and player b add up to 13 so it is fair

D Sample: Response 2

9. If "Take a spin" is now a fair game and Player A has won 5 out of 5 games, what is the likelihood that Player B will win game 6?

Use a cross to indicate the likelihood on the line below.

impossible unlikely equally likely likely certain

Explain your answer.

I pick impossible because the question at the top says player a won 5 out of 5 games, but it doesn't say a sixth game.