



Is it fair?

Sample responses



6

Mathematics

Queensland Comparable
Assessment Tasks
(QCATs) 2011

Contact information

Direct questions about receipt of QCAT materials or QCAT implementation to the Senior Operations Officer.
Phone: (07) 3120 6187 email: qcats.administrator@qsa.qld.edu.au

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Queensland Studies Authority PO Box 307 Spring Hill Qld 4004

Phone: (07) 3864 0299 Fax: (07) 3221 2553 Email: office@qsa.qld.edu.au Website: www.qsa.qld.edu.au

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C Sample: Response 1

Guide to making judgments — Year 6 Mathematics

Name

Focus: Apply knowledge of chance events, variation and bias to develop arguments and improve game fairness.

| Knowledge and understanding | Thinking and reasoning Reflecting | Communicating |
|--|---|--|
| <p>Uses possible outcomes and frequency to estimate likelihood as a common fraction.</p> <p>Compares and orders estimates of likelihood.</p> <p>Identifies differences between graphs.</p> <p>Questions 1–5</p> | <p>Reflects on learning and applies new understandings to:</p> <ul style="list-style-type: none"> develop mathematical arguments plan, evaluate and explain changes to game rules. <p>Questions 6–9</p> | <p>Communicates using mathematical language and representations to justify thinking and reasoning.</p> <p>Questions 1–3, 6, 8, 9</p> |
| <p>Explains differences between graphs using a comprehensive understanding of the factors influencing chance events and variation.</p> <p>Correctly compares and orders estimates of likelihood, and identifies outcomes with equal likelihoods. Considers chance events when explaining differences between graphs.</p> <p>Correctly expresses likelihoods as common fractions.</p> <p>Translates information from graphs into tables. Identifies obvious mathematical differences between graphs. Identifies a most and least likely outcome.</p> <p>Identifies numbers of outcomes from grid.</p> | <p>Justifies arguments using a comprehensive mathematical interpretation of the quantitative data and factors influencing chance events and bias.</p> <p>Makes changes to rules which make the game fair. Accurately judges Player B's chances of winning, and justifies using an understanding of chance events.</p> <p>Makes changes to rules which improve the fairness of the game. Considers chance events in arguments.</p> <p>Makes a relevant statement about why the game is unfair or why the rule changes are fair. Identifies likelihood of Player B winning.</p> <p>Makes a change to the rules.</p> | <p>Communicates and justifies thinking and reasoning using clear mathematical language.</p> <p>Communicates using appropriate mathematical language.</p> <p>Records possible outcomes and frequency in graphs.</p> |
| | | <p>A</p> <p>B</p> <p>C</p> <p>D</p> <p>E</p> |

Demonstrates most descriptors up to and including this level.

Ordering of estimates of likelihood in Q5 demonstrates a lapse in understanding of equal likelihood.

Demonstrates all descriptors up to and including this level.

Demonstrates most descriptors up to and including this level.

Explanations are brief and use some appropriate mathematical language.

Overall grade

This response demonstrates a sound level of achievement across all assessable elements. It is judged to be a C.

Collecting and analysing experimental data

Activity

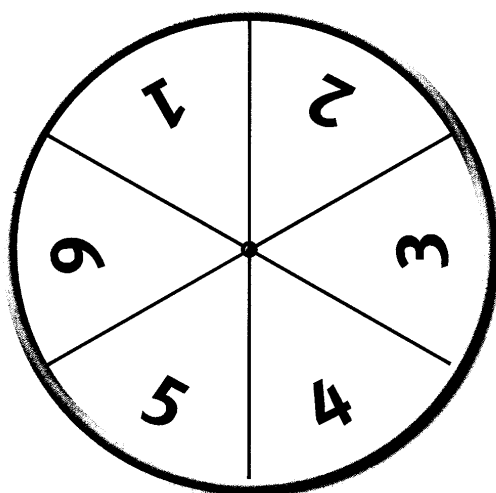
Play "Take a spin" in your pairs again. Complete Question 1 as you play.



1. Complete Graph 1 by using a cross (X) to record the difference for each spin while you play the game.

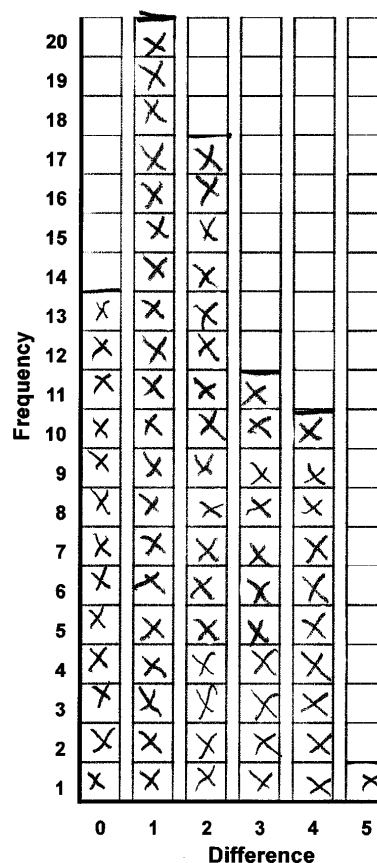
Stop recording if one of the differences reaches the top of the graph.
Finish the game you are playing if this happens.

Take a spin



| Game | Player A (0, 1 or 2) | Player B (3, 4 or 5) |
|------|-------------------------|-------------------------|
| 1 | ≡ ≡ | ≡ |
| 2 | ≡ ≡ | ≡ ≡ |
| 3 | ≡ ≡ | ≡ |
| 4 | ≡ ≡ | ≡ |
| 5 | ≡ ≡ | ≡ |
| 6 | | |

Graph 1: Frequency of each difference



C Sample: Response 1

Use Graph 1 on page 4 to answer Question 2.

2.
 - a) Complete the Frequency column in Table 1 below.
 - b) In the Likelihood column, express the likelihood of spinning each difference as a common fraction.
 - c) Order the differences from most likely to least likely in Diagram 1.

Table 1: Likelihood of spinning each difference

| Difference | Frequency | Likelihood |
|------------|-----------|-----------------|
| 0 | 13 | $\frac{13}{72}$ |
| 1 | 20 | $\frac{20}{72}$ |
| 2 | 17 | $\frac{17}{72}$ |
| 3 | 11 | $\frac{11}{72}$ |
| 4 | 10 | $\frac{10}{72}$ |
| 5 | 1 | $\frac{1}{72}$ |
| Total | 72 | |

Diagram 1: Order of likelihood

| Difference | Likelihood |
|------------|--------------|
| 1 | most likely |
| 2 | |
| 0 | |
| 3 | |
| 4 | |
| 5 | |
| | least likely |



Stop here: Wait for your teacher's directions.

Exploring outcomes and theoretical likelihood

This grid shows all **possible outcomes** when using two spinners and finding the difference. Use the grid to answer the questions below.

| | | Player B | | | | | |
|----------|---|----------|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| Player A | 1 | 0 | 1 | 2 | 3 | 4 | 5 |
| | 2 | 1 | 0 | 1 | 2 | 3 | 4 |
| | 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| | 4 | 3 | 2 | 1 | 0 | 1 | 2 |
| | 5 | 4 | 3 | 2 | 1 | 0 | 1 |
| | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

The difference of 4 has 4 possible outcomes.

These can be written as:

(1, 5) (2, 6) (5, 1) (6, 2)

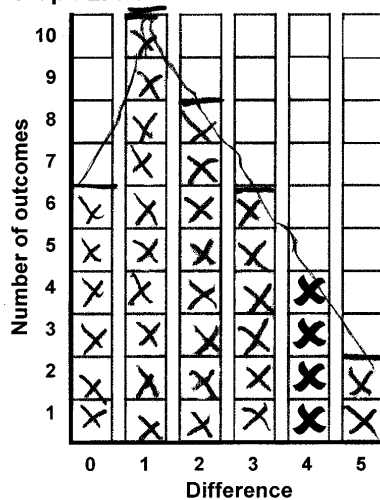
3. a) How many possible outcomes have a difference of 2? 8

- b) Write all the possible outcomes that have a difference of 3.

(1, 4) (5, 2) (6, 3) (2, 5) (4, 1) (3, 6)

- c) Complete Graph 2 by using a cross (X) to indicate the number of outcomes for each difference.

Graph 2: Number of outcomes for each difference



The possible outcomes for the difference of 4 have been completed for you.

C Sample: Response 1

Look at the shape of Graph 1 on page 4 and Graph 2 on page 6, then complete the sentence below.

4. The two graphs may not be the same shape because: my one says that
4 has more than it should and 5 has less than it
should. However 1 still got to the top first.

Use Graph 2 to answer the following.

5. a) Complete Table 2 below.
 b) Order the differences from most likely to least likely in Diagram 2.

Table 2: Likelihood of spinning each difference

| Difference | Number of outcomes | Likelihood |
|------------|--------------------|-----------------|
| 0 | 6 | $\frac{6}{36}$ |
| 1 | 10 | $\frac{10}{36}$ |
| 2 | 8 | $\frac{8}{36}$ |
| 3 | 6 | $\frac{6}{36}$ |
| 4 | 4 | $\frac{4}{36}$ |
| 5 | 2 | $\frac{2}{36}$ |
| Total | 36 | |

Diagram 2: Order of likelihood

| Difference | Likelihood |
|------------|--------------|
| 1 | most likely |
| 8 | |
| 3 | |
| 0 | |
| 4 | |
| 5 | |
| | least likely |



Stop here: Wait for your teacher's directions.

It's not fair!

During the group discussion (page 3), you talked about the question:

Is the game "Take a spin" fair or unfair?

6. Explain why the game is not fair.

Use the information you have collected to support your explanation.

I think the game is unfair, because player A has an unfair advantage. This is that player A has more outcomes than player B,

Applying your learning

In a fair game, each player has an equal chance of winning.

7. Make the game "Take a spin" fair.
Complete the rules below.

Take a spin

Rules for play

Getting ready

- Both players spin.
- The player who spins the lowest number will be Player A.

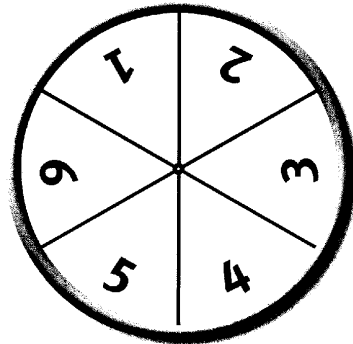
Playing the game

- Both players spin.
- When each spinner stops, find the difference between the numbers.

- If the difference is 0, 1, 4
then Player A scores a point
- If the difference is 2, 3, 5
then Player B scores a point
- Use a tally mark to record each point.

Winning the game

the first to 10 points wins the game



8. Explain how your changes to the game make it fair.

because they both have fair numbers and it means
that they both have equal chances of winning

C Sample: Response 1

9. If "Take a spin" is now a fair game and Player A has won 5 out of 5 games, what is the likelihood that Player B will win game 6?

Use a cross to indicate the likelihood on the line below.

X

impossible

unlikely

equally likely

likely

certain

Explain your answer.

because if player A won 5 out of 5 who could
player B win round 6?

C Sample: Response 2

Guide to making judgments — Year 6 Mathematics

Name

Focus: Apply knowledge of chance events, variation and bias to develop arguments and improve game fairness.

| Knowledge and understanding | Thinking and reasoning Reflecting | Communicating |
|--|---|--|
| <p>Uses possible outcomes and frequency to estimate likelihood as a common fraction.</p> <p>Compares and orders estimates of likelihood.</p> <p>Identifies differences between graphs.</p> <p>Questions 1–5</p> | <p>Reflects on learning and applies new understandings to:</p> <ul style="list-style-type: none"> develop mathematical arguments plan, evaluate and explain changes to game rules. <p>Questions 6–9</p> | <p>Communicates using mathematical language and representations to justify thinking and reasoning.</p> <p>Questions 1–3, 6, 8, 9</p> |
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| | | <p>A</p> <p>B</p> <p>C</p> <p>D</p> <p>E</p> |

Demonstrates all of the descriptors up to this level and aspects of the descriptor above.

The factors influencing chance events and variation are not drawn out enough in Q5 to be considered “comprehensive”.

Demonstrates most descriptors up to and including this level.

The explanation of why the rules are unfair, in Q6, lacks depth and is supported inappropriately by empirical data.

Demonstrates all descriptors up to this level.

Explanations are brief and use appropriate mathematical language.

Overall grade

This response demonstrates a high level of achievement in **knowledge and understanding** and a sound level of achievement in **thinking and reasoning, reflecting** and **communicating**.

The focus of this QCAT is on **thinking and reasoning** and **reflecting**.

On balance, it is judged to be a C.

Collecting and analysing experimental data

Activity

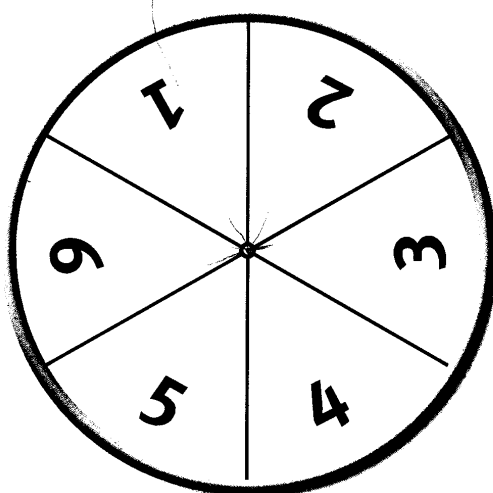
Play "Take a spin" in your pairs again. Complete Question 1 as you play.



- Complete Graph 1 by using a cross (X) to record the difference for each spin while you play the game.

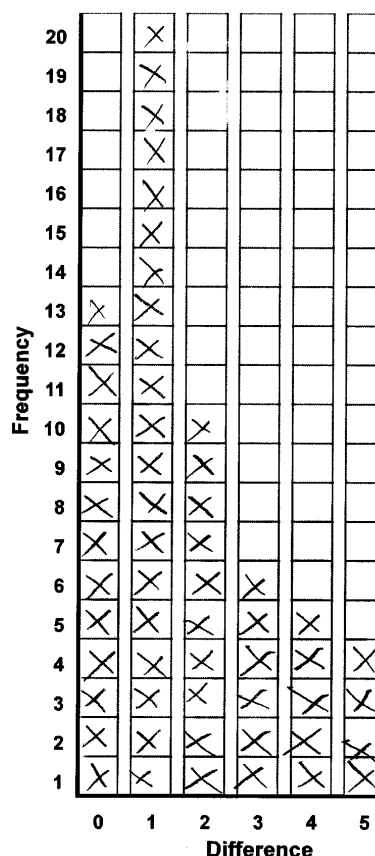
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Finish the game you are playing if this happens.

Take a spin



| Game | Player A (0, 1 or 2) | Player B (3, 4 or 5) |
|------|-------------------------|-------------------------|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |

Graph 1: Frequency of each difference



C Sample: Response 2

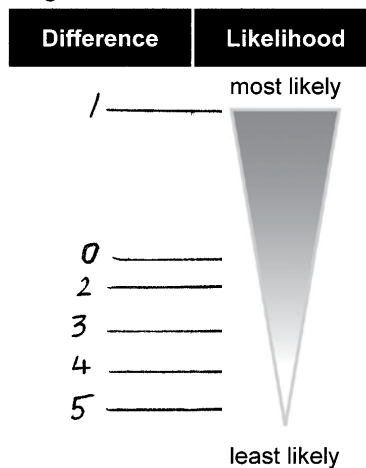
Use Graph 1 on page 4 to answer Question 2.

2. a) Complete the Frequency column in Table 1 below.
- b) In the Likelihood column, express the likelihood of spinning each difference as a common fraction.
- c) Order the differences from most likely to least likely in Diagram 1.

Table 1: Likelihood of spinning each difference

| Difference | Frequency | Likelihood |
|------------|-----------|-----------------|
| 0 | 13 | $\frac{13}{58}$ |
| 1 | 20 | $\frac{20}{58}$ |
| 2 | 10 | $\frac{10}{58}$ |
| 3 | 6 | $\frac{6}{58}$ |
| 4 | 5 | $\frac{5}{58}$ |
| 5 | 4 | $\frac{4}{58}$ |
| Total | 58 | |

Diagram 1: Order of likelihood



Stop here: Wait for your teacher's directions.

Exploring outcomes and theoretical likelihood

This grid shows all **possible outcomes** when using two spinners and finding the difference. Use the grid to answer the questions below.

| | | Player B | | | | | |
|----------|---|----------|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| Player A | 1 | 0 | 1 | 2 | 3 | 4 | 5 |
| | 2 | 1 | 0 | 1 | 2 | 3 | 4 |
| | 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| | 4 | 3 | 2 | 1 | 0 | 1 | 2 |
| | 5 | 4 | 3 | 2 | 1 | 0 | 1 |
| | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

The difference of 4 has 4 possible outcomes.

These can be written as:

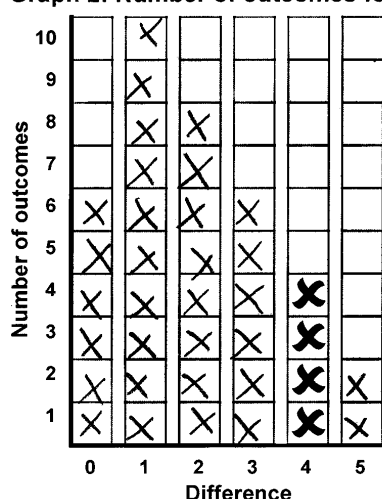
(1, 5) (2, 6) (5, 1) (6, 2)

3. a) How many possible outcomes have a difference of 2? eight
- b) Write all the possible outcomes that have a difference of 3.

(1, 4) (2, 5) (3, 6) (4, 1) (5, 2) (6, 3)

- c) Complete Graph 2 by using a cross (X) to indicate the number of outcomes for each difference.

Graph 2: Number of outcomes for each difference



The possible outcomes for the difference of 4 have been completed for you.

C Sample: Response 2

Look at the shape of Graph 1 on page 4 and Graph 2 on page 6, then complete the sentence below.

4. The two graphs may not be the same shape because: *one relied upon chance to be completed where as graph 2 relied upon all of the possible outcomes to be completed, and chance means you might get anything and wont always produce the same*

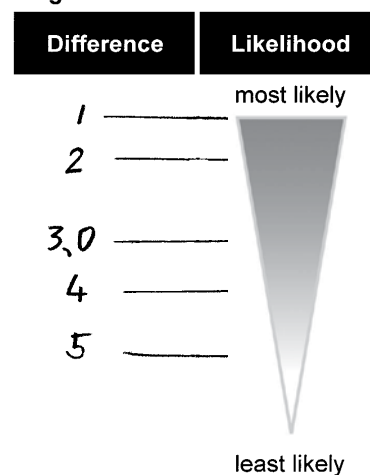
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| 3 | 6 | $\frac{6}{36}$ |
| 4 | 4 | $\frac{4}{36}$ |
| 5 | 2 | $\frac{2}{36}$ |
| Total | 36 | |

Diagram 2: Order of likelihood



Stop here: Wait for your teacher's directions.

It's not fair!

During the group discussion (page 3), you talked about the question:

Is the game "Take a spin" fair or unfair?

6. **Explain why the game is not fair.**

Use the information you have collected to support your explanation.

the game is not fair game because player A keep winning
all the games, but not player B. Player B is trying to win
but it doesn't work.

Applying your learning

In a fair game, each player has an equal chance of winning.

7. Make the game "Take a spin" fair.
Complete the rules below.

Take a spin

Rules for play

Getting ready

- Both players spin.
- The player who spins the lowest number will be Player A.

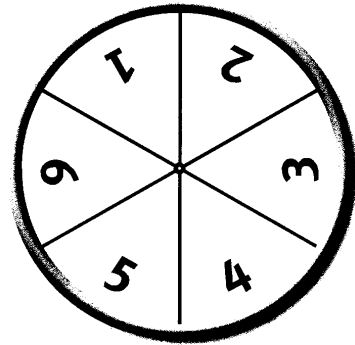
Playing the game

- Both players spin.
- When each spinner stops, find the difference between the numbers.

- If the difference is 1, 3, 5 or 0
then Player A scores a point
- If the difference is 2, 4, 5 or 0
then Player B scores a point
- Use a tally mark to record each point.

Winning the game

the first player to score 12 points wins



8. Explain how your changes to the game make it fair.

I changed the combination of numbers so that each player has two numbers that are not as likely to come up as a difference and two numbers that are likely to come up as a difference

C Sample: Response 2

9. If "Take a spin" is now a fair game and Player A has won 5 out of 5 games, what is the likelihood that Player B will win game 6?

Use a cross to indicate the likelihood on the line below.

×

impossible

unlikely

equally likely

likely

certain

Explain your answer.

if player A has already won 5/5 games then it is
most likely that player A will win again