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| INVESTIGATION | **MATHEMATICS** |
|  | **Level 4** |

# Patterns and pavers

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| **Strand**DD00866_ | **Topics**  |
| Patterns and Algebra | Patterns and functions Equivalence and equations |
| **Outcomes PA 4.1, PA 4.2** |

## Investigation

The school has decided to pave around a line of trees planted in the school grounds. You have been asked to provide a series of designs for the principal using a variety of different shaped pavers. You will also need to determine how many pavers are required for the entire row of trees, and how many trees can be planted if there are a limited number of pavers available.

## Overview

The following table shows how this investigation is organised in phases associated with **thinking, reasoning and working mathematically.**

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| Thinking, reasoning and working mathematically |
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| 1. Identifying and describing |  | 2. Understanding and applying |  | 3. Communicating and justifying |
| Introducing the investigationStudents:choose the area in the school grounds to be paved.Providing students with a sample activityStudents:establish the relationship between the number of pavers and treesformulate a work plan and checklist. |  | Working out the numbersStudents: model the investigation using concrete materialsestablish a rulecreate a graphuse backtracking to determine an answer.Working through the investigationStudents:design a paving pattern for a row of treesdetermine the number of trees to be planted. |  | Presenting the designs to the principalStudents: collate findings into a bookletgive explanations about how values were determinedidentify how the designs could be used in other situations. |

## Core learning outcomes

This investigation focuses on the following core learning outcomes from the *Years 1 to 10 Mathematics Syllabus*:

**PA 4.1** Students identify and create representations of patterns and functions and apply backtracking to solve simple equations that involve combinations of the four operations.

**PA 4.2** Students create and interpret equations, explain the effect of order of operations, and justify solutions to equations.

Using this investigation

The sequence of activities suggested in this investigation provides opportunities for students to demonstrate learning described by core learning outcomes or aspects of core learning outcomes. The investigation may be modified to provide opportunities for students to demonstrate learning described by core learning outcomes at other levels.

Contribution to the attributes of a lifelong learner

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| **Knowledgeable person with deep understanding** | Students ‘know about’ patterns and algebra, and apply rules to make generalisations. |
| **Complex thinker** | Students develop a plan for the investigation and use a range of strategies to solve problems. |
| **Active investigator** | Students manipulate concrete materials and make appropriate representations to assist understandings. |
| **Effective communicator** | Students represent their mathematical ideas and reasoning in different ways and select the most appropriate for the principal. |
| **Reflective and self-directed learner** | Students follow the plan for their investigations and modify where necessary. They select the most efficient and suggest possible applications for the future. |

Core content

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| Patterns and Algebra — Patterns and functions: Level 4 | Patterns and Algebra — Equivalence and equations: Level 4 |
| Patterns* rules based on the position of terms (combinations of operations)
* calculator number patterns
* ordered pairs and graphs (with discrete data only)

**Functions*** input 🡪 output (with combinations of operations)
* rules relating two sets of data
* backtracking (inverse)
* with combinations of operations
* representations of relationships
* ordered pairs
* tables, line graphs, equations (number sentences)
* trends
* discrete data
* continuous data
* electronic, manual
 | Equivalence* order of operations
* methods for solving equations
* balance
* guess and check

 **Representations*** symbols
* equals (=)
* does not equal (≠)
* brackets
* for unknowns (shapes, boxes, question marks, spaces, lines)
* arrow diagrams
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## Resources

* pattern blocks
* graph paper
* concrete materials

Identifying and describing

### Introducing the investigation

Students:

* choose the area in the school grounds to be paved.

##### Students:

* work through the sample activity as a class and develop strategies that can be used. They can examine the following pattern in which 13 pavers surround two trees:

A landscaper needs to know how many pavers will be needed to surround 100 trees.

Focus questions could include:

– How can we represent this information graphically?

– If the landscaper orders a certain number of pavers (e.g. 150), how many trees can be planted?

* discuss the requirements of the investigation
* generate and trial different strategies used to solve the investigation, including drawing a table, writing an equation or using different shapes to discover alternative patterns
* identify that the investigation has two values that need to be determined
* identify and negotiate possible pathways for solving the investigation
* formulate a work plan that will be used to provide evidence that various mathematical strategies have been used to solve the investigation (see student work plan and checklist).

##### Example of a student work plan and checklist

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| **Strategies I have used to solve the investigation** | **Date** | **Evidence attached** | Teacher comment |
| Modelling with concrete materials |  |  |  |
| Drawing a diagram |  |  |  |
| Arranging data by using a table |  |  |  |
| Discovering a pattern and writing an equation |  |  |  |
| Working backwards to solve an equation |  |  |  |

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| Assessing learning |
| Sources of evidence could include:* plans of approaches to investigations
* structured whole- or small-group discussions.

When making judgments, teachers consider whether the student has:* understanding of a function as a mathematical relationship between two values
* used different methods and ways to solve equations
* used mental computation strategies and computation methods to solve equations.
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Understanding and applying

### Working out the numbers

Students:

* use concrete materials (pattern blocks) to represent the investigation and to count the number of pavers needed for each tree planted
* record the results in a table, then attempt to find a pattern within the numbers recorded — for example:

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| trees | 1x | 2x | 3x | 4x | 5x |
| pavers | 8x | 13x | 18x | 23x | 28x |

* establish that adding five pavers to the previous number of pavers will continue the pattern
* discuss the impracticalities of solving the investigation using this method (e.g. discover how long the task would take if they wanted to find the number of pavers for the 100th tree)
* establish that a rule can be applied to any number of trees
* work individually to find the rule using any combination of the four operations. They write this in words: five times the number of trees plus three. Symbols can then be substituted for words: 5X+ 3 = the number of pavers needed
* use the rule to establish how many pavers will be needed if 100 trees are planted
* use the results from the table to create a straight line graph. They determine the value of the x and y axes, and draw up the graph accordingly. The teacher assists the students to see that the graph can be used to work out how many trees or pavers will be needed for different jobs either larger or smaller than this investigation
* discuss whether the graph alone is an efficient way to help solve the investigation
* backtrack within the context for the investigation
* establish how many trees could be planted if the landscaper has a certain amount of pavers (e.g. 150).

### Working through the investigation

Students:

* design their own paving pattern for a row of trees (two or more tessellating shapes may be used)
* determine the number of trees to be planted
* use strategies outlined in the work plan and checklist to determine possible pathways to solving the investigation.

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| Assessing learning |
| Sources of evidence could include:* student explanations of work in progress
* sketches, graphs and tables
* student led questioning, and responses to questions.

When making judgments, teachers consider whether the student has:* created patterns and functions using rules
* used mental computation strategies and computation methods for the operations
* described functions as mathematical relationships between two values
* identified rules for patterns and functions
* understood, and can explain, the effects of the order of operations
* created equations with unknowns
* represented functions
* demonstrated how and when to apply backtracking
* used different methods to solve equations?
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Communicating and justifying

### Presenting the designs to the principal

Students:

* draw pictures of the intended designs
* give an explanation about how the number of pavers was determined
* establish how many trees can be planted when there is a limited number of pavers available (number subject to the teacher’s discretion)
* collate their findings into a booklet of designs that may be used by the principal when discussing the issue with the P&C for budgeting
* suggest how the designs could be used in other situations
* make recommendations to accept or reject the offer of the pavers and give reasons for the decision
* provide suggestions for other projects using larger and smaller numbers of trees or pavers.

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| Assessing learning |
| Sources of evidence could include:* reports on investigations
* plans of approaches to further investigations.

When making judgments, teachers consider whether the student has:* justified solutions to equations
* interpreted equations with unknowns.
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Links

This investigation could be connected to core learning outcomes from other strands in the Mathematics key learning area and to core learning outcomes from other key learning areas — for example, Studies of Society and Environment and Technology.

### Mathematics

##### Strand Measurement

**M 4.1** Students choose appropriate units when estimating and measuring and explain relationships between dimensions when investigating areas, volumes of prisms and lengths of boundaries of rectangles.

Students:

* establish the area needed for each tree, and determine the correct length and width for each line of pavers
* estimate how much soil will be needed in relation to the area, and the volume of soil that will be required for each tree
* measure the length of each boundary to determine how many trees are to be planted
* measure the length of irrigation pipe to sustain trees.

##### Strand Space

**S 4.1** Students analyse the geometric properties of a range of 3D and 2D shapes to classify shapes into families and their subgroups and justify reasoning.

Students:

* examine a range of 3D and 2D regular polygons and construct and/or deconstruct shapes to analyse, identify or describe the defining geometric properties (students can make moulds to be filled with material to make their own pavers)
* classify families of shapes using geometric properties
* classify shapes into subgroups and give reasons for classifications
* identify shapes embedded within other shapes; describe the defining geometric properties such as rotational symmetry and the sum of internal angles; give reasons for classifications
* discuss the lines and angles found on each face of each 3D paver.

### Studies of Society and Environment

##### Strand Place and Space

**P.S 4.3** Students participate in a field study to recommend the most effective ways to care for a place.

Students:

* participate in a field study to recommend which native trees should be planted in the garden area
* examine the soil, pests and watering requirements that will be needed for the native trees.

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| For more information, refer to the elaborations inthe *Years 1 to 10 Studies of Society and Environment Sourcebook Guidelines,* which are available online from the QSA website: www.qsa.qld.edu.au |

### Technology

##### Strands Technology Practice (4.1, 4.2, 4.3, 4.4), Materials (4.1, 4.2)

Students:

* gather ideas and data about pavers and the materials used to make them
* prepare plans for the construction of pavers made from a wide range of different materials
* seek advice about the construction and design of pavers from a variety of experts
* gather feedback from others regarding design proposals, materials used and choices of colours and designs
* use their own and others’ knowledge to determine which equipment and techniques can be used to manipulate the materials to meet design specifications.

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| For more information, refer to the elaborations inthe *Years 1 to 10 Technology Sourcebook Guidelines*, which are available online from the QSA website: www.qsa.qld.edu.au |