QCATs
Queensland Comparable Assessment Tasks

2009

9

MATHEMATICS

SAMPLE RESPONSES

Rainwater

This booklet is designed to help teachers make overall, on-balance judgments by providing examples of student responses. The responses are not an exhaustive set.

A samples

Queensland Government

Queensland Studies Authority
Partnership and innovation

Contact information:

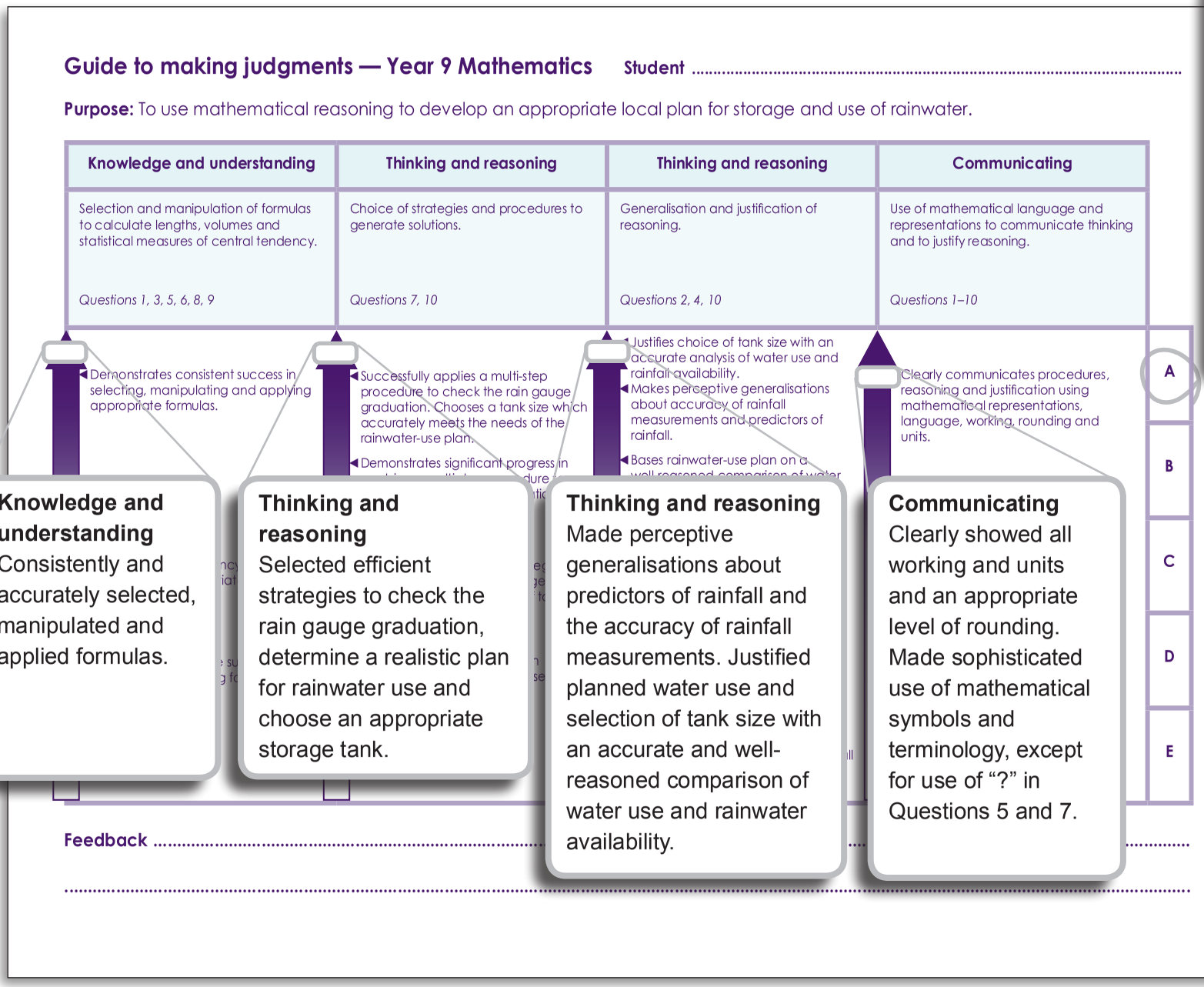
Information about QCATs is available on the QSA website <www.qsa.qld.edu.au>.

Direct questions concerning implementation or receipt of materials to:

Project Officer (Operations)
Phone: 07 3864 0299
Email: QCARadmin@qsa.qld.edu.au

Queensland Studies Authority Ground floor, 295 Ann Street Brisbane. PO Box 307 Spring Hill Qld 4004.
Phone: (07) 3864 0299 Fax: (07) 3221 2553 Email: office@qsa.qld.edu.au Website: www.qsa.qld.edu.au

A sample: Response 1



Guide to making judgments — Year 9 Mathematics **Student** .....

Purpose: To use mathematical reasoning to develop an appropriate local plan for storage and use of rainwater.

Knowledge and understanding	Thinking and reasoning	Thinking and reasoning	Communicating
Selection and manipulation of formulas to calculate lengths, volumes and statistical measures of central tendency.	Choice of strategies and procedures to generate solutions.	Generalisation and justification of reasoning.	Use of mathematical language and representations to communicate thinking and to justify reasoning.
Questions 1, 3, 5, 6, 8, 9	*Questions 7, 10*	*Questions 2, 4, 10*	*Questions 1–10*

Demonstrates consistent success in selecting, manipulating and applying appropriate formulas.

Successfully applies a multi-step procedure to check the rain gauge graduation. Chooses a tank size which accurately meets the needs of the rainwater-use plan.

Demonstrates significant progress in

Justifies choice of tank size with an accurate analysis of water use and rainfall availability.

Makes perceptive generalisations about accuracy of rainfall measurements and predictors of rainfall.

Bases rainwater-use plan on a

Clearly communicates procedures, reasoning and justification using mathematical representations, language, working, rounding and units.

A

B

C

D

E

Feedback .....

.....

Knowledge and understanding
Consistently and accurately selected, manipulated and applied formulas.

Thinking and reasoning
Selected efficient strategies to check the rain gauge graduation, determine a realistic plan for rainwater use and choose an appropriate storage tank.

Thinking and reasoning
Made perceptive generalisations about predictors of rainfall and the accuracy of rainfall measurements. Justified planned water use and selection of tank size with an accurate and well-reasoned comparison of water use and rainwater availability.

Communicating
Clearly showed all working and units and an appropriate level of rounding. Made sophisticated use of mathematical symbols and terminology, except for use of "?" in Questions 5 and 7.

Overall grade
As this response demonstrates a very high level of achievement in Knowledge and understanding, Thinking and reasoning and Communicating, it is judged to be an overall A.

A sample: Response 1

1. Calculate Birdsville's mean and median April rainfall for the period 1997 to 2006.

Show all working

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{0.0 + 52.8 + 2.0 + 86.4 + 1.0 + 0.0 + 6.7 + 1.0 + 0.0 + 0.0}{10}$$

$$= \frac{150.1}{10}$$

Show all working

$\left(\frac{n+1}{2}\right)$th score = median

$\frac{(10+1)}{2}$th score = median

$\frac{11}{2}$ " " = "

5.5th score = median

1 2 3 4 5 6
0.0, 0.0, 0.0, 0.0, 1.0, 1.0, ...

Mean April rainfall = 15.01 mm

Median April rainfall = 1.0 mm

2. The median is a more useful predictor of typical monthly rainfall than the mean. Use the April data for Birdsville to explain why.

As seen in the data from Birdsville the median is a more useful predictor of rainfall as it ~~takes~~ is very close to all the typical levels of rainfall. If the mean was used, the prediction would be inaccurate as the results are affected by the unusually high recordings of 52.8 in 1998 and 86.4 in 2000.

A sample: Response 1

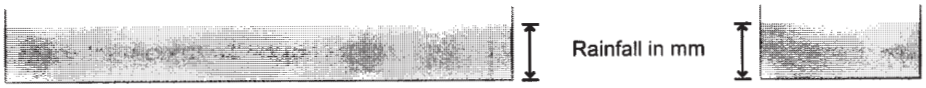
Measuring rainfall

A rain gauge is used to measure rainfall (usually in millimetres).

The level in the gauge represents the depth of rainwater that would build up on a flat horizontal surface if it didn't run off or soak in.

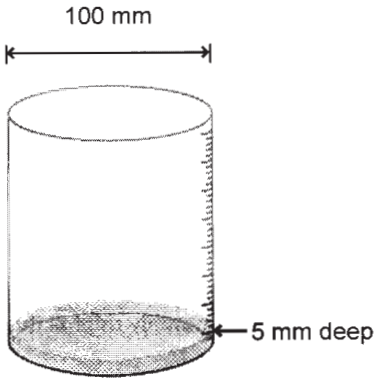
Any open container with straight vertical sides can be used as a simple rain gauge. The depth of water collected is a measure of the amount of rain.

Rain gauges A and B both measure the same amount of rainfall, even though A collects a greater volume of water because of its larger opening.



Rain gauge A Rain gauge B

3. If 5 mm of rain falls and is collected in a 100 mm diameter rain gauge, calculate the volume of rainwater in the gauge.



Volume of a cylinder
$= \pi r^2 h$

Show all working

$V = \pi r^2 \times h$

$= \pi \times (50\text{mm})^2 \times 5\text{mm}$

$= \pi \times 2500 \times 5 \text{ mm}^3$

$= \pi \times 12500 \text{ mm}^3$

$= 39269.91$ (2 dp)

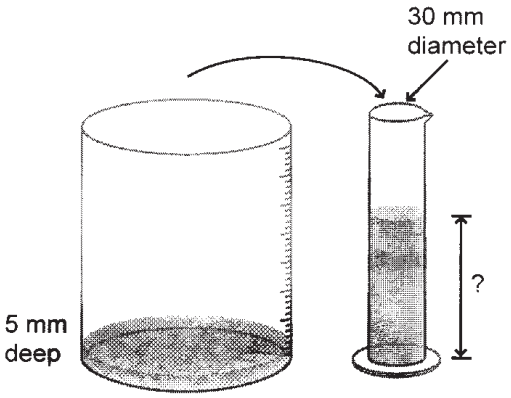
Volume of rainwater = ~~391~~ 39269.91 mm^3

A sample: Response 1

To improve the accuracy of rainfall measurement, the collected rainwater can be poured into a narrower cylinder.

4. How is the accuracy of rainfall measurements improved by pouring the collected rainwater into a narrower cylinder?

As the base gets narrower, the measurements for height become larger and clearer, thus allowing a more accurate reading.



5. Calculate the depth of the rainwater collected in Question 3 after it is poured into a narrow, 30 mm diameter cylinder.

Show all working

$39269.91 = \pi r^2 \times ?$

$= \pi \times (15^2) \times ?$

$= \pi \times 225 \times ?$

$= 706.86 \text{ (2dp)} \times ?$

$= ?$

$= 55.55 \text{ (2dp)}$

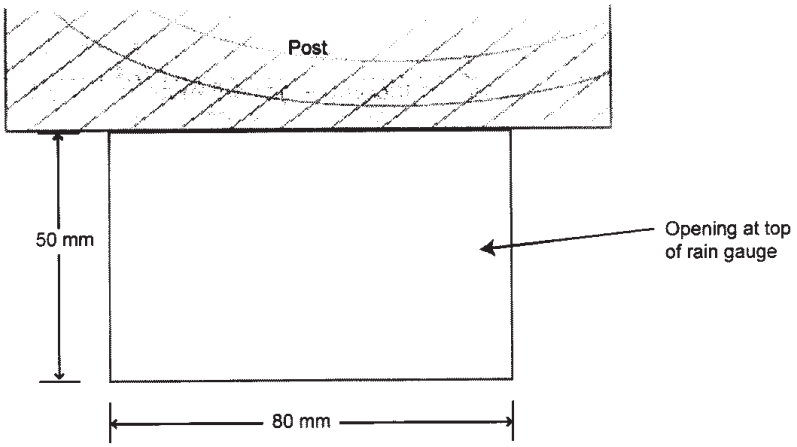
$39269.91 \div 706.86$

Depth of water in the narrow cylinder = 55.55 mm

A sample: Response 1

Not all rain gauges are cylinders.

The diagram below shows the top view of a rain gauge attached to a post.



Not to scale

6. If 5 mm of rain falls, calculate the volume of water collected in the rain gauge above.

Show all working

V = Area of ▭ × amount of rain

$= bh \times 5\text{mm}$

$= (50 \times 80) \times 5\text{mm}$

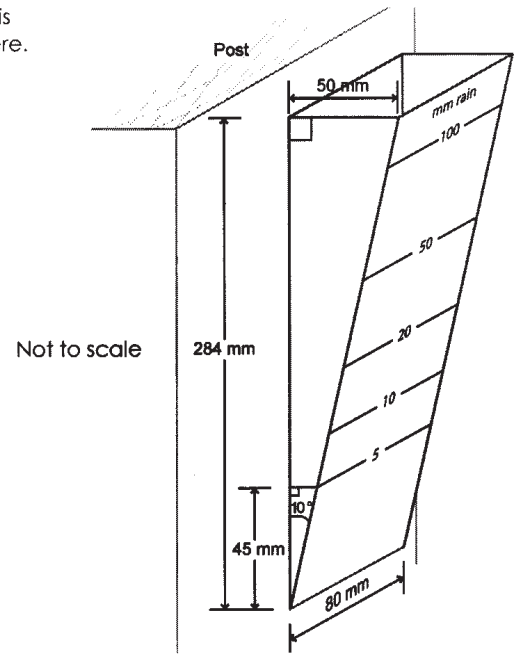
$= 4000 \times 5$

$= 20\,000\ \text{mm}^2$

Volume of rainwater collected = 20000 mm^3

A sample: Response 1

The rain gauge from page 8 is wedge-shaped, as shown here.



7. Use the volume calculated in Question 6 to check if the 5 mm graduation on the rain gauge is in the correct place.

Show all working

SOH/CAH/TOA

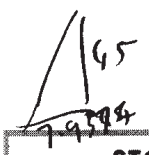
? 10° 45mm

tan 10° = opp/adj

= ?/45

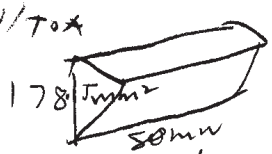
∴ tan 10° × 45 = ?/45 × 45

∴ ? = 7.9347 (4dp)



Area = (B+l)/2

= 7.9347 × 45 / 2 = 178.5307 (4dp) mm²



V = Area × Length

= 178.5307 × 80

= 14282.46 mm³

14282.46 < 20000

∴ graduation is too low

Tick the correct answer.

The 5 mm graduation is:

☐ in the correct place

☐ too high

☒ too low.

STOP HERE: WAIT FOR YOUR TEACHER'S DIRECTIONS

A sample: Response 1

Section 2: Planning a rainwater supply

Table 2 below shows typical roof areas for homes of different sizes.

Table 2

Typical roof areas	
Home type	**Roof area (m^2)**
2 bedroom home	100
3 bedroom home	150
4 bedroom home	200
5 bedroom home	250

Assume the roof is flat.

(This makes little difference to the amount of rain collected.)

From the table, choose a home that most closely matches the one you live in (or would like to live in).

My choice of home: 2 Bedroom

8. Calculate the amount of rainwater in litres (L) collected by the roof of your chosen home when one millimetre (1 mm) of rain falls.

Show all working

1 mm = 0.001 m
1 m^3 holds 1000 L

$V = Area \times rainfall$

$= 100 \times 0.001$

$= 0.1 m^3$

Amount of rainwater collected by the roof when 1 mm of rain falls = 100 L

A sample: Response 1

Your teacher has supplied you with median monthly rainfall data for:

Brisbane (location).

9 a. **Copy the median rainfall values for each month into column A of Table 3.**

b. **Using your answer to Question 8, calculate the expected amount of rainwater collected from your chosen roof during each month. Write these values in column B of Table 3.**

Table 3

Expected rainwater collected		
Month	**A** Median rainfall (mm)	**B** Rainwater collected by the roof (L)
Jan	116.7	11670 L
Feb	111.3	11130 L
Mar	92	9200 L
Apr	66	6600 L
May	71.4	7140 L
Jun	37.1	3710 L
Jul	40	4000 L
Aug	36	3600 L
Sep	26	2600 L
Oct	79.8	7960 L
Nov	102.2	10220 L
Dec	99.8	9980 L

A sample: Response 1

10. Explain how you could make the best use of the collected rainwater (Table 3) to reduce your need to use water from the local supply.

In your answer:

- ☑ record the number of people living in the house
- ☑ work out approximately how much water your household uses each month (refer to Table 4 below)
- ☑ compare the expected amount of rainwater collected (Table 3) to your monthly usage
- ☑ explain how you could use the collected rainwater to supply some or all of your needs
- ☑ select an appropriately sized tank to store rainwater for use in drier months (refer to Table 5 below)
- ☑ justify all your reasoning and show all calculations.

Answer on the following pages.
Check the box next to each point above as you complete it.

Table 4

Careful water use (using water wisely)	
Use	**Litres *per person* per month**
Drinking, cooking	**300**
Dish washing	**150**
Bathroom and toilet	**2500**
Washing clothes	**200**
A garden hose uses about 700 litres ***per hour***	

Table 5

Rainwater tank sizes and prices										
Tank capacity (litres)	500	1000	1500	2000	2500	3000	5000	10000	20000	50000
Installed cost	$500	$700	$750	$800	$1000	$1100	$1400	$2000	$3000	$7000

A sample: Response 1

① Approximate water usage (monthly, 2 person)

Water usage = 2 (300 + 150 + 2500 + 200)

= 2 (3150)

= 6300 for 2 people / month

Also 3 hours of using the garden hose in months that received less than 70mm of rain per month. This raises the water usage for those months to 8400 Litres.

I have done this as months that receive more than 70 mm per month can sustain the garden by itself.

② Rainfall (L) Av. Monthly usage (L) Leftover water (L)

	Rainfall (L)	Av. Monthly usage (L)	Leftover water (L)
Jan	11670	6300	5370
Feb	11130	6300	4830
Mar	9200	6300	2900
Apr	6600	8400	−1800
May	7140	6300	840
Jun	3710	8400	−4690
Jul	4000	8400	−4400
Aug	3400	8400	−5000
Sept	2600	8400	−5800
Oct	7960	6300	1660
Nov	10220	6300	3920
Dec	9980	6300	3680

Leftover in wet months = 23200L

Needed in dry months = 21690L

Left over = 1510L

Have you checked all the boxes in Question 10?
Continue your answer over the page if necessary.

A sample: Response 1

To account for the water demands in low rainfall months and to achieve minimum financial outlays, I will need to install a tank that can hold about 21690 litres

I would install a 20000L for $3000. (I could spend another $800 on a 20000L tank as well but it is not good value – I will need to be extra careful in September)

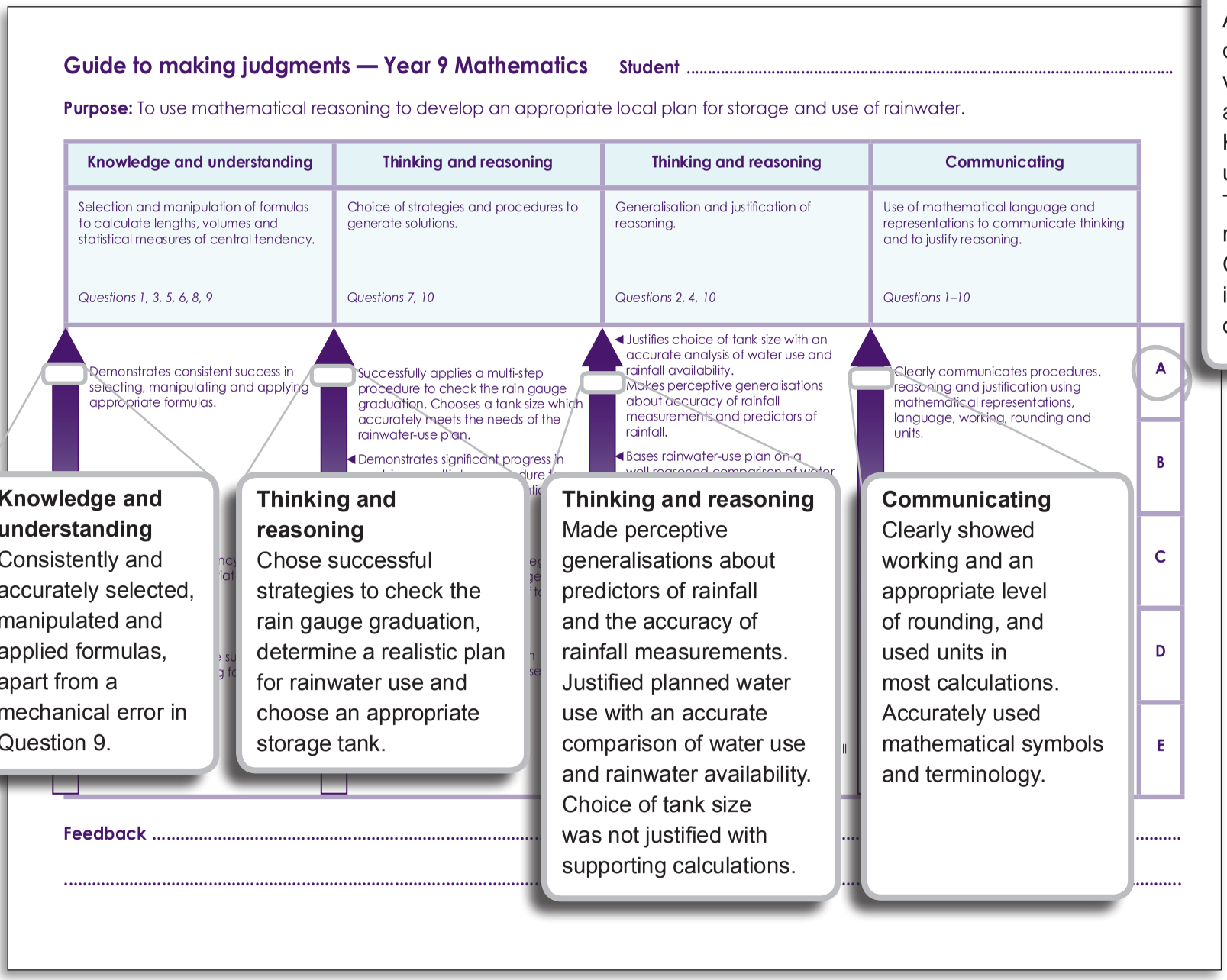
As my roof is collecting more water than I need, I can use my rain water for drinking, cooking, dishwashing, bathroom and toilet, washing clothes and watering the garden in dry months.

I can do this because I have chosen an appropriate sized rainwater tank to store enough water for the dry months.

I ~~will~~ should not need to use council water unless it is a very dry year, but I may sometimes be a bit short of rainwater in September.

Have you checked all the boxes in Question 10?
Continue your answer over the page if necessary.

A sample: Response 2



Guide to making judgments — Year 9 Mathematics **Student** .....

Purpose: To use mathematical reasoning to develop an appropriate local plan for storage and use of rainwater.

Knowledge and understanding	Thinking and reasoning	Thinking and reasoning	Communicating
Selection and manipulation of formulas to calculate lengths, volumes and statistical measures of central tendency.	Choice of strategies and procedures to generate solutions.	Generalisation and justification of reasoning.	Use of mathematical language and representations to communicate thinking and to justify reasoning.
Questions 1, 3, 5, 6, 8, 9	*Questions 7, 10*	*Questions 2, 4, 10*	*Questions 1–10*

Demonstrates consistent success in selecting, manipulating and applying appropriate formulas.

Successfully applies a multi-step procedure to check the rain gauge graduation. Chooses a tank size which accurately meets the needs of the rainwater-use plan.

◀ Demonstrates significant progress in

◀ Justifies choice of tank size with an accurate analysis of water use and rainfall availability.

Makes perceptive generalisations about accuracy of rainfall measurements and predictors of rainfall.

◀ Bases rainwater-use plan on a

Clearly communicates procedures, reasoning and justification using mathematical representations, language, working, rounding and units.

A

B

C

D

E

Feedback .....

.....

Knowledge and understanding
Consistently and accurately selected, manipulated and applied formulas, apart from a mechanical error in Question 9.

Thinking and reasoning
Chose successful strategies to check the rain gauge graduation, determine a realistic plan for rainwater use and choose an appropriate storage tank.

Thinking and reasoning
Made perceptive generalisations about predictors of rainfall and the accuracy of rainfall measurements. Justified planned water use with an accurate comparison of water use and rainwater availability. Choice of tank size was not justified with supporting calculations.

Communicating
Clearly showed working and an appropriate level of rounding, and used units in most calculations. Accurately used mathematical symbols and terminology.

Overall grade
As this response demonstrates a very high level of achievement in Knowledge and understanding, Thinking and reasoning and Communicating, it is judged to be an overall A.

A sample: Response 2

1. **Calculate Birdsville's mean and median April rainfall for the period 1997 to 2006.**

Show all working

mean = sum of scores / number of scores

= 150.10 / 10

= 15.01

Show all working

median = middle number

= 0.6, 0.6, 0.6, 0.6, 1.0, 1.0, 2.6, 6.4, 5.8, 86

= (1.0 + 1.0) / 2

= 1.0

Mean April rainfall = 15.01 Median April rainfall = 1.0 mm

2. **The median is a more useful predictor of typical monthly rainfall than the mean. Use the April data for Birdsville to explain why.**

This is because towns who receive unpredictable rainfall may receive very heavy rain one year and very little in most years. This makes the mean much bigger than most year's rainfall. The median is close to most years rainfall, so is a better prediction

A sample: Response 2

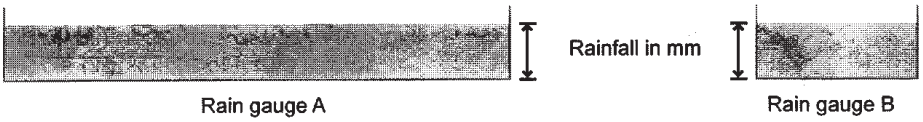
Measuring rainfall

A rain gauge is used to measure rainfall (usually in millimetres).

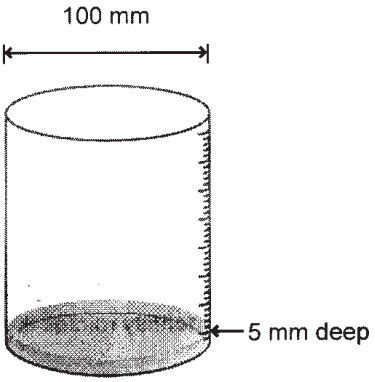
The level in the gauge represents the depth of rainwater that would build up on a flat horizontal surface if it didn't run off or soak in.

Any open container with straight vertical sides can be used as a simple rain gauge. The depth of water collected is a measure of the amount of rain.

Rain gauges A and B both measure the same amount of rainfall, even though A collects a greater volume of water because of its larger opening.



3. If 5 mm of rain falls and is collected in a 100 mm diameter rain gauge, calculate the volume of rainwater in the gauge.



Volume of a cylinder
$= \pi r^2 h$

Show all working

$V = \pi r^2 h$

$= \pi \times 50^2 \times 5$

$= 39269.9$

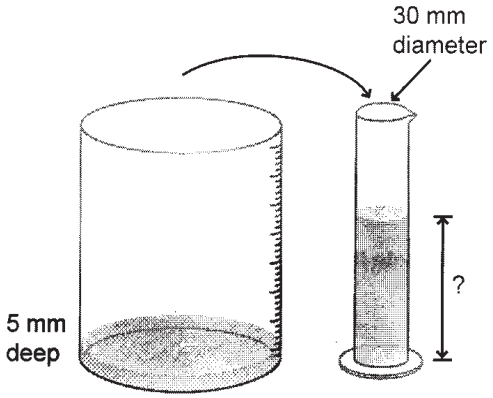
Volume of rainwater = 39269.9 mm^3

A sample: Response 2

To improve the accuracy of rainfall measurement, the collected rainwater can be poured into a narrower cylinder.

4. How is the accuracy of rainfall measurements improved by pouring the collected rainwater into a narrower cylinder?

Though the wider cylinder holds more water, the narrower one is easier to read because the mm marks are further apart.



5. Calculate the depth of the rainwater collected in Question 3 after it is poured into a narrow, 30 mm diameter cylinder.

Show all working

$V = \pi r^2 h$

$39269.9 = \pi \times 15\,mm^2 \times h$

$= 706.86 \times h$

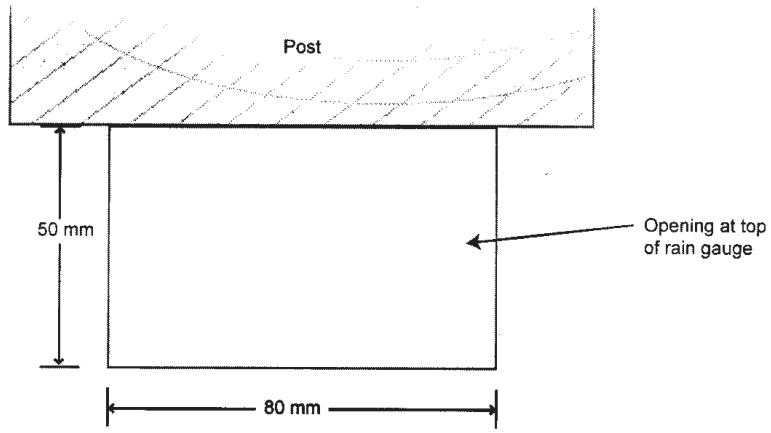
$\frac{39269.9}{706.86} = h = 55.55$

Depth of water in the narrow cylinder = 55.6 mm

A sample: Response 2

Not all rain gauges are cylinders.

The diagram below shows the top view of a rain gauge attached to a post.



Not to scale

6. If 5 mm of rain falls, calculate the volume of water collected in the rain gauge above.

Show all working

$V = AH$

$A = LW$

$= 50\text{mm} \times 80\text{mm}$

$= 4000\ \text{mm}^2$

$V = AH$

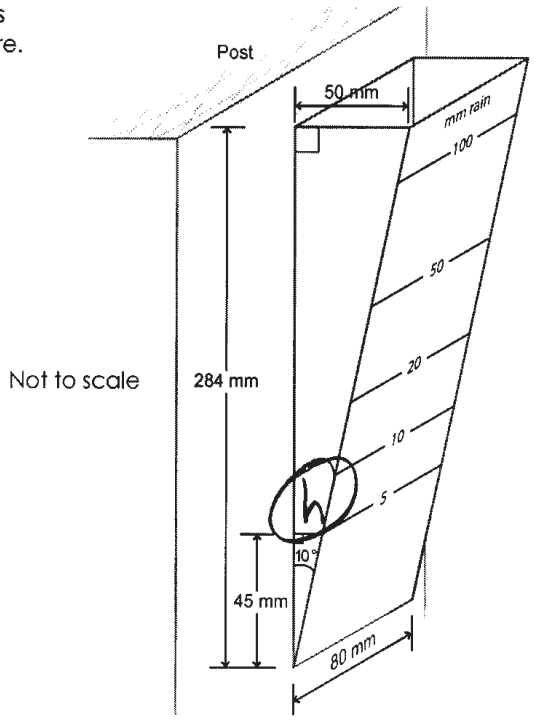
$= 4000\ \text{mm}^2 \times 5\text{mm}$

$= 20\ 000\ \text{mm}^3$

Volume of rainwater collected = 20 000 mm^3

A sample: Response 2

The rain gauge from page 8 is wedge-shaped, as shown here.



Not to scale

7. Use the volume calculated in Question 6 to check if the 5 mm graduation on the rain gauge is in the correct place.

Show all working

$\frac{h}{45} = \frac{50}{284}$

$h = \frac{50 \times 45}{284}$

$= 7.9 \text{ mm}$

$V = AH$

$A = \frac{b \times h}{2}$

$= \frac{45 \times 7.9}{2}$

$= 177.75$

$V = AH$

$= 177.75 \times 80 \text{ mm}$

$= 14220$

Tick the correct answer.

The 5 mm graduation is:

☐ in the correct place

☐ too high

☑ too low.

A sample: Response 2

Section 2: Planning a rainwater supply

Table 2 below shows typical roof areas for homes of different sizes.

Table 2

Typical roof areas	
Home type	**Roof area (m^2)**
2 bedroom home	100
3 bedroom home	150
4 bedroom home	200
5 bedroom home	250

Assume the roof is flat.

(This makes little difference to the amount of rain collected.)

From the table, choose a home that most closely matches the one you live in (or would like to live in).

My choice of home: 4 Bedroom

8. Calculate the amount of rainwater in litres (L) collected by the roof of your chosen home when one millimetre (1 mm) of rain falls.

Show all working

1 mm = 0.001 m
1 m^3 holds 1000 L

$V = AH$

$A = 200 m^2$

$V = 200 m^2 \times 1 mm$

$= 200 m^2 \times .001 m$

$= .2 m^3$

$= .2 m^3 \times 1000$

$= 200 L$

Amount of rainwater collected by the roof when 1 mm of rain falls = 200 L

A sample: Response 2

Your teacher has supplied you with median monthly rainfall data for:

Emerald (location).

9 a. Copy the median rainfall values for each month into column A of Table 3.

b. Using your answer to Question 8, calculate the expected amount of rainwater collected from your chosen roof during each month. Write these values in column B of Table 3.

Table 3

Expected rainwater collected		
Month	**A** Median rainfall (mm)	**B** Rainwater collected by the roof (L)
Jan	110.4	22080
Feb	96.7	19340
Mar	24.7	4940
Apr	34.8	6960
May	33.1	6620
Jun	14.9	1490
Jul	15.0	3000
Aug	15.0	3000
Sep	8.2	1640
Oct	49.6	9920
Nov	68.4	13760
Dec	94.7	18940

A sample: Response 2

10. Explain how you could make the best use of the collected rainwater (Table 3) to reduce your need to use water from the local supply.

In your answer:

- ☑ record the number of people living in the house
- ☑ work out approximately how much water your household uses each month (refer to Table 4 below)
- ☑ compare the expected amount of rainwater collected (Table 3) to your monthly usage
- ☑ explain how you could use the collected rainwater to supply some or all of your needs
- ☑ select an appropriately sized tank to store rainwater for use in drier months (refer to Table 5 below)
- ☑ justify all your reasoning and show all calculations.

Answer on the following pages.
Check the box next to each point above as you complete it.

Table 4

Careful water use (using water wisely)	
Use	**Litres *per person* per month**
Drinking, cooking	**300**
Dish washing	**150**
Bathroom and toilet	**2500**
Washing clothes	**200**
A garden hose uses about 700 litres ***per hour***	

Table 5

Rainwater tank sizes and prices										
Tank capacity (litres)	500	1000	1500	2000	2500	3000	5000	10000	20000	50000
Installed cost	$500	$700	$750	$800	$1000	$1100	$1400	$2000	$3000	$7000

A sample: Response 2

2 people living in this house.

Litres per person

300
150
2500
200
3150 L

2 people

2 × 3150 = 6300 L

Garden hose

2 hrs per week
4 wks per month
8 hrs per month

700 × 8 = 5600 L

Total = 6300 + 5600
= 11900 Litres per month.

Have you checked all the boxes in Question 10?
Continue your answer over the page if necessary.

A sample: Response 2

Comparing the results from table 3, on most months there would not be enough rainfall for all household water usage
On average, we can expect about 9307 L per month
(111690 L ÷ 12) (Total for year)
If we cut back on watering the garden ~~to~~ 4 hrs a month, that will save 2800 Litres

11900 −
2800
9100

9100 litres is less than my monthly rainfall, so if I get a tank to store the excess in the wet months, I should have enough without using the council water.
I would buy a 50 000 litre tank because this would hold enough of the excess for the dry months.
Even though it costs $7000, I would not need to use council water unless it was a very dry year.

Have you checked all the boxes in Question 10?
Continue your answer over the page if necessary.