Is it fair?

Teacher guidelines

6

Mathematics

Queensland Comparable
Assessment Tasks
(QCATs) 2011

Queensland Government
Queensland Studies Authority

Queensland Studies Authority

Contact information

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Contents

The 2011 QCATs

What are QCATs?

Queensland Comparable Assessment Tasks (QCATs) are designed to provide evidence of what students know, understand and can do in relation to a selection of Essential Learnings for English, mathematics and science in Years 4, 6 and 9, and to the Standards.

QCATs are authentic, performance-based assessments that:

- engage students in solving meaningful problems
- emphasise critical thinking and reasoning
- provide teachers, students and parents/carers with information about student progress and a focus for future teaching and learning.

Consistency of teacher judgments

QCATs support teachers in making consistent judgments about the quality of student work. Improved consistency of teacher judgments is achieved when teachers:

- engage in professional conversations about the quality of evidence in student responses
- reach consensus about the quality of student work
- adopt a consistent approach when using the Guide to making judgments (back page).

Information gathered may be used by teachers to promote, assist and improve key learning area programs, and to help students achieve the highest standards they can.

Additional resources

2011 QCATs Information statement
www.qsa.qld.edu.au/3163.html

Essential Learnings and Standards
www.qsa.qld.edu.au/574.html

Important dates

Friday 24 June	QCATs packages have arrived in schools
Monday 11 July ↓ Friday 16 September	Schools: • administer QCATs at any time during the school weeks of this period • grade QCATs • select five student samples that are representative of grades awarded
Monday 10 October	Schools are notified if selected to submit student samples for QSA's random sampling process
Monday 21 November	Final day for schools to submit student data to QSA
Friday 9 December*	Schools must retain all Student booklets until the end of the school year

* this date may vary from school to school

Getting ready

Student preparation

Students should have the opportunity to do their best work. For this to occur, student preparation should include:

- opportunities to engage with the Selected Essential Learnings (page 21) well in advance of participating in QCATs — if students have not engaged with the Selected Essential Learnings recently, review and consolidation may be necessary
- experience with the types of questions used within the QCAT.

Suggested learning experiences and resources are outlined in the document 2011 QCATs – Preparation.

The quality of information provided by the QCATs is enhanced by the level of interaction teachers have with their students before, during and after implementation.

Additional resources Queensland Comparable Assessment Tasks (QCATs)
www.qsa.qld.edu.au/3163.html

Catering for diversity — Special provisions

All students should have the opportunity to participate in school-based assessment. Schools are responsible for determining which students require special provisions.

The QCATs are designed to be part of a classroom assessment program, and principles of participation and equity apply. The Queensland Studies Authority (QSA) offers this general advice:

- Students who have been identified as having specific educational requirements may be assisted using those adjustments and supports usually available in the classroom. To make participation possible in all or part of the assessment task, such help may be in the form of inclusive learning technologies, reading support or the use of support personnel.
- Students for whom English is not their first language, and who are assessed as not achieving a reading level appropriate to complete the task, may be assisted by an interpreter or educational devices (e.g. pictures, electronic whiteboards, interactive devices) to allow participation in all or part of the task.
- In exceptional circumstances, where a student's learning difficulties have precluded them from engaging with the Selected Essential Learnings, the principal (in consultation with specialist and support staff and parents/carers) may make a decision about the participation of that student in the task. Some students may be given an opportunity to complete some aspects of the assessment.

Additional resources Inclusive strategies for implementing QCATs
www.qsa.qld.edu.au/3163.html

Equity
www.qsa.qld.edu.au/10188.html

Teacher preparation

Check the contents of QCAT packages as soon as they arrive at your school

- Check that you have the appropriate number of Student booklets (one per student) and Teacher guidelines (one per implementing teacher).
- Check for any defective Student booklets.
- Contact the QSA if any additional copies are required.

Familiarise yourself with the assessment

- Read all the documents provided.
- Review the Selected Essential Learnings (page 21).
- Complete a Student booklet yourself, and then refer to the Model response (page 23) so that you understand what students are required to do.
- Download and view Sample responses from the QSA Assessment Bank (see Additional resources below).

Plan implementation

- Discuss the assessment with your colleagues, and plan any teaching or revision that may be required.
- Set the times and dates for implementation, considering these points:
  - teachers have flexibility to implement the QCATs at any time during the designated period
  - QCATs may be completed in one, two or more sessions over one or more days
  - implementation times may differ for verified students, students with specific educational requirements or students who have English as a second language
  - QCATs will ideally replace an existing piece of assessment in the student portfolio of work for Semester 2.
- Plan:
  - any support required to enable students to do their best work (e.g. teacher aides or other support personnel)
  - any materials or equipment needed to implement the assessment.
- Decide:
  - how you will implement this task for all classes at this year level
  - the processes you will use to achieve consistency of teacher judgment
  - how you will select student samples for the QSA's random sampling process
  - when, how and who will submit your school's data.

Additional resources

Queensland Comparable Assessment Tasks (QCATs)
www.qsa.qld.edu.au/3163.html

Sample responses, QSA Assessment Bank
https://qcar.qsa.qld.edu.au/assessmentbank (registration required)

8 – Using Queensland Comparable Assessment Tasks (QCATs) to support learning
www.qsa.qld.edu.au/3166.html

Implementation

Setting up — Activity (Student booklet pages 3 & 4)

Equipment

One paperclip and pencil for each student in the class.

Tips for game play

- Explain how to use a paperclip and pencil as a spinner. Refer students to the image on the front of the QCAT.

- Discuss what to do when a paperclip lands on a line.
- Model how to decide who is Player A and Player B, and where students will record their points.
- Model the start of a game by taking a spin for each player, finding the difference and allocating a point.
- When modelling game play to your class, you could:
  - use a data projector and an electronic copy of the Student booklet, downloaded from the QSA Assessment Bank
  - copy the page of the Student booklet onto an overhead transparency and use an overhead projector.

Working with the Student booklet

Use advice given in the Annotated Student booklet (page 8) to set the conditions that ensure all students have the opportunity to do their best work.

Encourage students to interact with teachers to seek clarification when required, and with other students if appropriate to the task.

Suggested implementation timeline

Preparation

Setting the scene: Group discussion	15 minutes
Activity	10 minutes

The assessment task

Collecting and analysing experimental data	20 minutes
Exploring outcomes and theoretical likelihood	25 minutes
It's not fair!	20 minutes
Applying your learning	25 minutes

Annotated Student booklet

Suggested time: 25 minutes (Group discussion: 15 minutes, Activity: 10 minutes)

Read through Setting the scene with the whole class.

Engage students in a discussion about:

- the games of chance that students have played (e.g. *Snakes and ladders*)
- the meanings of the terms **chance**, **random**, **fair** and **unfair**
- how most games are designed to be fair.

Introduce the game "Take a spin" and explain the rules on page 3 of the Student booklet. See also Tips for game play on page 7 of this booklet.

Demonstrate how to use a paperclip and pencil as a spinner.

Explain that in this particular game, the **difference** is the larger score take away the smaller score.

Use the sample game on page 3 of the Student booklet to show how to keep score. Identify that Player A has won in this instance.

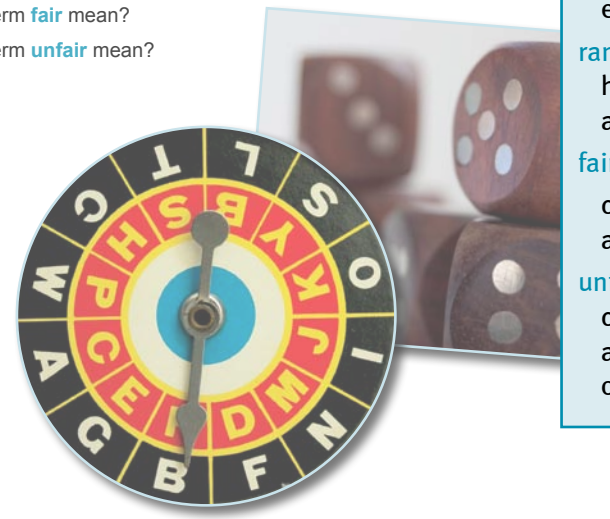
Setting the scene: Group discussion

Games of **chance** are games where an outcome is decided using devices such as dice, coins, cards, spinners, wheels or tickets. These devices produce a **random** result.

Games of chance can be **fair** or **unfair**.

Your teacher will lead a discussion about games of chance. Think about these ques

- What are some examples of games of chance?
- What does the term **fair** mean?
- What does the term **unfair** mean?



chance
: the likelihood or probability of an event happening

random
: happening without a pattern

fair
: chances of winning are equal

unfair
: chances of winning are biased towards one player

Activity

In the game "Take a spin":

- players use their spinners, and score a point based on the difference between the two spinners
- the first player to score 10 points wins the game.

Your teacher will now read through the rules for the game, and will organise you into pairs to play the game.

Organise students into pairs to play the game.

Students will need about 10 minutes to play at least two games.

Students will play again in Question 1.

Take a spin

Number of players: 2

Equipment: pencil, paperclip

Rules for play

Getting ready

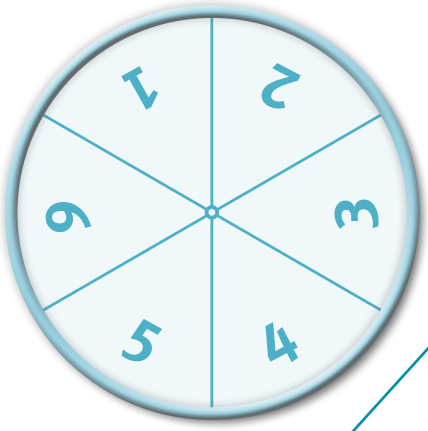
- Both players spin.
- The player who spins the lowest number will be Player A.

Playing the game

- Both players spin.
- When each spinner stops, find the difference between the numbers.
- If the **difference** is 0, 1 or 2, then Player A scores a point.
- If the **difference** is 3, 4 or 5, then Player B scores a point.
- Use a tally mark to record each point.

Winning the game

The first player to score 10 points wins the game.



Game	Player A (0, 1 or 2)	Player B (3, 4 or 5)
Sample	𝍸 𝍸	𝍸 |
1		
2		

When students play "Take a spin", each student must record game results for both players in their own Student booklet.

Students remain in their pairs for the next game (Question 1).

More group discussion

Your teacher will lead a discussion about the game "Take a spin". Think about this question:

Is the game "Take a spin" fair or unfair?

Read out the discussion question.

Ask students to share their thoughts. Discuss their responses to the discussion question.

Have students raise their hands if they won as Player A, and then raise their hands if they won as Player B.

Discuss their observations.

Lead students to the conclusion that the game is unfair.

In this assessment, you will:

- collect and analyse data
- think about what possible outcomes can occur
- find different likelihoods
- write an explanation
- plan and explain a new set of rules.

Stop here: Wait for your teacher's directions.

Stop here directions are placed at convenient points to finish a session, or to discuss the next part of the QCAT.

Work through the Guide to making judgments on the last page of the Student booklet with students to highlight the assessable elements for this QCAT.

Explain, in student-friendly terms, the task-specific descriptors. These identify what is being valued in the student responses.

Suggested time: 20 minutes

Explain that students will play "Take a spin" in their pairs again.

In addition to marking their tally marks, they must also fill in the graph to record each difference spun by the pair.

Model this process by taking a spin for each player, finding the difference, allocating a tally mark and marking the difference on the graph.

Collecting and analysing experimental data

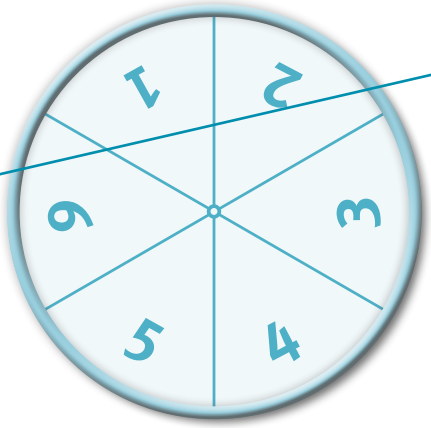
Activity

Play "Take a spin" in your pairs again. Complete Question 1 as you play.

1. **Complete Graph 1 by using a cross (✕) to record the difference for each spin while you play the game.**

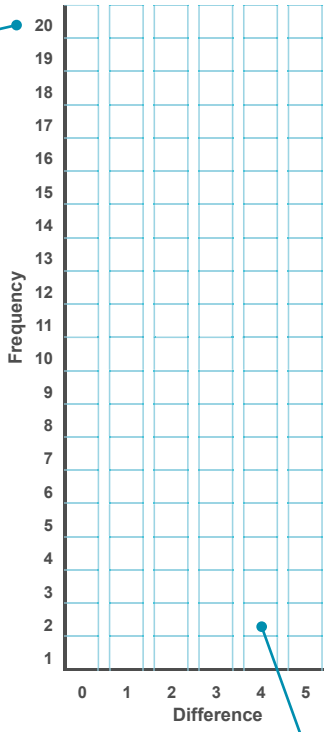
   Stop recording if one of the differences reaches the top of the graph.
   Finish the game you are playing if this happens.

Take a spin



Game	Player A (0, 1 or 2)	Player B (3, 4 or 5)
1		
2		
3		
4		
5		
6		

Graph 1: Frequency of each difference



Explain that students keep playing until they reach a frequency of 20 for any difference.

If this happens, they can finish the game they are playing, but stop recording results in the graph.

Completing this graph with a sizeable sample space is important for student success in Question 4.

4 | QCATs 2011 Student booklet Year 6 Mathematics

Students do not have to play 6 games. They may keep playing after completing the graph until you ask them to stop.

Emphasise to students that they are not to colour the squares in the graph — they should use a cross to mark the difference for each spin.

As you work through the QCAT with students, check for understanding of the task and provide clarification as needed.

Where students ask individual questions, answers should be shared with all students wherever possible.

Use Graph 1 on page 4 to answer Question 2.

Students work individually from this point on.

2. a) **Complete the Frequency column in Table 1 below.**

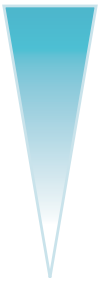
   b) **In the Likelihood column, express the likelihood of spinning each difference as a common fraction.**

   c) **Order the differences from most likely to least likely in Diagram 1.**

Table 1: Likelihood of spinning each difference

Difference	Frequency	Likelihood
0	.....	.....
1	.....	.....
2	.....	.....
3	.....	.....
4	.....	.....
5	.....	.....
Total	.....	

Diagram 1: Order of likelihood

Difference	Likelihood
	most likely  least likely

Explain that students will use Graph 1 when completing Table 1.

Provide students with the formula for probability, as you have taught it in class.

Explain that students will need to write the likelihood of each difference as a fraction.

Students do not need to simplify fractions.

Stop here: Wait for your teacher's directions.

What is being assessed

These questions gather evidence of students' understanding of:

- representing data graphically
- expressing experimental data for chance as a common fraction, using suitable mental computation
- comparing and ordering experimental data for chance from most likely to least likely, and attributing equal likelihood.

Suggested time: 25 minutes

possible outcomes
the results that may occur

Engage students in a discussion about the layout of the grid, including:

- the meaning of the term **possible outcomes**
- how to find the difference by looking at the intersection of the two spinner results on the grid
- the example shown to the right of the grid.

Exploring outcomes and theoretical likelihood

This grid shows all **possible outcomes** when using two spinners and finding the difference. Use the grid to answer the questions below.

Player A \ Player B	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

The difference of 4 has 4 possible outcomes.

These can be written as:

(1, 5) (2, 6) (5, 1) (6, 2)

Students should use this format to write responses for Question 3b.

Explain that although some pairs appear similar, they are different, e.g. (1, 5) and (5, 1). Relate each part of the ordered pair to what Player A spins and what Player B spins.

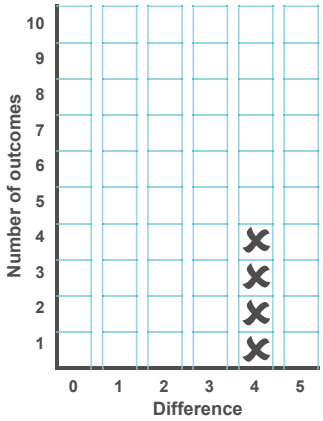
3. a) How many possible outcomes have a difference of 2? .....

b) Write all the possible outcomes that have a difference of 3.

.....

c) Complete Graph 2 by using a cross (✕) to indicate the number of outcomes for each difference.

Graph 2: Number of outcomes for each difference



The possible outcomes for the difference of 4 have been completed for you.

Explain that students will use the grid to complete Graph 2. Suggest to students that they cross out the differences in the grid as they count them. Use the difference of 4 as an example.

QCATs 2011 Student booklet Ye

What is being assessed

Question 3 gathers evidence of students' understanding of:

- the concepts of sample spaces and possible outcomes
- representing data graphically.

Explain that students will need to look back at Graph 1 on page 4. Emphasise that you are looking for mathematical explanations.

Look at the shape of Graph 1 on page 4 and Graph 2 on page 6, then complete the sentence below.

4. **The two graphs may not be the same shape because:** .....

.....

.....

.....

.....

.....

Explain that students will use Graph 2 when completing Question 5.

Use Graph 2 to answer the following.

5. a) **Complete Table 2 below.**

b) **Order the differences from most likely to least likely in Diagram 2.**

Provide students with the formula for probability, as you have taught it in class.

Explain that students will need to write the likelihood of each difference as a fraction.

Students do not need to simplify fractions.

Table 2: Likelihood of spinning each difference

Difference	Number of outcomes	Likelihood
0	.....	.....
1	.....	.....
2	.....	.....
3	.....	.....
4	.....	.....
5	.....	.....
Total	.....	

Diagram 2: Order of likelihood

Difference	Likelihood
	most likely least likely

What is being assessed

Questions 4 and 5 gather evidence of students' understanding of:

- comparing theoretical and experimental probability
- identifying causes of variation in data collections
- using mathematical language to communicate reasoning
- expressing theoretical probability as a common fraction, using suitable mental computation
- comparing and ordering estimates of probability from most likely to least likely, and attributing equal likelihood.

Suggested time: 20 minutes

Remind students of the discussion held during Setting the scene, about the meaning of the terms **fair** and **unfair**, and whether the game is fair or unfair.

It's not fair!

During the group discussion (page 3), you talked about the question:

Is the game "Take a spin" fair or unfair?

6. **Explain why the game is not fair.**

Use the information you have collected to support your explanation.

Explain that students will need to use the information they have gathered so far to complete the question.

In doing so, do not:

- direct students to what information they should use
- interpret the data for students.

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What is being assessed

This question gathers evidence of students' understanding of:

- reflecting upon learning within the QCAT
- developing an argument to justify a solution
- using mathematical language to communicate reasoning.

Stop here: Wait for your teacher's directions.

Suggested time: 25 minutes

Explain that students will need to use everything they have learned so far to complete Questions 7, 8 and 9.

Explain to students that there is more than one way to make the game fair.

In doing so, do not:

- direct students to the thinking and reasoning processes they should use to make the game fair
- interpret the data for students.

Remind students to reflect upon and evaluate their rule changes to determine whether they are fair.

If students are having difficulty, advise them to look at how the Rules for play are written on page 3 of the Student booklet.

Applying your learning

In a fair game, each player has an equal chance of winning.

7. **Make the game "Take a spin" fair.**
Complete the rules below.

Take a spin

Rules for play

Getting ready

- Both players spin.
- The player who spins the lowest number will be Player A.

Playing the game

- Both players spin.
- When each spinner stops, find the difference between the numbers.
- If the **difference** is .....
  then Player A scores .....
- If the **difference** is .....
  then Player B scores .....
- Use a tally mark to record each point.

Winning the game

.....

.....

8. **Explain how your changes to the game make it fair.**

.....

.....

.....

.....

.....

9. **If "Take a spin" is now a fair game and Player A has won 5 out of 5 games, what is the likelihood that Player B will win game 6?**

Use a cross to indicate the likelihood on the line below.

Explain your answer.

.....

.....

.....

.....

Teachers could follow up this QCAT by:

- comparing a range of rule changes suggested by students in the class
- having students explain their rules to another student, then collect data to evaluate whether the rules were fair
- collating student data into a class set
- discussing small versus large data trials
- converting the likelihoods to decimals and percentages
- predicting expected frequencies before conducting similar chance experiments with both small and large numbers of trials, using appropriate digital technologies
- creating and comparing other methods of displaying the data.

What is being assessed

These questions gather evidence of students' understanding of:

- reflecting upon learning within the QCAT
- evaluating multiple rule options to make the game fair
- developing an argument to justify a response
- using mathematical language to communicate reasoning.

Making judgments

Use the Guide to making judgments (GTMJ) on the back page to grade student responses.

The Model response (page 23) and Sample responses are provided for reference purposes only. They each demonstrate possible responses and should be used to support the GTMJ.

Making judgments is not about determining whether one student's work is better than that of another. Rather, you should make standards-based judgments by matching evidence in student responses to descriptors in the GTMJ.

Read and consider all of the evidence in the student's responses before making and recording a judgment about the quality of the performance for each assessable element.

Additional resources Sample responses, QSA Assessment Bank
https://qcar.qsa.qld.edu.au/assessmentbank (registration required)

Using the GTMJ

This QCAT uses a continua-style GTMJ, where descriptors are placed along a continuum within each column.

Record a nil award of "N" only when there is insufficient evidence to make a judgment for an overall grade.

In the following diagrams:

- Diagram 1: Understanding the GTMJ points out the different parts of the GTMJ continua model
- Diagram 2: Using the GTMJ — the judgment process gives steps to follow when grading student responses.

Diagram 1: Understanding the GTMJ

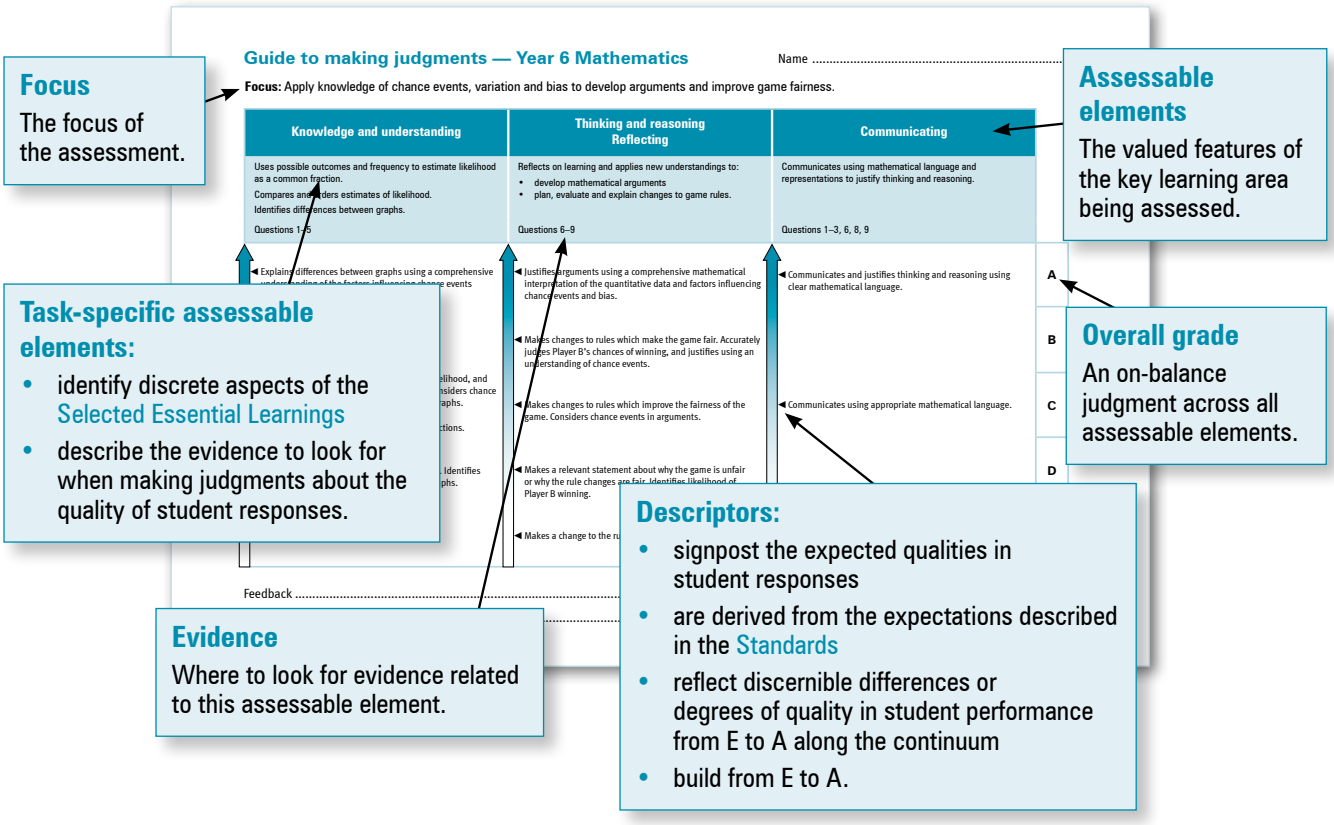


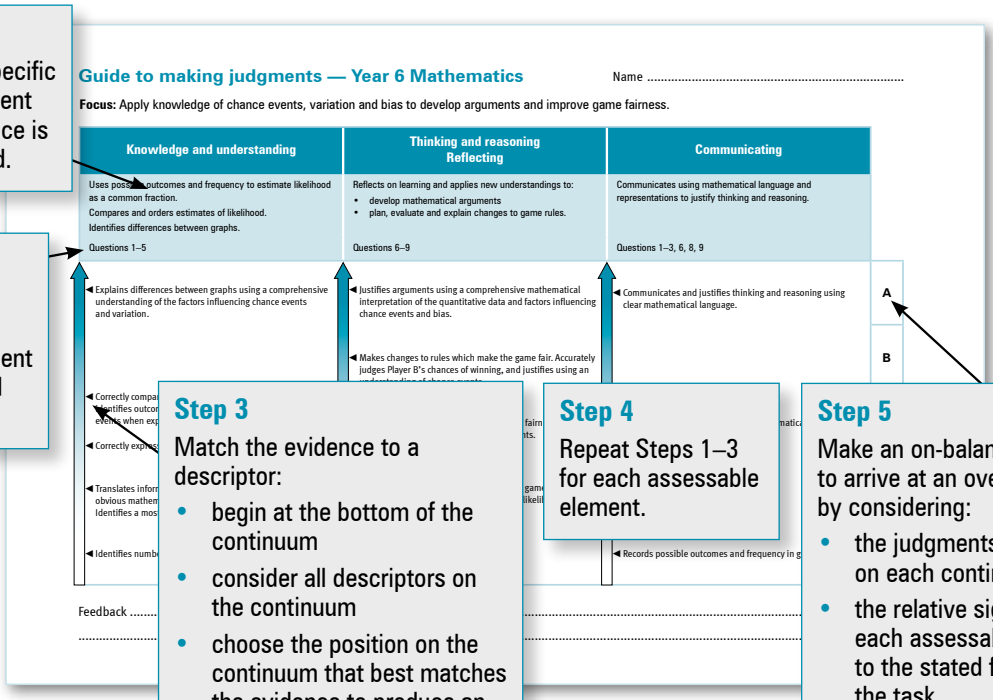
Diagram 2: Using the GTMJ — the judgment process

Step 1

Read the task-specific assessable element for which evidence is being considered.

Step 2

Locate all the evidence for an assessable element in the completed Student booklet.



Step 3

Match the evidence to a descriptor:

- begin at the bottom of the continuum
- consider all descriptors on the continuum
- choose the position on the continuum that best matches the evidence to produce an on-balance judgment for the assessable element
- look at Model and Sample responses for clarification if necessary.

Step 4

Repeat Steps 1–3 for each assessable element.

Step 5

Make an on-balance judgment to arrive at an overall grade by considering:

- the judgments recorded on each continuum
- the relative significance of each assessable element to the stated focus of the task.

Using feedback

Assessment alone will not contribute significantly to improved learning — it is what teachers and students do with the information gathered that makes the difference. Providing quality and useful feedback is a crucial step in using assessment information to support future learning.

Assessment feedback goes beyond a simple mark or grade. Comments on the strengths of students' achievements, and on areas for improvement, provide quality feedback that can be used to inform future teaching and learning. Assessment feedback is most helpful if the specific elements of the knowledge and skills are identified and specific suggestions are provided.

The information gathered from the implementation, marking and moderation of QCATs should feed back into future planning of teaching and learning.

Feedback to help students learn

Quality feedback to a student:

- focuses on their achievement in relation to either the assessable elements with their task-specific descriptors or the Selected Essential Learnings (page 21) and their associated questions
- includes strengths of achievements
- identifies areas for improvement and strategies for future learning
- is communicated in student-friendly language
- is appropriate (e.g. in quantity and detail) to the student's age and their capacity to respond
- includes the use of Sample responses to provide examples of the quality of work corresponding to each standard.

Feedback to help teacher planning

Individual and collective student performance on QCATs, along with other school-based assessment, can be used to inform teaching and learning.

Additional resources

Using feedback to inform teaching and learning
www.qsa.qld.edu.au/3163.html

Sample responses, QSA Assessment Bank
https://qcar.qsa.qld.edu.au/assessmentbank (registration required)

Resources

Selected Essential Learnings

This QCAT will assess what students know, understand and can do in relation to the following selection of Essential Learnings.

Mathematics Essential Learnings by the end of Year 7	
Assessable elements The valued features of the key learning area about which evidence of learning is collected and assessed.	**Ways of working** The processes students use to develop and demonstrate their knowledge and understanding. Students are able to:
Thinking and reasoning	• plan activities and investigations to explore concepts through selected pathways, and plan strategies to solve mathematical questions, problems and issues • select and use suitable mental and written computations, estimations, representations and technologies to generate solutions and to check for reasonableness • develop arguments to justify predictions, inferences, decisions and generalisations from solutions • evaluate thinking and reasoning, to determine whether mathematical ideas, strategies and procedures have been applied effectively
Communicating	• communicate thinking and justify reasoning and generalisations, using mathematical language, representations and technologies
Reflecting	• reflect on learning, apply new understandings and identify future applications.
	Knowledge and understanding The essential concepts, facts and procedures.
Knowledge and understanding	***Number*** **Numbers, key percentages, common and decimal fractions and a range of strategies are used to generate and solve problems.** • Whole numbers, including positive and negative numbers, and common and decimal fractions can be ordered and compared using a number line. ***Chance and data*** **Probability of events can be calculated from experimental data. Data can be summarised and represented to support inferences and conclusions.** • Events have different likelihoods of occurrence and estimates of probability can be expressed as percentages, common fractions or decimal fractions between 0 and 1. • Experimental data for chance events can be compared with theoretical probability. • Sample data drawn from a given population can be summarised, compared and represented in a variety of ways. • Measures of location such as mean, median and mode, and frequency and relative frequency, can be used to explore distributions of sample data. • Variation and possible causes of bias can be identified in data collections.

Source: QSA 2007, www.qsa.qld.edu.au/7296.html

Connection to the Australian Curriculum

This QCAT connects to the following proficiencies and content descriptions of the Australian Curriculum.

The Australian Curriculum: Mathematics		Version 1.2
Proficiency strands		Assessed proficiencies are **highlighted**.
Understanding		Students build a robust knowledge of adaptable and transferable mathematical concepts. They **make connections between related concepts and progressively apply the familiar to develop new ideas.** They develop an understanding of the relationship between the "why" and the "how" of mathematics. Students build understanding when they connect related ideas, when they represent concepts in different ways, when they identify commonalities and differences between aspects of content, when they **describe their thinking mathematically** and when they interpret mathematical information.
Fluency		Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students are fluent when they **calculate answers** efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly use facts, and when they can manipulate expressions and equations to find solutions.
Problem Solving		Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and **communicate solutions** effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and **plan their approaches**, when they apply their existing strategies to seek solutions, and when they **verify that their answers are reasonable**.
Reasoning		Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, **evaluating**, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and **justify strategies used and conclusions reached**, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false and when they compare and contrast related ideas and explain their choices.
Content strands		**Content descriptions**
Number and Algebra	Year 6	***Fractions and decimals*** • Compare fractions with related denominators and locate and represent them on a number line.
Statistics and Probability	Year 6	***Chance*** • Describe probabilities using fractions, decimals and percentages. • Conduct chance experiments with both small and large numbers of trials using appropriate digital technologies. • Compare observed frequencies across experiments with expected frequencies. ***Data representation and interpretation*** • Interpret and compare a range of data displays, including side-by-side column graphs for two categorical variables. • Interpret secondary data presented in digital media and elsewhere.

Source: Australian Curriculum, Assessment and Reporting Authority (ACARA) 2011, www.australiancurriculum.edu.au/Mathematics
Resources: QSA 2011, www.qsa.qld.edu.au/13656.html

Model response

This Model response gives one example of a very high quality response for each question. The Sample responses, available for download from the QSA Assessment Bank, demonstrate the quality of student responses for each standard, A to E.

Collecting and analysing experimental data

Activity

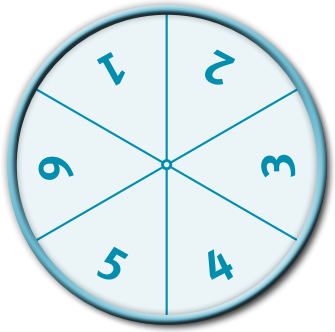
Play "Take a spin" in your pairs again. Complete Question 1 as you play.

Play game

1. **Complete Graph 1 by using a cross (✕) to record the difference for each spin while you play the game.**

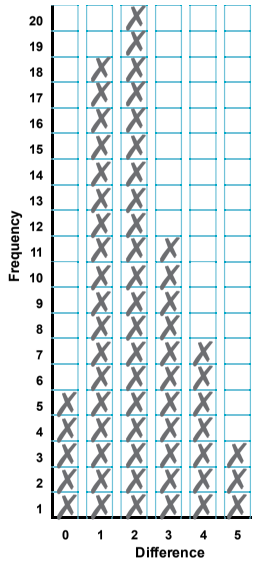
   Stop recording if one of the differences reaches the top of the graph.
   Finish the game you are playing if this happens.

Take a spin



Game	Player A (0, 1 or 2)	Player B (3, 4 or 5)
1	𝍸 𝍸	II
2	𝍸 𝍸	I
3	𝍸	𝍸 𝍸
4	𝍸 𝍸	III
5	𝍸 𝍸	III
6		

Graph 1: Frequency of each difference



Use Graph 1 on page 4 to answer Question 2.

2. a) **Complete the Frequency column in Table 1 below.**

   b) **In the Likelihood column, express the likelihood of spinning each difference as a common fraction.**

   c) **Order the differences from most likely to least likely in Diagram 1.**

Table 1: Likelihood of spinning each difference

Difference	Frequency	Likelihood
0	5	$\frac{5}{64}$
1	18	$\frac{18}{64}$
2	20	$\frac{20}{64}$
3	11	$\frac{11}{64}$
4	7	$\frac{7}{64}$
5	3	$\frac{3}{64}$
Total	64	

Diagram 1: Order of likelihood

Difference	Likelihood
2	most likely
1	
3	
4	
0	
5	least likely

Stop here: Wait for your teacher's directions.

Note to teachers: This is a model response for the data collected in Question 1.

Model response

Note to teachers: This is a model response for the data collected in Question 1.

Exploring outcomes and theoretical likelihood

This grid shows all **possible outcomes** when using two spinners and finding the difference.
Use the grid to answer the questions below.

Player A \ Player B	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

The difference of 4 has 4 possible outcomes.
These can be written as:
(1, 5) (2, 6) (5, 1) (6, 2)

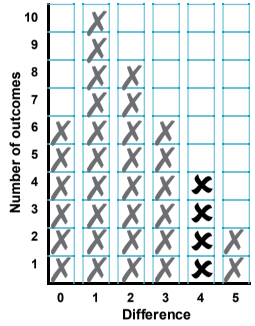
3. a) **How many possible outcomes have a difference of 2?** 8

   b) **Write all the possible outcomes that have a difference of 3.**

   (1, 4) (2, 5) (3, 6) (4, 1) (5, 2) (6, 3)

   c) **Complete Graph 2 by using a cross (✗) to indicate the number of outcomes for each difference.**

Graph 2: Number of outcomes for each difference



The possible outcomes for the difference of 4 have been completed for you.

Look at the shape of Graph 1 on page 4 and Graph 2 on page 6, then complete the sentence below.

4. **The two graphs may not be the same shape because:** Graph 1 shows actual frequencies while playing the game, while Graph 2 shows the theoretical number of outcomes possible. A difference of 1 is what you would expect to see spun the most, but 2 was what came up the most in this small sample. The graphs might look the same shape if I had a much larger sample size.

Use Graph 2 to answer the following.

5. a) **Complete Table 2 below.**

   b) **Order the differences from most likely to least likely in Diagram 2.**

Table 2: Likelihood of spinning each difference

Difference	Number of outcomes	Likelihood
0	6	$\frac{6}{36}$
1	10	$\frac{10}{36}$
2	8	$\frac{8}{36}$
3	6	$\frac{6}{36}$
4	4	$\frac{4}{36}$
5	2	$\frac{2}{36}$
Total	36	

Diagram 2: Order of likelihood

Difference	Likelihood
1	most likely
2	
0, 3	
4	
5	least likely

Stop here: Wait for your teacher's directions.

Model response

It's not fair!

During the group discussion (page 3), you talked about the question:

Is the game "Take a spin" fair or unfair?

6. Explain why the game is not fair.

Use the information you have collected to support your explanation.

If you look at the table on page 7, you can see that the game is not fair because the outcomes that player A can score points on have 24 out of 36 chances of being spun, where as the outcomes that player B can score points on only have 12 out of 36 chances. Player A is twice as likely to win as player B.

Stop here: Wait for your teacher's directions.

Model response

Applying your learning

In a fair game, each player has an equal chance of winning.

7. **Make the game "Take a spin" fair.**
Complete the rules below.

Take a spin

Rules for play

Getting ready

- Both players spin.
- The player who spins the lowest number will be Player A.

Playing the game

- Both players spin.
- When each spinner stops, find the difference between the numbers.
- If the **difference** is 0, 2, 4
  then Player A scores 1 point
- If the **difference** is 1, 3, 5
  then Player B scores 1 point
- Use a tally mark to record each point.

Winning the game

the first person to reach 10 points is the winner.

8. **Explain how your changes to the game make it fair.**

I looked at the table on page 7 and thought about various combinations of differences, because each player had to have an equal chance of winning. I eventually figured out what possible outcomes went together so that each player had an 18 out of 36 chance of winning.

9. **If "Take a spin" is now a fair game and Player A has won 5 out of 5 games, what is the likelihood that Player B will win game 6?**
Use a cross to indicate the likelihood on the line below.

Explain your answer.

If the game is fair, both players have an equal chance of winning. The results of previous games do not affect the outcome of future games. Spinners don't remember what they just got.

Note to teachers: There are at least two other methods of making the game fair.

- The differences each player has and the number of points they win each time remain the same, but Player A has to reach 20 points to win the game, while Player B only has to reach 10 points.
- The differences each player has and the target score remain the same, but Player B scores 2 points each time their differences are spun.

Notes

Guide to making judgments — Year 6 Mathematics

Name .....

Focus: Apply knowledge of chance events, variation and bias to develop arguments and improve game fairness.

Knowledge and understanding	Thinking and reasoning Reflecting	Communicating	
Uses possible outcomes and frequency to estimate likelihood as a common fraction. Compares and orders estimates of likelihood. Identifies differences between graphs. Questions 1–5	Reflects on learning and applies new understandings to: • develop mathematical arguments • plan, evaluate and explain changes to game rules. Questions 6–9	Communicates using mathematical language and representations to justify thinking and reasoning. Questions 1–3, 6, 8, 9	
◀ Explains differences between graphs using a comprehensive understanding of the factors influencing chance events and variation.	◀ Justifies arguments using a comprehensive mathematical interpretation of the quantitative data and factors influencing chance events and bias.	◀ Communicates and justifies thinking and reasoning using clear mathematical language.	**A**
	◀ Makes changes to rules which make the game fair. Accurately judges Player B's chances of winning, and justifies using an understanding of chance events.		**B**
◀ Correctly compares and orders estimates of likelihood, and identifies outcomes with equal likelihoods. Considers chance events when explaining differences between graphs. ◀ Correctly expresses likelihoods as common fractions.	◀ Makes changes to rules which improve the fairness of the game. Considers chance events in arguments.	◀ Communicates using appropriate mathematical language.	**C**
◀ Translates information from graphs into tables. Identifies obvious mathematical differences between graphs. Identifies a most and least likely outcome.	◀ Makes a relevant statement about why the game is unfair or why the rule changes are fair. Identifies likelihood of Player B winning.		**D**
◀ Identifies numbers of outcomes from grid.	◀ Makes a change to the rules.	◀ Records possible outcomes and frequency in graphs.	**E**

Feedback .....

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