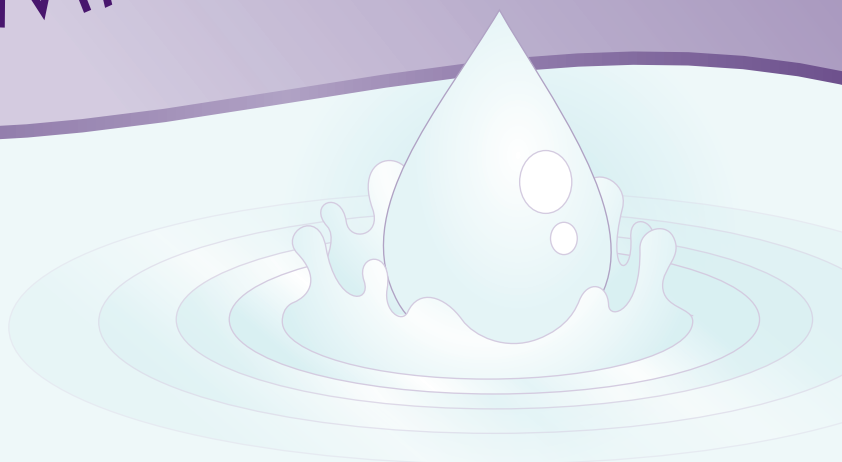


# MATHEMATICS

9

## SAMPLE RESPONSES



### Rainwater

This booklet is designed to help teachers make overall, on-balance judgments by providing examples of student responses. The responses are not an exhaustive set.

C samples

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## Guide to making judgments — Year 9 Mathematics

**Purpose:** To use mathematical reasoning to develop an appropriate local plan for storage and use of rainwater.

Knowledge and understanding	Thinking and reasoning	Thinking and reasoning	Communicating
<p>Selection and manipulation of formulas to calculate lengths, volumes and statistical measures of central tendency.</p> <p>Questions 1, 3, 5, 6, 8, 9</p>	<p>Choice of strategies and procedures to generate solutions.</p> <p>Questions 7, 10</p>	<p>Generalisation and justification of reasoning.</p> <p>Questions 2, 4, 10</p>	<p>Use of mathematical language and representations to communicate thinking and to justify reasoning.</p> <p>Questions 1–10</p>
<p><b>Knowledge and understanding</b></p> <p>Consistently and accurately selected, manipulated and applied formulas, apart from a mechanical error in Question 8.</p>	<p><b>Thinking and reasoning</b></p> <p>Demonstrated some success in planning for rainwater use and choosing a storage tank.</p>	<p><b>Thinking and reasoning</b></p> <p>Made a reasonable generalisation about predictors of rainfall. Rainwater use plan was based on available rainwater, with minimal consideration of saving water from wetter months for use in drier months.</p>	<p><b>Communicating</b></p> <p>Demonstrated clear working in most calculations, with appropriate use of units and rounding. Made accurate use of mathematical symbols but explanations were not always clear.</p>

**Overall grade**  
Although this response demonstrates a very high level of achievement in Knowledge and understanding and a high level of achievement in Communicating, the sound level of achievement in Thinking and reasoning suggests an overall C. This on-balance judgment is made in light of the emphasis on Thinking and reasoning in the purpose of the assessment.

## C sample: Response 1

1. Calculate Birdsville's mean and median April rainfall for the period 1997 to 2006.

Show all working

$$\begin{array}{r}
 0.0 \\
 0.0 \\
 52.8 \\
 2.0 \\
 86.6 \\
 1.0 \\
 0.0 \\
 6.7 \\
 1.0 \\
 + 0.0 \\
 \hline
 150.1 \\
 \\
 \frac{150.1}{10} = 15.01
 \end{array}
 \quad \left. \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array} \right\} 10 \text{ numbers.}$$

Mean April rainfall = 15.01 mm

Show all working

$$\begin{array}{c}
 10 \text{ numbers} \\
 \swarrow \quad \searrow \\
 5 \quad \quad 5
 \end{array}$$

$$0.0, 0.0, 0.0, 0.0, \textcircled{1.0}, \textcircled{1.0}, 2.0, 6.7, 52.8, 86.6$$

Half of 1.0 and 1.0 = 1.0

Median April rainfall = 1.0 mm

2. The median is a more useful predictor of typical monthly rainfall than the mean. Use the April data for Birdsville to explain why.

This would be because the mean is showing the average, but isn't descriptive. This is because there are only 2 numbers equal to or higher than 15.01 out of 10 and there are eight numbers below it. There are 7 numbers which are nearly a tenth of the mean which indicates that the average rainfall is more likely to be the median, not the mean. (The mean is only increased because of 2 unexpectedly high numbers)

## C sample: Response 1

### Measuring rainfall

A rain gauge is used to measure rainfall (usually in millimetres).

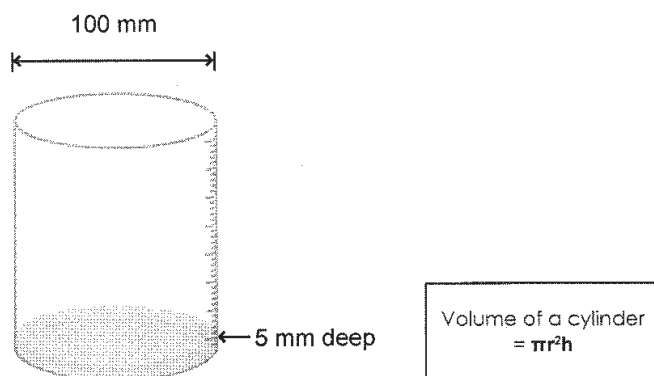
The level in the gauge represents the depth of rainwater that would build up on a flat horizontal surface if it didn't run off or soak in.

Any open container with straight vertical sides can be used as a simple rain gauge. The depth of water collected is a measure of the amount of rain.

Rain gauges A and B both measure the same amount of rainfall, even though A collects a greater volume of water because of its larger opening.



3. If 5 mm of rain falls and is collected in a 100 mm diameter rain gauge, calculate the volume of rainwater in the gauge.



Show all working



$$\pi r^2 = \pi 50^2$$

$$= 7853.98$$

$$\pi r^2 h = 7853.98 \times 5$$

$$= 39269.91$$

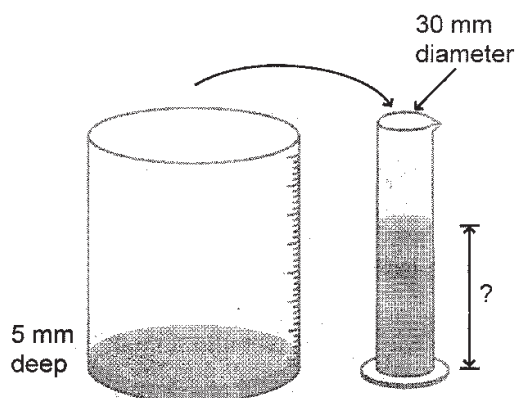
Volume of rainwater = 39269.91 mm<sup>3</sup>

## C sample: Response 1

To improve the accuracy of rainfall measurement, the collected rainwater can be poured into a narrower cylinder.

4. How is the accuracy of rainfall measurements improved by pouring the collected rainwater into a narrower cylinder?

The accuracy of rainfall measured has improved because the narrower gauge is more precise.



5. Calculate the depth of the rainwater collected in Question 3 after it is poured into a narrow, 30 mm diameter cylinder.

Show all working

$$\begin{array}{r} \textcircled{100} \quad \textcircled{30} \\ \text{A} \quad \text{B} \\ \hline \downarrow \\ \text{5mm} \\ \text{Deep} \end{array}$$

$\textcircled{15}$  ← radius

$$\pi r^2 = \pi 15^2$$

$$= 706.86$$


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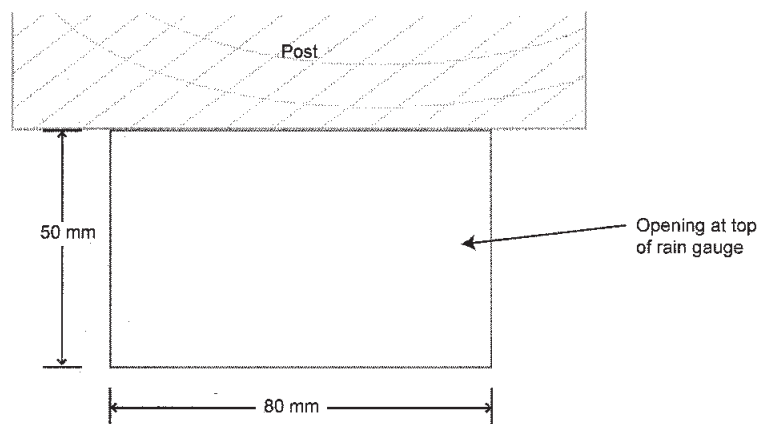

$$\frac{39269.91}{706.86} = 55.56$$

Depth of water in the narrow cylinder = 55.56 mm.

## C sample: Response 1

Not all rain gauges are cylinders.

The diagram below shows the top view of a rain gauge attached to a post.



Not to scale

6. If 5 mm of rain falls, calculate the volume of water collected in the rain gauge above.

Show all working

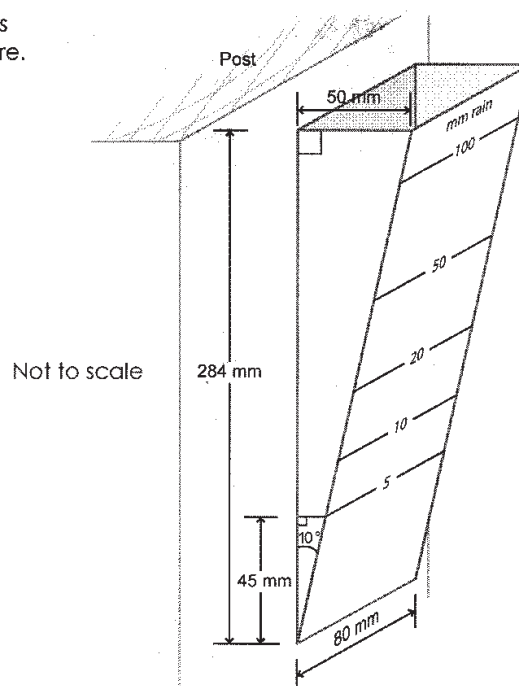
$$\begin{aligned} \text{Size, } L \times H \\ &= 50 \times 80 \\ &= 4000 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of gauge} \times \text{vol of water} &= 4000 \times 5 \\ &= 20000 \text{ mm}^3 \end{aligned}$$

$$\text{Volume of rainwater collected} = 20000 \text{ mm}^3$$

## C sample: Response 1

The rain gauge from page 8 is wedge-shaped, as shown here.



7. Use the volume calculated in Question 6 to check if the 5 mm graduation on the rain gauge is in the correct place.

Show all working

$$\text{Top Area} = 4000 \text{ mm}^2$$

$$\text{Height} = 284 \text{ mm}$$

$$4000 \times 284 = 1,136,000$$

$$1,136,000 \div 2 = 568,000$$

↓  
capacity × 2

$$80 \times 45 = 3600 \text{ mm}^2 \text{ (bottom)}$$

↓  
capacity

$$\frac{3600}{9} = 400$$

$$45 \times 50 = \frac{2250}{9} = 250$$

Tick the correct answer.

The 5 mm graduation is:

☒ in the correct place

☐ too high

☐ too low.

## C sample: Response 1

### Section 2: Planning a rainwater supply

Table 2 below shows typical roof areas for homes of different sizes.

Table 2

Typical roof areas	
Home type	Roof area (m <sup>2</sup> )
2 bedroom home	100
3 bedroom home	150
4 bedroom home	200
5 bedroom home	250

Assume the roof is flat.  
(This makes little difference to the amount of rain collected.)

From the table, choose a home that most closely matches the one you live in (or would like to live in).

My choice of home: 4 Bedroom Home

8. Calculate the amount of rainwater in litres (L) collected by the roof of your chosen home when one millimetre (1 mm) of rain falls.

Show all working

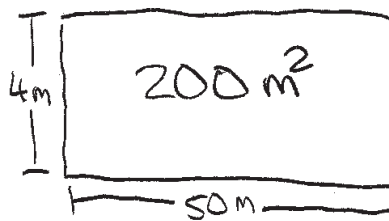
Roof area = 200 m<sup>2</sup>.  
Rainfall = 1 mm.

0.001 m's worth of rain falls.

1 mm = 0.001 m  
1 m<sup>3</sup> holds 1000 L

$200 \times 1000 = 200000$  L of rain can be held.

$\frac{0.2}{0.2} (0.001 \text{ m} \times 200) = 1 \text{ mm}$   
 $0.2 \times 1000 \text{ L} = 20 \text{ L}$



Amount of rainwater collected by the roof when 1 mm of rain falls = 20 L

## C sample: Response 1

Your teacher has supplied you with median monthly rainfall data for:

..... Redlands ..... (location).

- 9 a. Copy the median rainfall values for each month into column A of Table 3.
- b. Using your answer to Question 8, calculate the expected amount of rainwater collected from your chosen roof during each month.  
Write these values in column B of Table 3.

Table 3

Expected rainwater collected		
Month	A Median rainfall (mm)	B Rainwater collected by the roof (L)
Jan	126.2	2524
Feb	127.7	2554
Mar	145	2900
Apr	73.1	1462
May	105.5	2110
Jun	52.4	1048
Jul	52	1040
Aug	33.7	674
Sep	25.4	508
Oct	74.1	1482
Nov	102	2040
Dec	118.7	2374.

## C sample: Response 1

10. Explain how you could make the best use of the collected rainwater (Table 3) to reduce your need to use water from the local supply.

In your answer:

- ☐ record the number of people living in the house
- ☐ work out approximately how much water your household uses each month (refer to Table 4 below)
- ☐ compare the expected amount of rainwater collected (Table 3) to your monthly usage
- ☐ explain how you could use the collected rainwater to supply some or all of your needs
- ☐ select an appropriately sized tank to store rainwater for use in drier months (refer to Table 5 below)
- ☐ justify all your reasoning and show all calculations.

Answer on the following pages.  
Check the box next to each point above as you complete it.

Table 4

Careful water use (using water wisely)	
Use	Litres per person per month Household
Drinking, cooking	$300 \times 3 = 900\text{ L}$
Dish washing	$150 \times 3 = 450\text{ L}$
Bathroom and toilet	$2500 \times 3 = 7500\text{ L}$
Washing clothes	$200 \times 3 = 600\text{ L}$
A garden hose uses about 700 litres per hour	TOTAL $9450\text{ L}$ per month

Table 5

Rainwater tank sizes and prices										
Tank capacity (litres)	500	1000	1500	2000	2500	3000	5000	10000	20000	50000
Installed cost	\$500	\$700	\$750	\$800	\$1000	\$1100	\$1400	\$2000	\$3000	\$7000

**C sample: Response 1**

The approximate monthly water usage for my household is 9450l per month. I could make the best use of rainwater falling on my roof by buying a 3000 litre capacity tank. I chose a 3000l tank because in my area the most rain we expect to get is 2900l in March, which is just under 3000l. My house's approximate water usage is 9450l, so that can be cut down to around 6500l which is nearly  $\frac{2}{3}$  of my normal usage. The tank is at a cost of \$1100 but in the long term it can save me a lot more than \$1100. The water can be stored for drier months between June and October because January, February and March has heavy rainfall. Without using all the rainwater at once, I can use it wisely by only using a bit each month, to keep the tank constantly filled with water even in the drier months. This can be achieved by using 1000l each month unless the tank is overflowing.

Have you checked all the boxes in Question 10?  
Continue your answer over the page if necessary.

**C sample: Response 1**

Table 3 shows that during June – September only around 500–1000L of rain falls, which would be enough to keep the tank constantly full if I only use 1000 a month, (with the exception of wet months when more water can be used). This is how I can save water by using rainwater and contribute to <sup>shire</sup> SE QCD's drought.

Have you checked all the boxes in Question 10?  
Continue your answer over the page if necessary.

# C sample: Response 2

**Overall grade**  
As this response demonstrates a sound level of Knowledge and understanding, Thinking and reasoning and Communicating, it is judged to be an overall C.

## Guide to making judgments — Year 9 Mathematics Student

**Purpose:** To use mathematical reasoning to develop an appropriate local plan for storage and use of rainwater.

Knowledge and understanding	Thinking and reasoning	Thinking and reasoning	Communicating
Selection and manipulation of formulas to calculate lengths, volumes and statistical measures of central tendency. Questions 1, 3, 5, 6, 8, 9	Choice of strategies and procedures to generate solutions. Questions 7, 10	Generalisation and justification of reasoning. Questions 2, 4, 10	Use of mathematical language and representations to communicate thinking and to justify reasoning. Questions 1–10

← Demonstrates consistent success in selecting, manipulating and applying appropriate formulas.

← Demonstrates proficiency in selecting and applying appropriate formulas.

← Successfully applies a multi-step procedure to check the rain gauge graduation. Chooses a tank size which accurately meets the needs of the rainwater-use plan.

← Demonstrates significant progress in applying a multi-step procedure to check the rain gauge graduation. Determines a realistic plan for rainwater use.

← Chooses an appropriate strategy to plan rainwater use and storage, with some success. Links choice of tank size to rainwater-use plan.

← Justifies choice of tank size with an accurate analysis of water use and rainfall availability.

← Makes perceptive generalisations about accuracy of rainfall measurements and predictors of rainfall.

← Bases rainwater-use plan on a well-reasoned comparison of water needs and available rainwater. Justifies choice of tank size with a partial analysis of water use and rainwater availability.

← Makes a plausible generalisation about accuracy of rainfall measurements or predictors of rainfall. Bases rainwater-use plan on a comparison of water needs and available rainwater.

← Clearly communicates procedures, reasoning and justification using mathematical representations, language, working, rounding and units.

← Shows working and units in calculations. Communication of explanations and justifications is variable.

### Knowledge and understanding

Demonstrated proficiency in successfully selecting and applying formulas, apart from conflicting working and an error in Question 1.

### Thinking and reasoning

Demonstrated some success in planning for rainwater use.

### Thinking and reasoning

Gave a partial explanation about predictors of rainfall. Based rainwater use plan on available rainwater, with no consideration of saving water from wetter months.

### Communicating

Showed working and units in most calculations, with an appropriate level of rounding. Correctly used mathematical symbols but explanations were not always clear.

## C sample: Response 2

1. Calculate Birdsville's mean and median April rainfall for the period 1997 to 2006.

Show all working

$$0 + 0 + 52.8 + 2 + 86.6 + 1 + 0 + 6.7 +$$

$$1 + 0 = 120.1$$

$$120.1 \div 10 = 12.01$$

Show all working

$$0, 0, 0, 0, 1, 2, 6.7, 52.8, 86.6,$$

$$\frac{86.6 + 1}{2} = 43.8$$

$$11 \div 2 = 1$$

Mean April rainfall = 12.01mm

Median April rainfall = 1mm

2. The median is a more useful predictor of typical monthly rainfall than the mean. Use the April data for Birdsville to explain why.

Because it doesn't make a difference if there are higher numbers it's just the middle of all the figures.

## C sample: Response 2

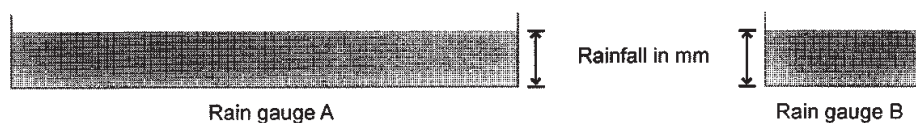
### Measuring rainfall

A rain gauge is used to measure rainfall (usually in millimetres).

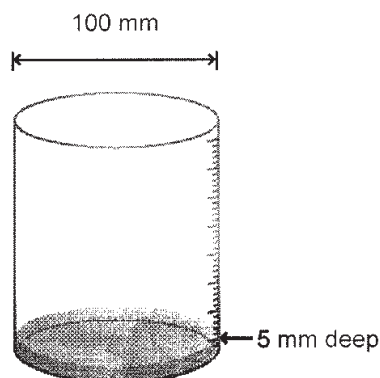
The level in the gauge represents the depth of rainwater that would build up on a flat horizontal surface if it didn't run off or soak in.

Any open container with straight vertical sides can be used as a simple rain gauge. The depth of water collected is a measure of the amount of rain.

Rain gauges A and B both measure the same amount of rainfall, even though A collects a greater volume of water because of its larger opening.



3. If 5 mm of rain falls and is collected in a 100 mm diameter rain gauge, calculate the volume of rainwater in the gauge.



Volume of a cylinder  
=  $\pi r^2 h$

Show all working

~~$\pi \times 50^2 \times 5 = 39269.9 \text{ mm}^3 \text{ or } 39269.9 \text{ mm}^3$~~

$$\begin{aligned} V &= \pi r^2 h \\ V &= \pi \times 50^2 \times 5 \\ V &= 39269.9 \text{ mm}^3 \end{aligned}$$

~~$$\begin{aligned} V &= \pi r^2 h \\ V &= \pi \times 50^2 \times 5 \\ V &= \pi (50^3) \times 5 \end{aligned}$$~~

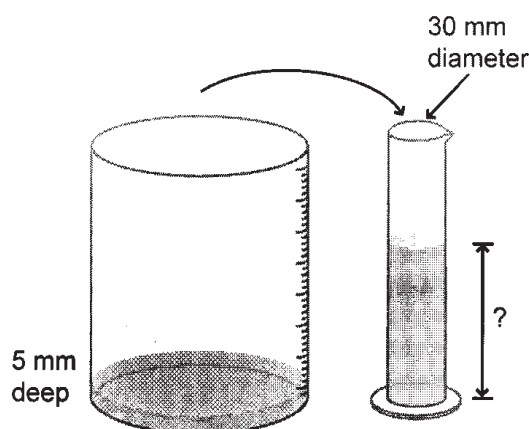
Volume of rainwater = ~~39269.9~~ <sup>36269.9</sup> .....  $\text{mm}^3$ .

## C sample: Response 2

To improve the accuracy of rainfall measurement, the collected rainwater can be poured into a narrower cylinder.

4. How is the accuracy of rainfall measurements improved by pouring the collected rainwater into a narrower cylinder?

~~Not very~~ as it is just an estimate. It may  
of been slightly more accurate because you  
are able to get clearer measurement.



5. Calculate the depth of the rainwater collected in Question 3 after it is poured into a narrow, 30 mm diameter cylinder.

Show all working

~~1. 39269.0 mm<sup>3</sup>~~  
~~2. 39269.0 mm<sup>3</sup> ÷ (1.5 × 100)~~  
~~1.5 × 100 = 150~~  
~~39269.0 ÷ 150 = 261.79~~  
~~2. 39269.0 mm<sup>3</sup> = 261.79 × 150 = 39268.5~~

$$\begin{aligned} 5 \div 100 &= 0.05 \\ 30 \div 20 &= 1.5 \\ 1.5 \times 100 &= 150 \text{ mm} \end{aligned}$$

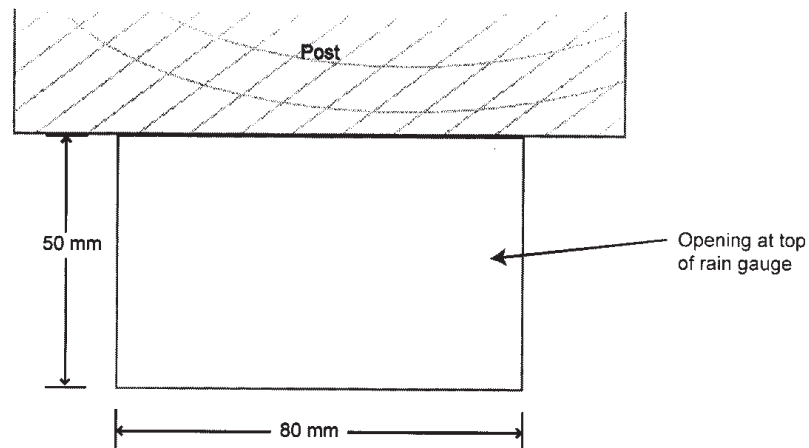
150 mm  
~~150.41~~  
~~150.41~~

Depth of water in the narrow cylinder = ..... mm.

## C sample: Response 2

Not all rain gauges are cylinders.

The diagram below shows the top view of a rain gauge attached to a post.



Not to scale

6. If 5 mm of rain falls, calculate the volume of water collected in the rain gauge above.

Show all working

$$V = L \times W \times H$$

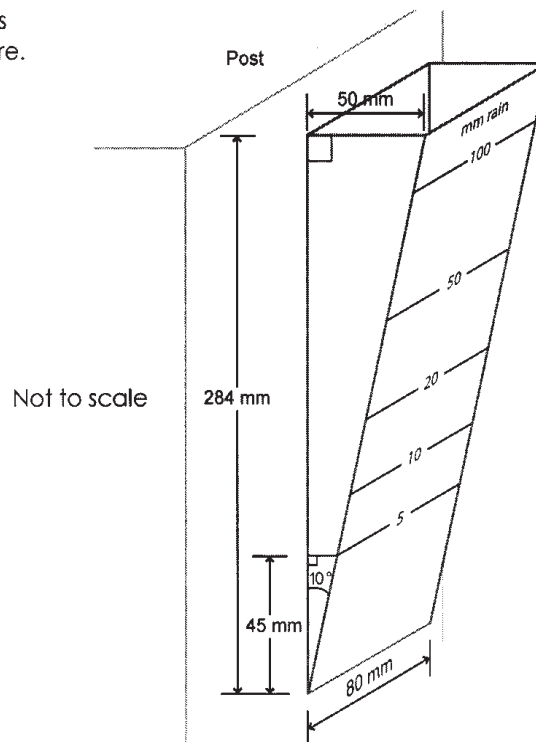
$$V = 80 \times 50 \times 5$$

$$V = 20000$$

Volume of rainwater collected = .....20000..... mm<sup>3</sup>.

## C sample: Response 2

The rain gauge from page 8 is wedge-shaped, as shown here.



7. Use the volume calculated in Question 6 to check if the 5 mm graduation on the rain gauge is in the correct place.

Show all working

~~$V = \frac{1}{2} \times W \times H$~~   $V = 20000$

~~$V =$~~   $V = \frac{L \times W \times H}{2}$

$V = \frac{80 \times 45 \times 50}{2}$

$V = 90000$

Tick the correct answer.

The 5 mm graduation is:

- ☐ in the correct place
- ☒ too high
- ☐ too low.

## C sample: Response 2

### Section 2: Planning a rainwater supply

Table 2 below shows typical roof areas for homes of different sizes.

Table 2

Typical roof areas	
Home type	Roof area (m <sup>2</sup> )
2 bedroom home	100
3 bedroom home	150
4 bedroom home	200
5 bedroom home	250

Assume the roof is flat.  
(This makes little difference to the amount of rain collected.)

From the table, choose a home that most closely matches the one you live in (or would like to live in).

My choice of home: ...<sup>4</sup>... bed room.....

8. Calculate the amount of rainwater in litres (L) collected by the roof of your chosen home when one millimetre (1 mm) of rain falls.

Show all working

$$200 \text{ m}^2 \approx 10 \times 20$$

$$\cancel{200 \text{ m}^2} \times 10 \times 20 \times \frac{0.001}{1000} = 0.2 \text{ m}^3$$

$$\cancel{200 \text{ m}^2}$$

$$200 \times 1000 = 200000 \text{ L}$$

$$0.2 \times 1000 = 200 \text{ L}$$

1 mm = 0.001 m  
1 m<sup>3</sup> holds 1000 L

Amount of rainwater collected by the roof when 1 mm of rain falls .....<sup>200</sup>..... L

## C sample: Response 2

Your teacher has supplied you with median monthly rainfall data for:

Redlands (location).

- 9 a. Copy the median rainfall values for each month into column A of Table 3.
- b. Using your answer to Question 8, calculate the expected amount of rainwater collected from your chosen roof during each month. Write these values in column B of Table 3.

Table 3

Expected rainwater collected		
Month	A Median rainfall (mm)	B Rainwater collected by the roof (L)
Jan	126.2	<del>11620L</del> 25290L
Feb	127.7	25540L
Mar	145	29000L
Apr	73.1	14620L
May	105.5	21100L
Jun	52.4	10480L
Jul	52	10400L
Aug	33.7	6740L
Sep	25.4	5080L
Oct	74.1	14820L
Nov	102	20200L
Dec	118.7	23740 /

## C sample: Response 2

~~$$1) 300 + 150 + 2500 + 200 + 10000 + 12600 + 12000$$

$$= 15150 \times 4 \text{ people}$$~~

$$1) 300 + 150 + 2500 + 200$$

$$= 3150 \div 4 \text{ people}$$

$$= 12600 + 12000 \text{ (hose)}$$

$$= 24600 \text{ total house usage per month}$$

$$2) \frac{10000}{24600} \text{ tank needed}$$

3) Use tank water ~~the~~ instead of using a garden hose only water plants one day a week and wash car from a bucket

4) tank full ~~at~~ every month but August and September  
 24600 use the tank water only for hosing in these months every other months use it will try to not use it all only about 80% incase of emergency

Have you checked all the boxes in Question 10?  
 Continue your answer over the page if necessary.