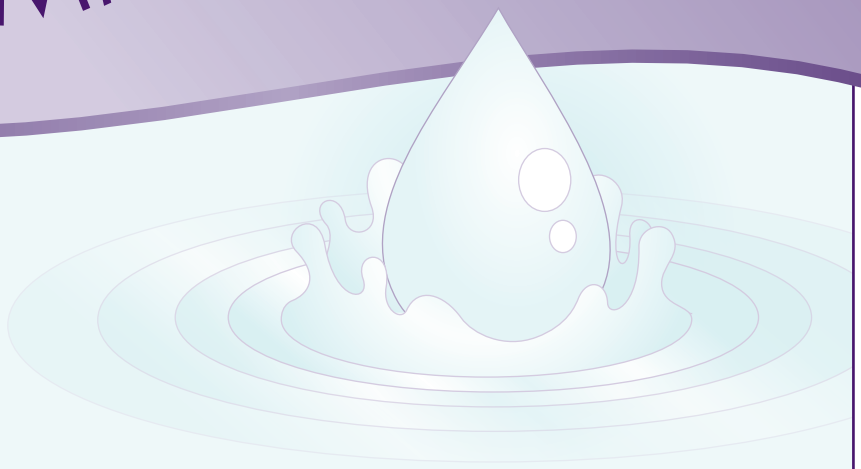
QCATs
Queensland Comparable Assessment Tasks
9
2009
MATHEMATICS
SAMPLE RESPONSES



Rainwater

This booklet is designed to help teachers make overall, on-balance judgments by providing examples of student responses. The responses are not an exhaustive set.

B samples

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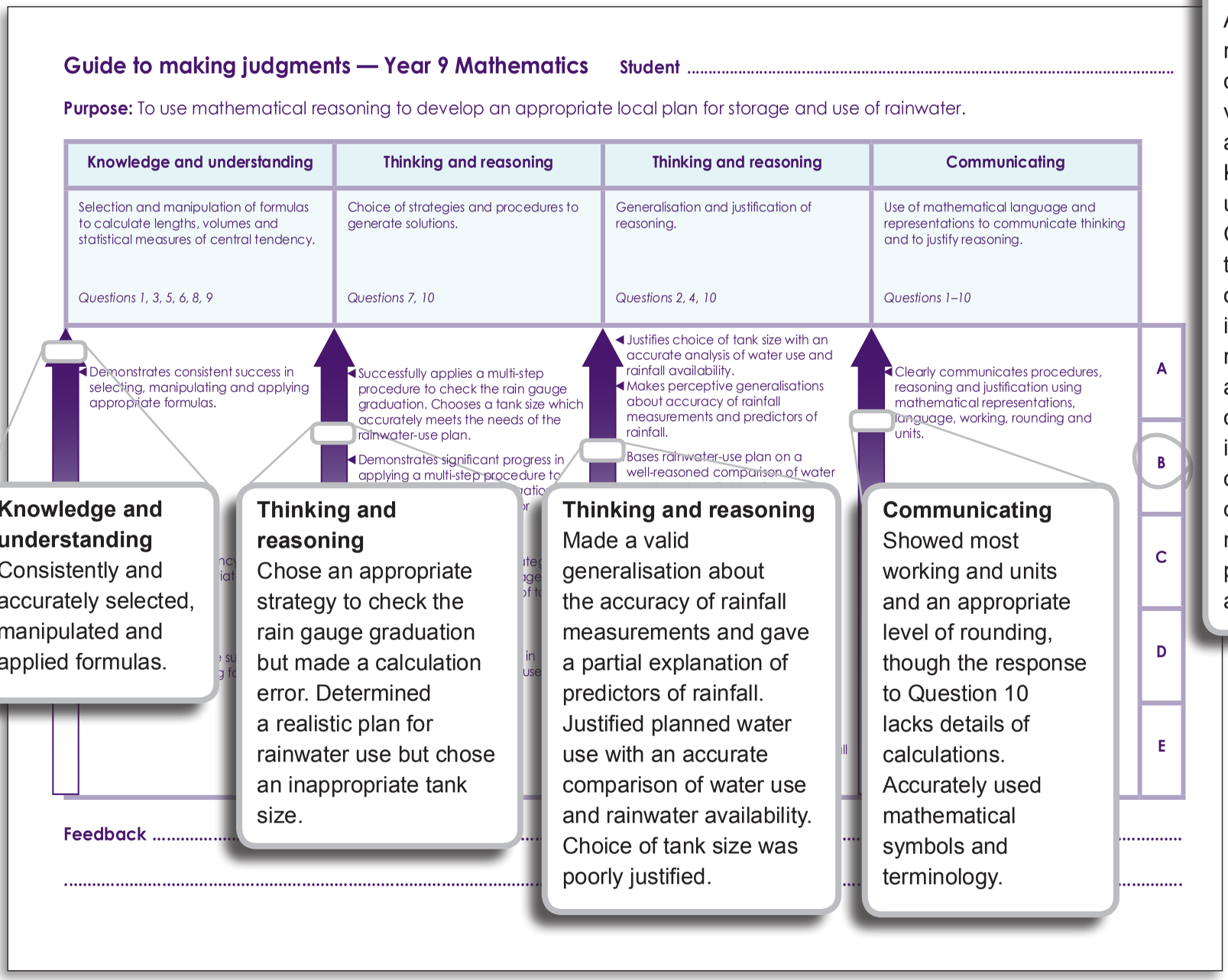
B sample: Response 1



Knowledge and understanding
Consistently and accurately selected, manipulated and applied formulas.

Thinking and reasoning
Chose an appropriate strategy to check the rain gauge graduation but made a calculation error. Determined a realistic plan for rainwater use but chose an inappropriate tank size.

Thinking and reasoning
Made a valid generalisation about the accuracy of rainfall measurements and gave a partial explanation of predictors of rainfall. Justified planned water use with an accurate comparison of water use and rainwater availability. Choice of tank size was poorly justified.

Communicating
Showed most working and units and an appropriate level of rounding, though the response to Question 10 lacks details of calculations. Accurately used mathematical symbols and terminology.

Overall grade
Although this response demonstrates a very high level of achievement in Knowledge and understanding and Communicating, the high level of achievement in Thinking and reasoning suggests an overall B. This on-balance judgment is made in light of the emphasis on Thinking and reasoning in the purpose of the assessment.

B sample: Response 1

1. Calculate Birdsville's mean and median April rainfall for the period 1997 to 2006.

Show all working

0.0
52.8
2.0
86.6
1.0
0.0
6.7
1.0
0.0
150.1

$\frac{150.1}{10} = 15.01$

Show all working

0/0, 0/6, 0/0, 0/0, 1.0
1.0, 2/0, 6/7, 52.8, 86/6

Mean April rainfall = 15.01 mm

Median April rainfall = 1.0

2. The median is a more useful predictor of typical monthly rainfall than the mean. Use the April data for Birdsville to explain why.

The majority of the records are low, therefore having a low result for monthly rainfall. The mean shows what we would get if distributed evenly throughout the years whereas the median shows what we are more likely to get ~~see~~ every month.

B sample: Response 1

Measuring rainfall

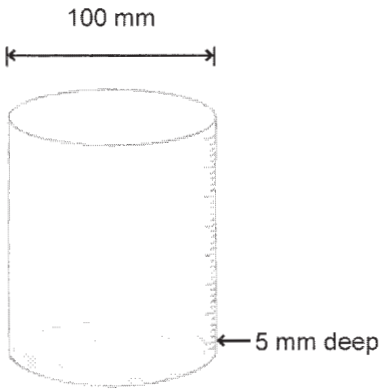
A rain gauge is used to measure rainfall (usually in millimetres).

The level in the gauge represents the depth of rainwater that would build up on a flat horizontal surface if it didn't run off or soak in.

Any open container with straight vertical sides can be used as a simple rain gauge. The depth of water collected is a measure of the amount of rain.

Rain gauges A and B both measure the same amount of rainfall, even though A collects a greater volume of water because of its larger opening.

3. If 5 mm of rain falls and is collected in a 100 mm diameter rain gauge, calculate the volume of rainwater in the gauge.



Volume of a cylinder
$= \pi r^2 h$

Show all working

$= \pi r^2 h$

$= \pi \times 50 \times 50 \times 5$

$= 39269.908$

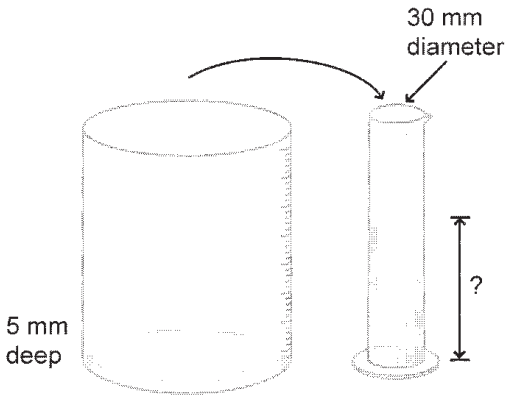
Volume of rainwater = 39269.9 mm³

B sample: Response 1

To improve the accuracy of rainfall measurement, the collected rainwater can be poured into a narrower cylinder.

4. How is the accuracy of rainfall measurements improved by pouring the collected rainwater into a narrower cylinder?

The accuracy is improved because the narrow cylinder has a scale which is easier to read as the numbers would be further apart.



5. Calculate the depth of the rainwater collected in Question 3 after it is poured into a narrow, 30 mm diameter cylinder.

Show all working

$\pi r^2 H = V$

$\pi \times 15 \times 15 \times H = V$

$706.858 \times H = V$

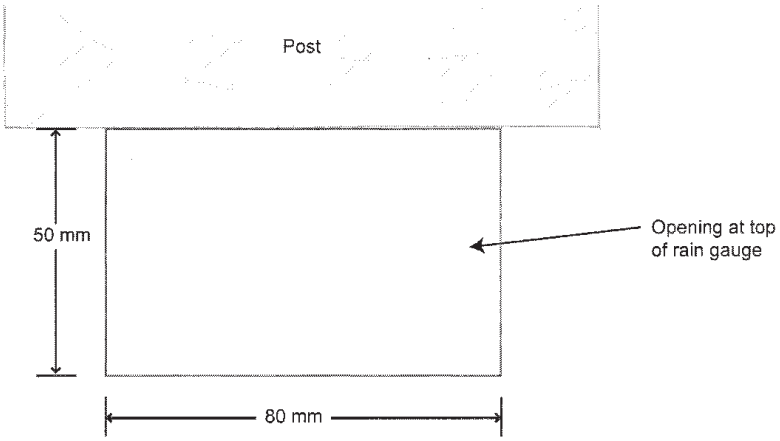
$H = \frac{39269.9}{706.858}$

Depth of water in the narrow cylinder = 55.55 mm

B sample: Response 1

Not all rain gauges are cylinders.

The diagram below shows the top view of a rain gauge attached to a post.



Not to scale

6. If 5 mm of rain falls, calculate the volume of water collected in the rain gauge above.

Show all working

$V = \text{area} \times H$

$= L \times w \times H$

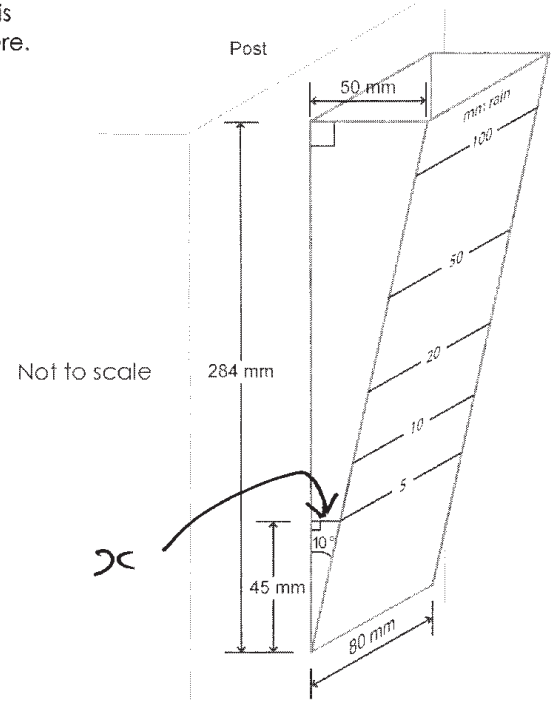
$= 80 \times 50 \times 5$

$= 20\,000$

Volume of rainwater collected = 20 000 mm^3

B sample: Response 1

The rain gauge from page 8 is wedge-shaped, as shown here.



7. Use the volume calculated in Question 6 to check if the 5 mm graduation on the rain gauge is in the correct place.

Show all working

$\frac{50}{x} = \frac{284}{45}$

$\frac{x}{50} = \frac{45}{284}$

$x = 50 \times 0.158$

$= 7.9$

$V = \text{area} \times H$

$= \frac{b \times h}{2} \times H$

$= \frac{45 \times 7.9}{2} \times 50$

$= 8887.5$

(should be 20 000)

Tick the correct answer.

The 5 mm graduation is:

- ☐ in the correct place
- ☐ too high
- ☑ too low.

B sample: Response 1

Section 2: Planning a rainwater supply

Table 2 below shows typical roof areas for homes of different sizes.

Table 2

Typical roof areas	
Home type	**Roof area (m^2)**
2 bedroom home	100
3 bedroom home	150
4 bedroom home	200
5 bedroom home	250

Assume the roof is flat.

(This makes little difference to the amount of rain collected.)

From the table, choose a home that most closely matches the one you live in (or would like to live in).

My choice of home: 2 bedroom house

8. **Calculate the amount of rainwater in litres (L) collected by the roof of your chosen home when one millimetre (1 mm) of rain falls.**

Show all working

1 mm = 0.001 m
1 m^3 holds 1000 L

$V = \text{area} \times D$

$= 100 \times 0.001$

$= 0.1 m^3$

$= 100$ L

Amount of rainwater collected by the roof when 1 mm of rain falls = 100 L

B sample: Response 1

Your teacher has supplied you with median monthly rainfall data for:

Redcliffe (location).

9 a. **Copy the median rainfall values for each month into column A of Table 3.**

b. **Using your answer to Question 8, calculate the expected amount of rainwater collected from your chosen roof during each month. Write these values in column B of Table 3.**

Table 3

	Expected rainwater collected	
Month	**A** Median rainfall (mm)	**B** Rainwater collected by the roof (L)
Jan	85.6	8560
Feb	110.4	11 040
Mar	103.8	10 380
Apr	70.7	7070
May	90.9	9090
Jun	45.2	4520
Jul	38.2	3820
Aug	31.0	3100
Sep	~~71.4~~ 27.2	2720
Oct	71.4	7140
Nov	91.4	9140
Dec	125.8	12580

B sample: Response 1

10. Explain how you could make the best use of the collected rainwater (Table 3) to reduce your need to use water from the local supply.

In your answer:

- ☑ record the number of people living in the house
- ☑ work out approximately how much water your household uses each month (refer to Table 4 below)
- ☑ compare the expected amount of rainwater collected (Table 3) to your monthly usage
- ☑ explain how you could use the collected rainwater to supply some or all of your needs
- ☑ select an appropriately sized tank to store rainwater for use in drier months (refer to Table 5 below)
- ☑ justify all your reasoning and show all calculations.

Answer on the following pages.
Check the box next to each point above as you complete it.

Table 4

Careful water use (using water wisely)	
Use	**Litres *per person* per month**
Drinking, cooking	**300**
Dish washing	**150**
Bathroom and toilet	**2500**
Washing clothes	**200**
A garden hose uses about 700 litres ***per hour***	

Table 5

Rainwater tank sizes and prices										
Tank capacity (litres)	500	1000	1500	2000	2500	3000	5000	10000	20000	50000
Installed cost	$500	$700	$750	$800	$1000	$1100	$1400	$2000	$3000	$7000

B sample: Response 1

5 people live in my house so altogether we use 15750 L + water used from the garden hose. In a typical year we have 1097mm of rain which is 109770 L. The most rain in a month is 12500 L in December, so I won't have enough rainwater for everything. The average (collection in a month is 9147.5 Litres which is enough for everything if we use ~~cooncit~~ the local water supply to the toilet.

I would use rain water for:

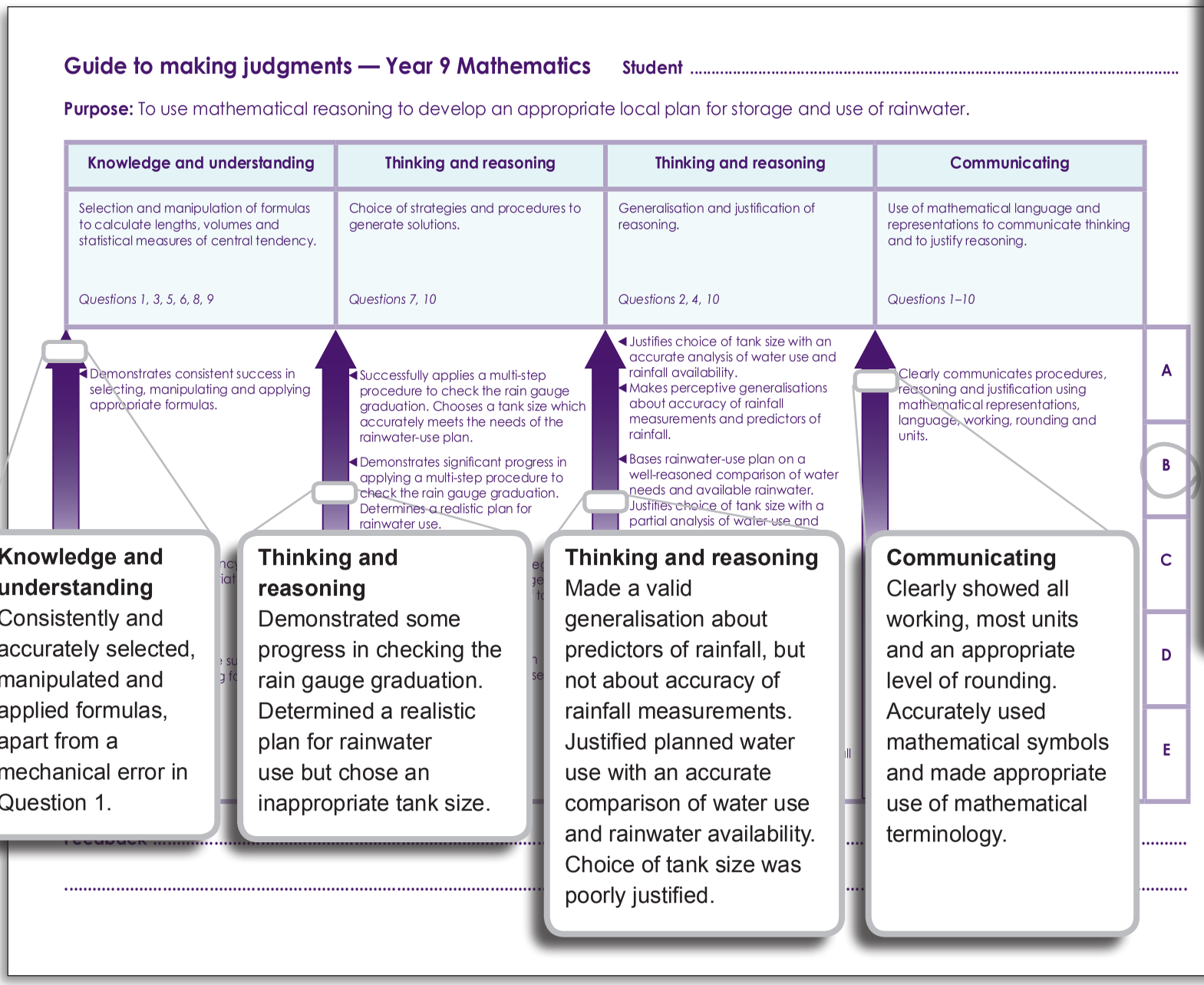
- Drinking + cooking
- Dishwashing
- Washing clothes
- Hose $1\frac{1}{2}$ hours a week

Total = 7450 L

I would get a 10 000 L tank as its cheaper than a 5000 and a 2000.

Have you checked all the boxes in Question 10?
Continue your answer over the page if necessary.

B sample: Response 2



Guide to making judgments — Year 9 Mathematics

Student .....

Purpose: To use mathematical reasoning to develop an appropriate local plan for storage and use of rainwater.

Knowledge and understanding	Thinking and reasoning	Thinking and reasoning	Communicating
Selection and manipulation of formulas to calculate lengths, volumes and statistical measures of central tendency.	Choice of strategies and procedures to generate solutions.	Generalisation and justification of reasoning.	Use of mathematical language and representations to communicate thinking and to justify reasoning.
Questions 1, 3, 5, 6, 8, 9	*Questions 7, 10*	*Questions 2, 4, 10*	*Questions 1–10*

- Demonstrates consistent success in selecting, manipulating and applying appropriate formulas.
- Successfully applies a multi-step procedure to check the rain gauge graduation. Chooses a tank size which accurately meets the needs of the rainwater-use plan.
- Demonstrates significant progress in applying a multi-step procedure to check the rain gauge graduation. Determines a realistic plan for rainwater use.
- Justifies choice of tank size with an accurate analysis of water use and rainfall availability.
- Makes perceptive generalisations about accuracy of rainfall measurements and predictors of rainfall.
- Bases rainwater-use plan on a well-reasoned comparison of water needs and available rainwater.
- Justifies choice of tank size with a partial analysis of water use and
- Clearly communicates procedures, reasoning and justification using mathematical representations, language, working, rounding and units.

A B C D E

Knowledge and understanding
Consistently and accurately selected, manipulated and applied formulas, apart from a mechanical error in Question 1.

Thinking and reasoning
Demonstrated some progress in checking the rain gauge graduation. Determined a realistic plan for rainwater use but chose an inappropriate tank size.

Thinking and reasoning
Made a valid generalisation about predictors of rainfall, but not about accuracy of rainfall measurements. Justified planned water use with an accurate comparison of water use and rainwater availability. Choice of tank size was poorly justified.

Communicating
Clearly showed all working, most units and an appropriate level of rounding. Accurately used mathematical symbols and made appropriate use of mathematical terminology.

Overall grade
Although this response demonstrates a very high level of achievement in Knowledge and understanding and Communicating, the high level of achievement in Thinking and reasoning suggests an overall B. This on-balance judgment is made in light of the emphasis on Thinking and reasoning in the purpose of the assessment.

B sample: Response 2

1. **Calculate Birdsville's mean and median April rainfall for the period 1997 to 2006.**

Show all working

$$\text{mean} = \frac{150.1}{10}$$

$$= 15.01$$

Show all working

0.0, 0.0, 0.0, 0.0, 1.0, 1.0, 2.0, 6.7, 52.8, 86.6

$$\text{median} = \frac{5^{th} + 6^{th} \text{ term}}{2}$$

$$= \frac{1.0 + 2.0}{2}$$

$$= 1.5$$

Mean April rainfall = 15.01 Median April rainfall = 1.5

2. **The median is a more useful predictor of typical monthly rainfall than the mean. Use the April data for Birdsville to explain why.**

Because there are two years (1998 + 2000) that got a ridiculous amount of rain, that has made the mean much larger. The median hasn't been affected by these years and is close to most years rainfall.

B sample: Response 2

Measuring rainfall

A rain gauge is used to measure rainfall (usually in millimetres).

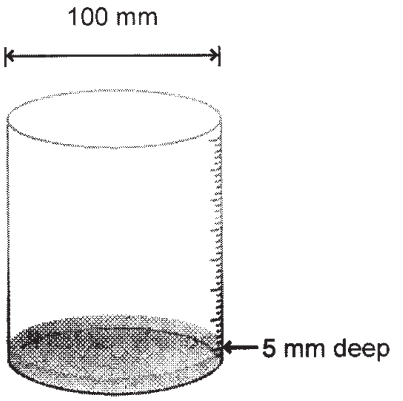
The level in the gauge represents the depth of rainwater that would build up on a flat horizontal surface if it didn't run off or soak in.

Any open container with straight vertical sides can be used as a simple rain gauge. The depth of water collected is a measure of the amount of rain.

Rain gauges A and B both measure the same amount of rainfall, even though A collects a greater volume of water because of its larger opening.

Rain gauge A Rain gauge B

3. If 5 mm of rain falls and is collected in a 100 mm diameter rain gauge, calculate the volume of rainwater in the gauge.



Volume of a cylinder
$= \pi r^2 h$

Show all working

$V = Ah$

$= \pi r^2 h$

$= (\pi \times 50^2) \times 5$

$= 39\,269.91 \text{ mm}^3$

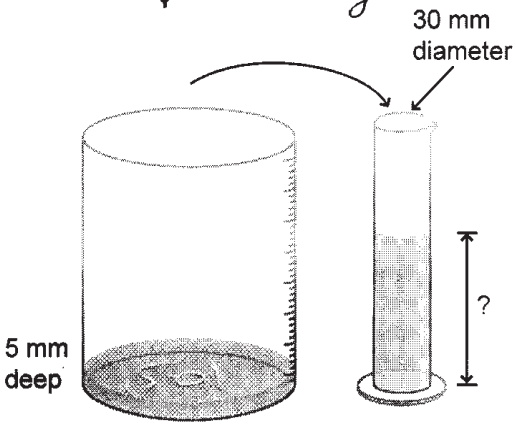
Volume of rainwater = 39 269.91 mm^3

B sample: Response 2

To improve the accuracy of rainfall measurement, the collected rainwater can be poured into a narrower cylinder.

4. How is the accuracy of rainfall measurements improved by pouring the collected rainwater into a narrower cylinder?

Because the area of the base of the narrow cylinder is smaller than the large cylinder, so the rainwater collected will be more level which makes it easier to read against the mm marks



5. Calculate the depth of the rainwater collected in Question 3 after it is poured into a narrow, 30 mm diameter cylinder.

Show all working

$V = Ah$

$= \pi r^2 h$

$39.269.91 = (\pi \times 15^2) \times h$

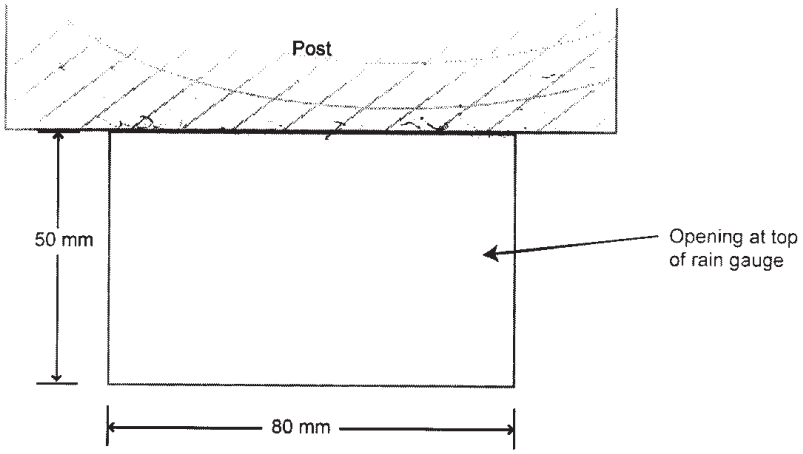
$\frac{39269.91}{\pi \times 15^2} = h$

Depth of water in the narrow cylinder = 55.56 mm

B sample: Response 2

Not all rain gauges are cylinders.

The diagram below shows the top view of a rain gauge attached to a post.



Not to scale

6. **If 5 mm of rain falls, calculate the volume of water collected in the rain gauge above.**

Show all working

$V = Ah$

$= L \times W \times h$

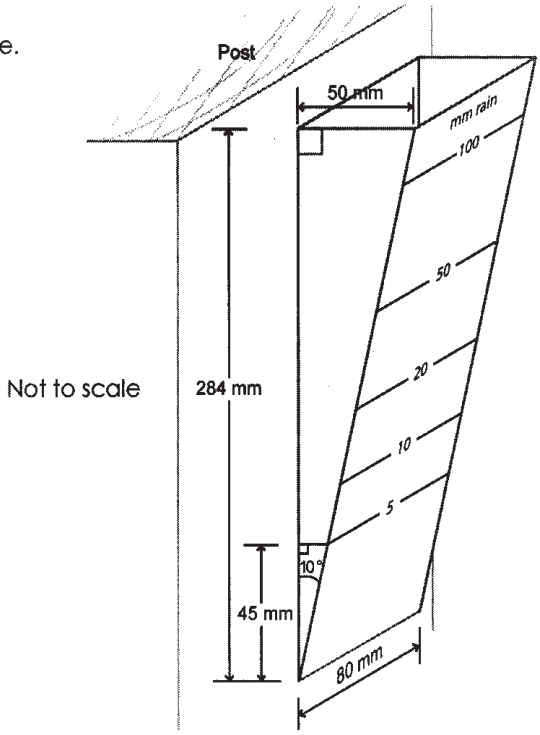
$= 80 \times 50 \times 5$

$= 20\,000\ mm^3$

Volume of rainwater collected = 20 000 mm^3

B sample: Response 2

The rain gauge from page 8 is wedge-shaped, as shown here.



Not to scale

7. Use the volume calculated in Question 6 to check if the 5 mm graduation on the rain gauge is in the correct place.

Show all working

$V = AH$

$= \left(\frac{b \times h}{2}\right) H$

$= \frac{50 \times 284}{2} \times 80$

$= 568000$

$= \frac{b \times 45}{2} \times 80$

$=$

Tick the correct answer.

The 5 mm graduation is:

- ☐ in the correct place
- ☐ too high
- ☐ too low.

B sample: Response 2

Section 2: Planning a rainwater supply

Table 2 below shows typical roof areas for homes of different sizes.

Table 2

Typical roof areas	
Home type	**Roof area (m^2)**
2 bedroom home	100
3 bedroom home	150
4 bedroom home	200
5 bedroom home	250

Assume the roof is flat.

(This makes little difference to the amount of rain collected.)

From the table, choose a home that most closely matches the one you live in (or would like to live in).

My choice of home: ....5 Bedroom house

8. Calculate the amount of rainwater in litres (L) collected by the roof of your chosen home when one millimetre (1 mm) of rain falls.

Show all working

1 mm = 0.001 m
1 m^3 holds 1000 L

$V = Ah$

$= 250 \times h$

$= 250 \times 0.001$

$= 0.25\ m^3$

$L = 1000 \times 0.25$

$= 250\ L$

Amount of rainwater collected by the roof when 1 mm of rain falls = ...250..... L

B sample: Response 2

Your teacher has supplied you with median monthly rainfall data for:

.....Maryborough..... (location).

9 a. Copy the median rainfall values for each month into column A of Table 3.

b. Using your answer to Question 8, calculate the expected amount of rainwater collected from your chosen roof during each month. Write these values in column B of Table 3.

Table 3

Expected rainwater collected		
Month	**A** Median rainfall (mm)	**B** Rainwater collected by the roof (L)
Jan	117.4 mm	29 350
Feb	121.5 mm	30 375
Mar	87.9 mm	21 975
Apr	62.9 mm	15 725
May	87.1 mm	21 775
Jun	36.9 mm	9 225
Jul	35.0 mm	8 750
Aug	31.2 mm	7 800
Sep	30.1 mm	7 525
Oct	77.2 mm	19 300
Nov	87.0 mm	21 750
Dec	97.2 mm	24 300

B sample: Response 2

10. Explain how you could make the best use of the collected rainwater (Table 3) to reduce your need to use water from the local supply.

In your answer:

- ☒ record the number of people living in the house
- ☒ work out approximately how much water your household uses each month (refer to Table 4 below)
- ☒ compare the expected amount of rainwater collected (Table 3) to your monthly usage
- ☒ explain how you could use the collected rainwater to supply some or all of your needs
- ☒ select an appropriately sized tank to store rainwater for use in drier months (refer to Table 5 below)
- ☒ justify all your reasoning and show all calculations.

Answer on the following pages.
Check the box next to each point above as you complete it.

Table 4

Careful water use (using water wisely)	
Use	**Litres *per person* per month**
Drinking, cooking	**300**
Dish washing	**150**
Bathroom and toilet	**2500**
Washing clothes	**200**
A garden hose uses about 700 litres ***per hour***	

Table 5

Rainwater tank sizes and prices										
Tank capacity (litres)	500	1000	1500	2000	2500	3000	5000	10000	20000	50000
Installed cost	$500	$700	$750	$800	$1000	$1100	$1400	$2000	$3000	$7000

B sample: Response 2

Drinking, cooking = 300 × P
= 300 × 5
= 1500 L

* P = Number of people

Dishwashing = 150 × P
= 150 × 5
= 750 L

Bathroom + toilet = 2500 × P
= 2500 × 5
= 12 500 L

Clothes = 200 × P
= 200 × 5
= 1000 L

Hose = 700 L

Total = 16450 L

The average rainfall is about 18 000 litres per month so we should have enough if we store some for the drier months

Have you checked all the boxes in Question 10?
Continue your answer over the page if necessary.

B sample: Response 2

Our household would have to have some water stored for the months of April, July, August and September, however we could have leftover water from Jan, Feb, March, May, Nov, Dec, and Oct.

As the average monthly rainfall is 18000 L, a 20 000 tank would be enough, at a cost of $3000.

Have you checked all the boxes in Question 10?
Continue your answer over the page if necessary.