About space

Spatial thinking involves making 'mind pictures' (visualisations) of objects and shapes in various orientations for different purposes, and making mental images of locations, directions and movements in and through environments. This thinking is used on a daily basis in all our lives and, to be used effectively, requires knowledge and understanding of the associated language.

Spatial reasoning is the basis for the resolution of many design and packaging problems encountered in the everyday lives of people and in advertising, commercial, technological and industrial fields. Many commercial enterprises rely on spatial thinking and reasoning for cost-effective packaging and enticing visual displays that promote products for purchase and/or use. Town planners and architects use spatial thinking and reasoning to plan interesting and strong structures as well as making effective use of space for work and leisure.

The Space strand is about concepts related to shape, line, location, direction and movement. It includes using geometric terms and properties to identify and represent everyday shapes and objects in different ways; solving problems about shapes using visualisation (mind pictures) of shapes and objects from different viewpoints and locations; and reading, understanding, creating and using maps, plans and diagrams.

Learning in the Measurement strand can complement many investigations developed for the Space strand of the Mathematics key learning area. Links can also be made to other topics in Mathematics and to other key learning areas. For example, interpretations of data displays require understandings of the representations of space, as do activities in the fields of science and technology.

Learning about space

Learning in this strand of Mathematics involves physical, social and cognitive learning. The physical component develops fine and gross motor skills, coordination and spatial awareness. These help students create mind pictures. Gross motor skills and spatial awareness are developed and refined as students negotiate pathways, locate objects and positions in space, and use their bodies to represent shapes. Fine motor skills and coordination are enhanced as students manipulate a range of shapes and objects, and construct and deconstruct shapes and objects using equipment, including electronic technologies and mathematical instruments. Visuomotor skills, such as perceiving an object or shape against a background of images, identifying an object or shape after seeing only one small portion, identifying a shape or object regardless of its orientation, and being able to identify a similar object or shape by its properties are developed and refined.

There are many social interactions involved in spatial learning. Learning opportunities that encourage students to describe what they see when they are looking at a representation of a shape or object encourages them to develop a language for spatial thinking. Students should be asked to share those descriptions for others to act on. They should also be encouraged to make mind pictures when they are giving and receiving information. For example, mind pictures may be used when giving directions for others to locate objects or follow a pathway, and when receiving information to construct an object or shape similar to one hidden behind a screen or barrier. These interactions rely on common understandings of spatial language.

Students' cognitive learning is developed as they learn the universally accepted language associated with shape, line, location, direction and movement. This language enables students to engage in mathematical conversations using common and shared understandings. It is enhanced through the development of new ways of learning and explanations of mathematical ideas.

The continua of cognitive learning described in this strand reflect the van Hiele framework of the development of geometric thinking. This model describes five levels that characterise the way in which students develop their thinking about geometric relationships. These are:

- Visualisation: Students identify, name, compare and operate on geometric shapes without being able to identify the geometric properties of specific shapes. They may be able to recognise some of the obvious properties of some shapes; however, they do not use these to recognise or sort shapes.
- Analysis: Students begin to analyse the geometric properties of shapes and the relationships among the attributes of shapes. They discover properties and rules through observation.

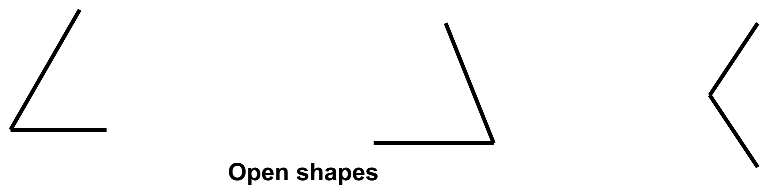
- Informal deduction: Students discover and formulate generalisations about previously learned properties and rules, and develop and follow logical arguments that describe these.
- Deduction: Students go beyond identifying properties of shapes and prove theorems deductively. They are also able to understand the structure of the geometric system.
- Rigour: This is the highest level of the framework in which students are able to establish theorems in different systems, and compare and analyse these systems. (Crowley 1987, pp. 1–3)

In the Years 1 to 10 Mathematics Syllabus, the learning described by the core learning outcomes take learners to the informal deduction level of the van Hiele framework.

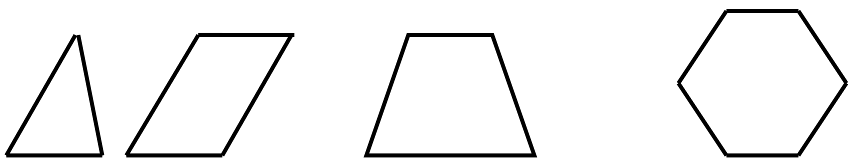
Shape and line

On entering school, students have already developed some ideas about shapes and movement in and through space. These ideas are derived from students' explorations of, and interactions with, their environments, construction and manipulation of objects during play, movement through space during physical activities and routines, and from interactions with adults. Students need to actively use shapes and objects to fully understand what they are learning and experiencing — seeing and naming shapes are not enough.

Activities that involve the construction and deconstruction of shapes and objects using a variety of materials are necessary to develop spatial thinking and reasoning. During these activities, students' attention can be drawn to the properties of the shapes and objects; they are able to see how lines can join together to make angles and when they close to form shapes. Through such modelling and experimentation, students will come to know the difference between figures that are 'open' and 'closed.'

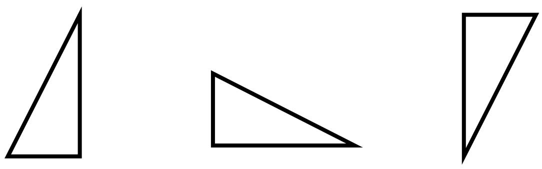


Open shapes



Closed shapes

Students need to see a range of representations of a shape or object to develop constant understandings about its form. With experience, they will be able to identify a shape or object, regardless of the orientation, from its properties or characterising features rather than recognising it only when it has the familiar or traditional orientation and dimensions. For example, students commonly identify B, the typical orientation, as a right-angled triangle.



A **B** **C**

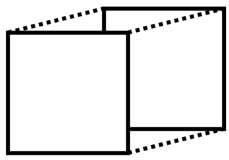
Experiences that involve viewing shapes from different orientations and only parts of shapes and objects encourage spatial thinking and reasoning. To identify the significant geometric properties of a shape and to name it, students will need to select from prior knowledge and mind pictures created over time, and combine this with the new visual information.

These experiences train students' perceptual skills and assist them to focus on the properties of shapes as they develop geometric understandings.

Geometric terms and properties

To build on the understandings developed by students during their early childhood years, teachers need to provide learning opportunities that promote explicit links between 3D shapes and objects and 2D representations. For example, when using a 3D shape to create a print, it is the 2D face of the shape that is represented. It is important that students' visual attention is focused on the same property being labelled or described by a peer or teacher.

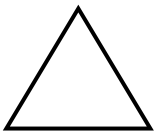
As understandings of the relationships between 3D and 2D shapes develop, representations will come closer to looking like the actual shape. To begin this process, students may simply be connecting overlays as shown below.



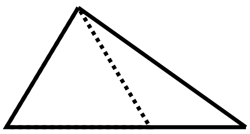
The use of different dotted paper, including isometric paper, allows students to experiment with representations and enlargements of 3D shapes and objects. This encourages students to focus on the properties of shapes and changes rather than on the drawing process.

When students are using properties to define families and subgroups of families, they should be encouraged to experiment with the construction and representation of different shapes and to describe the results after enlargement and reduction of one or all the properties. They can then decide whether or not the shape can still be categorised as a member of the same subgroup.

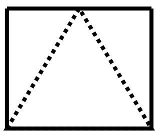
For example, if sides of a triangle are lengthened, it can still be classified in the family of triangles; if another side is added, it no longer belongs to the triangle family.



Triangle

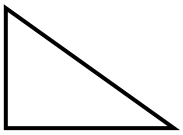


Triangle with sides lengthened

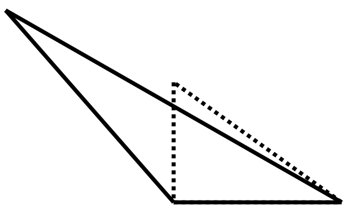


Side added

In the subgroup of right-angled triangles, when the right angle is increased in size, the shape can no longer be classified in that subgroup but can still be classified in the family of triangles.

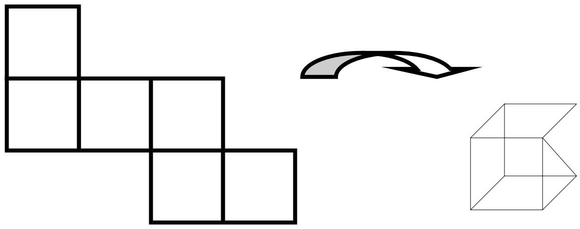


Subgroup: right-angled triangle



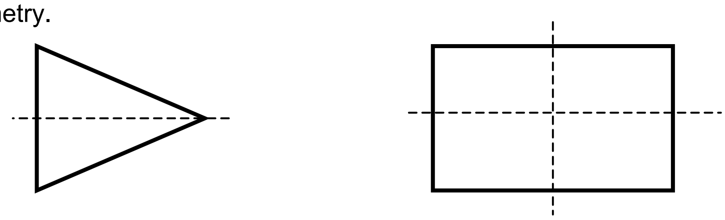
Increase in size of angle

Students need opportunities to investigate a range of nets that can be used to construct the same shape. For example, a cube can be constructed using the 'crucifixion net', which is often the only net they experience. Manipulative construction materials and equipment will provide opportunities for students to construct and deconstruct a cube to find other nets that will make a cube. They can then trace around the construction equipment, draw the different nets freehand or use dotted or grid paper appropriate to their level of development to record the different nets for a shape.



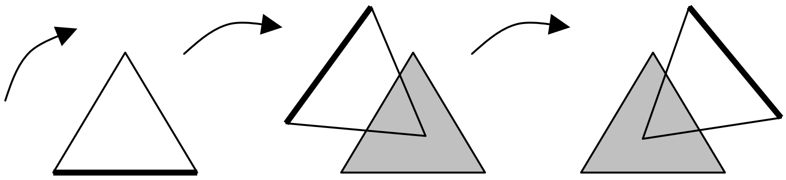
Symmetry: A figure is said to be symmetrical when it has parts that correspond with each other in terms of size and shape. There are two types of symmetry: bilateral and rotational.

A shape has **bilateral symmetry** when one half of a figure is the mirror image of the other half. The lines that divide the figure into halves are lines of symmetry. For example, an isosceles triangle has one line of symmetry while a rectangle has two lines of symmetry. Bilateral symmetry is also referred to as line symmetry.

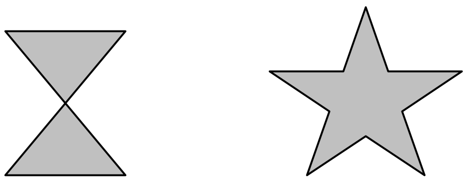


To develop mind pictures that enable them to visualise and make judgments about the symmetry of figures, students need opportunities to physically fold and cut many different shapes. They should also differentiate between shapes that are symmetrical and those that are asymmetrical.

A shape has **rotational symmetry** when it coincides with an outline of itself more than once before one complete turn around a central point. The triangle in the following diagram will match itself three times during the rotation.



The following diagrams are examples of shapes that have rotational symmetry.



It is possible for plane and solid geometric shapes to have no symmetry, one form of symmetry or both bilateral and symmetrical symmetry. Symmetry can be found in art, architecture and nature.

With increasing knowledge of geometric terms and properties, students are more able to question their own and others' solutions to geometric problems. They could develop logical chains of reasoning to make generalisations about shapes, lines and angles. Students should communicate their solutions to geometric problems using geometric terms and symbols supported by appropriate terms and representations.

Visualisations and representations

Spatial reasoning forms the foundation on which much of the learning in Mathematics and other key learning areas occurs. It is based on students being able to imagine and create 'static' and 'moving' mind pictures (visual representations) that are useful for solving spatial problems. For example, imagining a cube when someone talks about it may result in a 'static' mind picture; a 'moving' mind picture involves being able to imagine folding a net to identify the shape it makes. Another example is imagining a piece of a puzzle (static) and rotating the image (moving) in the mind to decide where it could fit into the puzzle. Spatial reasoning also includes being able to position oneself in relation to objects and locations as occurs in visualisations related to travelling to or from destinations.

Visualisation skills can be improved with specific training. To create mind pictures, students need to combine movement with touch and sight. This can be achieved through active engagement in learning and by teachers encouraging students to make mind pictures at particular times during the learning process and as reflection on learning. Students may need training in making and recalling mind pictures for particular investigations. They should use touch as well as sight to identify the geometric property that is the focus of the visualisation to ensure that the mind pictures created are the ones that are desired. To assist the recall of such images, teachers can prompt memory by referring to the relevant tactile experiences.

Students need many opportunities to investigate different materials and equipment with which to create representations when working with spatial concepts. When engaging with these resources, students are able to move shapes and objects through space and to view them from different viewpoints — 'bird's-eye' view, side view, front view, back view. Initially, they may need to have the concrete shape or object present when creating their representations. As their understandings develop, students are able to rely on their visual memory of the shape or object and its movement to get a mental image that matches the viewpoint of the representation required.

Flips, slides and turns: Students will find imagining the movements of flips, slides and turns easier when they have had opportunities to develop mind pictures and the ability to move and rotate these mind pictures. Without these experiences, they will rely on the physical movement of the shapes and objects and will be simply representing the result of an action rather than being able to think and reason spatially. The ability to make predictions of the results of moving or rotating shapes and objects relies on their ability to move the mind pictures of those shapes and objects.

Location, direction and movement

Learning in this topic promotes the development of the language, knowledge procedures and strategies necessary to construct and read maps, plans and grids, and to identify and describe locations, directions and movements. It supports the development of spatial orientation, which is essential for knowing where we are and how to move around.

Students need to position themselves, physically in space and relate their bodies to the different objects and locations within proximity. They need to be aware that one position can be described differently depending on the objects and/or locations that the position is related to. For example, when sitting in class, a student can be in front of one student, next to another, between two students and behind another student. They can also be close to the door, or far from the whiteboard. These positions need to be experienced frequently and described in relation to different objects and locations in a range of familiar settings and environments before they start to experience learning experiences with less familiar locations such as bush walking.

Students need experiences positioning themselves and finding pathways to the same location from different directions. Students may experience difficulty identifying the location and finding a pathway to that location if they are in a different position in relation to it. For example, young students may know how to get to their classroom from the front gate but cannot find their way when they are positioned on the opposite side of the classroom and approaching it from the library or the playground. It is possible that older students may experience similar confusion when orientating themselves in an unfamiliar environment such as a camping ground.

Angles: Students require learning opportunities that allow them to experience angles in different contexts. The built environment provides opportunities for students to identify angles. Examples include square corners where two walls meet and acute angles where staircases meet the ground. Seeing an angle as a square corner can also connect to students' visual imagery of a square. Students may find it more difficult to see the corner of a road as an angle as this is more abstract. In these situations, they see angles as two lines joining at one point rather than as the amount of turn.

When students have opportunities to physically experience an angle, for example, to use their arms and legs to make angles and to see the change in an angle as they move a limb, they will see an angle as the space within the two lines, the acute angle. They also need to understand that there is an angle outside the two lines, the obtuse angle. Similar angles that pivot or hinge, such as those made by scissors or a swinging door, can also be investigated.

Students need assistance to make connections between the different experiences used to develop their conceptual understanding of angles and to learn about ways to measure angles. Those students who have experiences in making angles using turns will make strong connections between those experiences and measuring angles using a 360-degree protractor.

Teachers can introduce the idea of angles as the amount of turn by relating it to students' understandings of fractions. Once students have a sound understanding of angles, they learn to relate the amount of turn to the degrees of angles within a circle. For example, a quarter turn is the same as 90 degrees, half a turn is 180 degrees, which is a straight line from the start point, a three-quarter turn is 270 degrees, and a full turn, 360 degrees.

When asked to estimate the size of an angle, students draw on different experiences and the visual images and personal referents they have established. For example, they may use the half turn (180 degrees) to assist with an estimate of an angle that is
170 degrees, or they may imagine the corner of a room (90 degrees) to assist with the estimation of an angle that is 100 degrees. Another strategy that may be used is to imagine making the angle using the 360-degree protractor.

Real-world examples will provide students with opportunities to develop connections between the language of space and their life situations, to develop strategies to solve life situations beyond those presented in the classroom, and to enhance their spatial thinking and reasoning.

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