

About number

Part 1: Number concepts and operations

All people need to develop a good sense of number, that is, ease and familiarity with and intuition about numbers. This requires a sound grasp of number concepts and notation, familiarity with number patterns and relationships, a working repertoire of number skills and, most importantly, confidence in one's capacity to deal with numerical situations.

A National Statement on Mathematics for Australian Schools, 1991

The Number strand of the *Years 1 to 10 Mathematics Syllabus* focuses on three key and connected elements of mathematics:

- Number concepts, which develops numeration and number sense, including the subsets of numbers within the set of rational numbers, the base ten system, and the uses and purposes of money in our society
- Addition and subtraction, which promotes the connections between these concepts and uses understandings of number to support mental computation strategies and other computation methods
- Multiplication and division, which promotes the connections between these concepts and uses understandings of number to support mental computation strategies, fractional and proportional thinking, and other computation methods.

Students draw on numeration and their number sense when thinking, reasoning and working mathematically in the other strands of the syllabus — Patterns and Algebra, Measurement, Chance and Data, and Space.

When thinking, reasoning and working mathematically in the Number strand across all levels, students use their understandings of numbers and the number system. These understandings include knowing:

- counting sequences
- the position and order of numbers
- the quantity of a number
- place value.

The various representations of numbers, for example, whole numbers, common fractions, decimal fractions, percentages, integers and index notation, should be considered as integral to the aspects listed above.

These understandings equip students with the strategies to work flexibly with numbers in a range of situations. They apply their understandings of numbers and the number system when they:

- identify and describe the mathematics of a situation
- understand and apply mathematical knowledge, procedures and strategies
- communicate and justify pathways through, and solutions to, real-life and purely mathematical problems involving addition, subtraction, multiplication or division, or combinations of these
- represent numbers using counting sequences, quantity, position and order, or place value to comfortably engage in different situations. For example, students could use:
 - the counting sequence for $27 + 3$
 - position, order and quantity for 3×27 — that is, $(3 \times 25) + (3 \times 2)$ because 25 is positioned near 27
 - place value for 3×27 — that is, 3 times the number of tens (3×20) + 3 times the number of ones (3×7).

Counting sequence

As students learn to count, they learn:

- the names of numbers associated with our number system
- the order of numbers when counting
- that one number name corresponds to one item (one-to-one correspondence)
- that items can be counted in any order
- the patterns used in counting (e.g. in 1s, 2s, 3s, 5s, 10s, 100s ...) and relate these to decimal and common fractions
- the position of numbers in relation to other numbers
- to count on and back from any number
- different representations of a counting sequence (e.g. positioning whole numbers, decimal fractions, common fractions, positive or negative numbers on a number line, in a five or ten frame, or a number track)
- that zero is not a counting number.

Students use one-to-one correspondence when they identify the quantity of a collection.

They use their knowledge of the count as one of the first strategies for addition and subtraction; that is, they count on in 1s, 2s or 3s and count back in 1s, 2s or 3s.

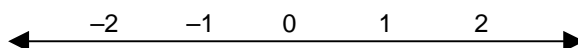
As students think and reason about the patterns in the counting sequence, they begin to use skip counting as one of the first strategies for multiplication; that is, repeated addition (e.g. counting in 2s is 2, 2 + 2, 4 + 2, 6 + 2 ...). When they identify the rule for skip counting such as adding 2, they are engaging in algebraic thinking.

Knowing and understanding the counting sequence contributes to understandings of the position and order of numbers that are essential in learning about the subsets of numbers such as odd and even numbers, multiples, factors, prime and composite numbers, square and triangular numbers.

Position and order of numbers

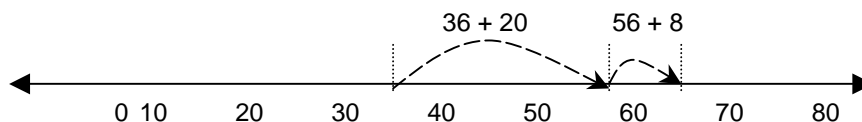
As students learn about position and order, they:

- use counting skills to position and order numbers on number tracks and number lines
- identify the usefulness of nearby numbers for calculating (e.g. when adding 37 to another number, the nearness of either 35 or 40 could be used to support the calculation)
- know that zero is the marker between positive numbers and negative numbers

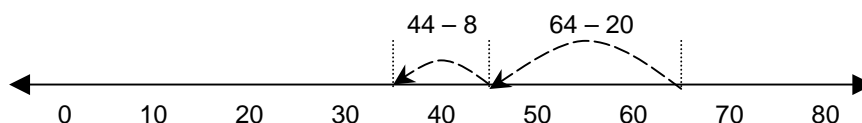


Students need to understand that zero is an integer with no magnitude and is neither negative nor positive

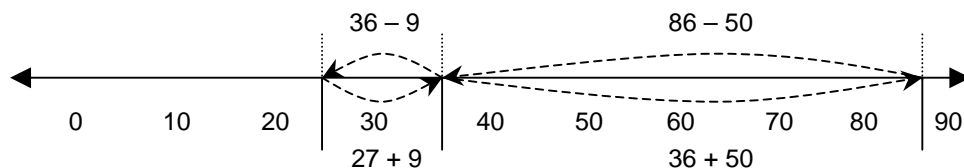
- use a number line representation when:
 - adding, for example, 36 + 28



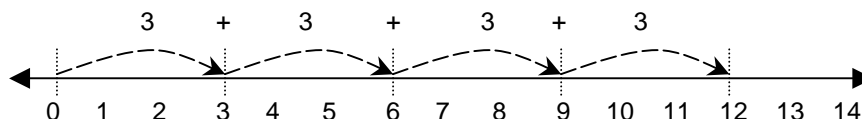
- subtracting, for example, 64 – 28



- backtracking, for example, $86 - 59$



- using the linear model for multiplication, for example, 4×3 is interpreted as $3 + 3 + 3 + 3$



- use nearby numbers as their preferred reference points (e.g. 5, 25, 50 or 10, 100, 1000) for mental calculations. Students develop preferred reference points based on their knowledge of the number system, and select and use these appropriately depending on the situation
- develop flexible and efficient strategies for estimation, approximation and mental computation based on the position and order of numbers, quantity, count and place value.

Students need to develop efficient strategies for positioning numbers, emphasising the symmetry around the zero, and identifying parameters such as 3.5 is between 3 and 4; similarly, -3.5 is between -3 and -4 .

Representations of numbers on number lines are essential for working with integers and are useful for mental computation. Number lines may be either horizontal or vertical.

When calculating mentally, students make *sensible adjustments* of numbers in different ways to ease the cognitive load and to make the calculation manageable. Some strategies include:

- using their preferred reference points such as rounding up to the next decade or hundred

$$47 + 53 = 47 + 3 + 50 = 100$$
- adjusting both numbers by adding to one number and subtracting the same quantity from the other to get to preferred reference points for calculations

$$47 + 53 = 50 + 50 = 100$$
- adjusting only one number in a computation and readjusting the answer

$$47 + 53 = (47 + 3) + 53 = 103 - 3 = 100$$
- using the knowledge of the count and the position of a number relative to other numbers when estimating and approximating such as referring to 254 789 as about 255 000, or just more than a quarter of a million
- developing preferred reference points for fractions that are comfortable to work with (e.g. 1, 0, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$) in given situations.

The quantity of a number

As students learn about the quantity of a number, they:

- refer to the total of any collection regardless of how it is made up (e.g. 2 bananas and 2 apples total 4 pieces of fruit; 1 banana, 1 apple, 1 pear, 1 orange total 4 pieces of fruit)
- identify the total in a collection as the last number counted
- know that any number can be broken into different combinations of smaller numbers (e.g. 100 is 4 groups of 25, 10 groups of 10, or 5 lots of 20; and extensions of these ideas such as 1000 being 4 groups of 250 ...) as required by the situation
- visualise representations of small numbers (e.g. by either making ‘mind pictures’ of 1, 2, 3 and other small numbers using a five and ten frame, or visualising domino patterns of small numbers)
- compare small numbers in relation to 5 or 10 using a five or ten frame and extend this thinking to larger numbers (e.g. 57 is 2 more than 55 or 3 less than 60; 85 is 15 less than 100)

- visually identify small numbers within a collection without counting, that is, subitising — for example, seeing ● ● ● ● ● as three counters and two counters without counting
- use one-to-one correspondence to count a collection
- know that zero (0) is the representation of an empty set
- know that the quantity in a fraction of a collection varies in relation to the whole collection (e.g. $\frac{1}{4}$ of 16 is a smaller quantity than $\frac{1}{4}$ of 28)
- know that the quantity of a percentage is determined in relation to one whole (e.g. the quantity calculated as 75% of a whole will vary depending on the number the whole represents)
- use different representations of quantity (e.g. blank number line, blank hundred grid)
- use symbolic conventions for representing numbers (e.g. squares, cubes, powers).

Students need to represent numbers where appropriate using concrete materials and make connections with verbal, symbolic and visual representations including virtual images.

In the upper levels, for accuracy and brevity, students use representations that include abbreviations for everyday use such as 20 K for 20 000, \$1.5m for \$1.5 million and 3b for 3 billion, and index notation (e.g. 5^3 , 9^5) and scientific notation for very large and very small numbers (e.g. the distance to the nearest star to Earth — $4.157 \times 10^{10} = 41\,570\,000\,000$ km; $10^{-5} = 0.00001$).

Conservation of whole numbers refers to the understanding that changes in arrangements or representations of numbers do not alter the quantity (or cardinality) of a collection.

- With common and decimal fractions, the size of a fractional piece of a whole may change depending on the size of the whole. For example, $\frac{1}{2}$ the length of the straight on a running track is shorter than $\frac{1}{2}$ the distance around the track; $\frac{1}{2}$ a small pizza is smaller than $\frac{1}{2}$ a family-size pizza.

Place value

As students learn about place value for the base ten system they:

- know the relationship between the places (e.g. ones, tens, hundreds, tenths, hundredths ...; 10 in a place has the same value as 1 in the next place to the left)
- know that the position of a digit determines its value
- know that zero (0) indicates there is none of a particular place (e.g. in 207, there are 2 hundreds, no tens, and 7 ones)
- know that the decimal point is the mark to identify the ones place; that there is symmetry around the ones place with each place to the left being ten times greater, and each place to the right being ten times smaller.

In the Space strand, students learn that zero is also used to indicate the prime meridian (zero degrees longitude) and the equator (zero degrees latitude). In the Chance and Data strand, students may encounter 'zero gradients'.

The connections between the count, position and order, quantity, and place value, when made explicit to students, promote the development of understandings that enable flexibility when working with numbers.

Developing number facts strategies

Students are able to advance their sense of number to multiplicative thinking once they understand that it is easier to work with groups than to use repeated addition. To do this, they need a range of strategies. Some of these are exemplified in the core content of the syllabus and in the advice paper on mental computation, which is available online from the QSA website.

Students combine their sense of number with mental computation strategies to estimate and approximate. They rely on their number sense and their feel for numbers to check the reasonableness of answers.

The main computation strategies to be used are included in the core content of the syllabus under the heading *Connections*. These are the use of addition for subtraction and multiplication for division as each is the inverse of the other.

Mental computation strategies are powerful when learning number facts as they reduce the cognitive load. That is, instead of learning and recalling each fact in isolation, students can apply one or a series of mental computation strategies to a range of familiar and unfamiliar situations. For example, it should be made explicit to students that, when they double, they are multiplying by 2; when they double doubles, they are multiplying by 4; and when they double, double, doubles, they are multiplying by 8. (Mental computation strategies for the four operations are listed in the core content of the syllabus.)

For students to be successful in mathematics, it is essential that they have a sound understanding of numbers. To develop this understanding, classroom activities should build on students' prior learnings, require students to think, reason and work mathematically, and engage students in real-life investigations. Students should also be challenged to pose questions or problems that require the application of numbers and computations similar to those they encounter in investigations.

Resources

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About number

Part 2: Mental computation

Mental computation is the most common form of computation used in everyday life. It is used for quick calculations and estimations, and to check the reasonableness of answers obtained with calculators or other resources. Mental computation is integral to mathematics learning.

National and international literature on this topic suggests that mental computation is strongly linked to the development of number sense. To help them understand how numbers work, students should be provided with opportunities to develop their own mental computation strategies. By gaining experience in dealing with numbers, students develop confidence in *their* ability to make sense of number operations.

Mental computation defined

Mental computation refers to the process of working out and obtaining exact or approximate answers mentally, possibly aided by jottings on paper to assist short-term memory.

When calculating mentally, students select from a range of strategies depending on the numbers being used, the situation encountered, and whether an exact or approximate answer is required. As they develop their repertoire of strategies, students select those that are more efficient and effective, and less reliant on jottings and external devices.

When teaching mental computation in the classroom, the primary learning focus is on the strategies used to obtain answers. Students' use of particular mental strategies is closely linked to their developing understandings of *number concepts*, which includes knowledge of numbers, the relationships between them, and the flexibility of their use. Teachers can build on the already developed strategies that students bring to the mathematics classroom.

Mental computation in a learning environment is more than daily consolidation and practice drills where the focus tends to be on speed rather than on strategic thinking. Such an approach constitutes testing rather than teaching, and is usually referred to as *mental arithmetic*.

Facts about mental computation

- Mental computation contributes to the development of:
 - a deeper understanding of numbers and the relationships between them
 - student-generated strategies for working with numbers
 - creative and diverse thinking and reasoning
 - knowledge, procedures and strategies associated with estimation and problem solving
 - connections between mathematical concepts and procedures across areas of mathematics.
- Calculations in everyday life are predominantly 'done in the head'.
- Mental computation is the simplest way to perform many arithmetical calculations.
- Students are able to generate and use preferred strategies that are efficient and effective for them.
- Mental computation relies on conceptual understandings of number and ways of working with numbers rather than the memorisation of rules and facts.
- An overemphasis on written computation, particularly on standard algorithms, hinders students' effective use of mental strategies.
- Efficiency in mental computation is related to the range of strategies students know and are able to apply in different situations.
- Efficiency in mental computation is also related to the selection and use of strategies that minimise demands on short-term memory.
- Students may use preferred number reference points to assist their computations (e.g. compatible numbers that add to 10, multiples of 10, or multiples of 5).

Mental computation process

In any situation requiring a mental computation, an individual needs to:

- decide whether an exact or approximate answer is required
- decide which operation/s is/are appropriate
- select an appropriate strategy
- carry out the computation
- check whether the result obtained is reasonable in terms of the demands of the situation.

Individuals should be able to explain the thinking and reasoning they have used for their computations.

The selection of mental, written or technological computation methods (e.g. calculators), or a combination of these, may be influenced by the following:

- a student's repertoire of mental computation strategies
- a student's depth of understanding about, and flexible use of, numbers
- the numerical demands of the situation encountered. For example, the numbers involved or the type of answer required such as one requiring four decimal places
- the availability of calculative tools such as writing materials or electronic calculating devices and/or a student's preferred learning style, and the demands of the situation encountered
- a student's confidence with the mathematics embedded in the situation.

Generalising using mental computation strategies

Mental computation strategies can be used with whole numbers, common fractions, decimal fractions and percentages across strands and in a range of contexts.

Diversity characterises the use of mental computation strategies for calculating exact and approximate answers. A particular situation may be solved using many different strategies. For example, $46 + 38$ may be solved by:

- adding $46 + 30 + 8$
- adding $40 + 30, 6 + 8$; then $70 + 14$
- adding $46 + 40$ and subtracting 2.

The strategies listed above would apply directly to the addition of percentages. They could also be applied to decimal fractions. For example, $4.6 + 3.8$ may be solved by:

- adding $4.6 + 3 + 0.8$
- adding $4 + 3, 0.6 + 0.8$; then $7 + 1.4$
- adding $4.6 + 4$ and subtracting 0.2.

Common fractions can also be added mentally using different strategies. For example, $1\frac{1}{2} + \frac{3}{4}$ may be solved by:

- adding $1\frac{1}{2} + \frac{1}{2}$ to make 2, then adding $\frac{1}{4}$
- adding $\frac{1}{4} + \frac{3}{4}$ to make 1, then adding $1\frac{1}{4}$
- adding 3 halves ($1\frac{1}{2}$) and one half, then adding $\frac{1}{4}$ ($\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$).

Some commonly used strategies to work out basic facts and extensions of these strategies to larger numbers are listed below.

Addition

- count-on (in 1s, 2s):
For example, for $6 + 2$ and $2 + 6$, when students think '6 count on 2' or '2 count on 6 in 2s', they could make generalisations about addition and, by extending the strategy to larger numbers, work out:
 - $60 + 20$
 - $36 + 2$
 - $360 + 20$
 - $3600 + 2000$.

- doubles:

For example, if students know **double 4**, they could generalise:

- double 40
- double 400
- double 4000.

If students know the doubles of some common fractions, such as double $\frac{1}{2}$ is 1 and double $\frac{1}{4}$ is $\frac{1}{2}$, they can combine this with their knowledge of doubles of whole numbers to double mixed numbers.

In real-life situations, if students know how to double mixed numbers when measurement is needed, then they will be less reliant on jottings and external devices, and their mental computation skills will be enhanced. For example:

- doubling $2\frac{1}{2}$ hours
- doubling $1\frac{1}{2}$ cups (for recipes)
- doubling $3\frac{1}{2}$ metres.

- near doubles:

For example, if students know that **6 + 7** and **7 + 6** are near $6 + 6$ or $7 + 7$, they could generalise to work out:

- $60 + 70$ using double 60 and adding 10
- $600 + 700$ using double 600 and adding 100
- $60 + 70$ using double 70 and subtracting 10
- $600 + 700$ using double 700 and subtracting 100.

- make to 10:

For example, if students know **3 + 7** and **7 + 3**, they could generalise:

- $30 + 70$
- $700 + 300$.

They could also apply this strategy using other combinations of numbers to 10. For example, if students can work out $9 + 6$ by thinking 'this becomes $10 + 5$ when 1 from the 6 is used with the 9 to make 10', they could apply this strategy to work out:

- $39 + 26$ thinking 'make 39 to 40, and add 25'.

Other student-generated strategies may include:

- breaking up numbers — for example:
 - working out $39 + 25$ by breaking up both numbers and thinking ' $30 + 20 + 9 + 5$ '
 - working out $0.5 + 0.65$ by breaking up one number and thinking ' $0.5 + 0.5 + 0.15$ '
 - working out $\frac{7}{8} + \frac{1}{2}$ by breaking up $\frac{7}{8}$ into $\frac{4}{8}$ and $\frac{3}{8}$ and thinking ' $\frac{4}{8}$ is $\frac{1}{2}$; $\frac{1}{2}$ add $\frac{1}{2}$ makes 1, then add $\frac{3}{8}$ '
- making numbers manageable — for example:
 - working out $28 + 29$ by thinking ' $30 + 30 - 2 - 1$ ' (rounding and adjusting)
 - working out $6.3 + 1.9$ by thinking $6.3 + 2 - 0.1$
 - approximating $\frac{9}{10} + \frac{7}{8}$ by thinking, 'both are close to 1, therefore the answer is a little less than 2'
- compatible pairs — for example, working out $4 + 5 + 6$ by thinking ' $4 + 6$ makes 10, then add 5'; working out $24 + 35 + 76$ by thinking ' $24 + 76$ makes 100, and then add 35'
- using extensions of doubles — for example, knowing double 15, double 25, double 33, double 45, double 75 ... and:
 - working out $25 + 27$ by thinking 'double 25 add 2 more'
 - working out $76 + 82$ by thinking 'double 75 add 1 and 7 more'.

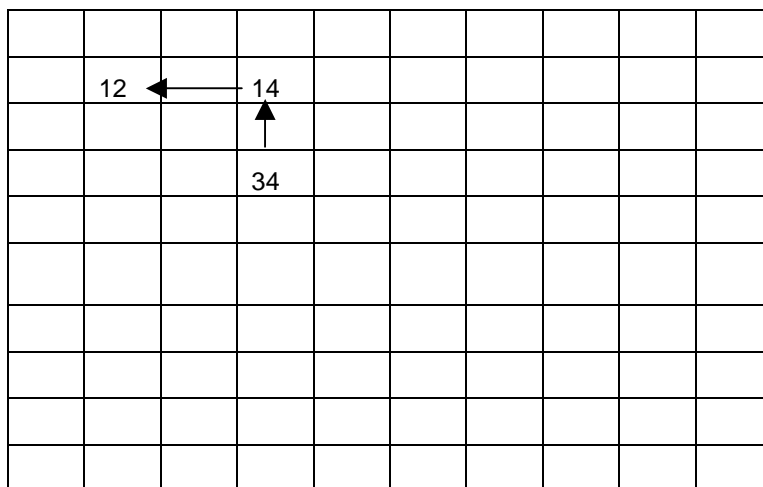
Subtraction

- count-back (in 1s, 2s):
For example, if students know $6 - 2$, they could make generalisations about subtraction and, by extending this strategy to larger numbers, work out:
 - $16 - 2$
 - $36 - 2$
 - $360 - 20$
 - $3600 - 200$.
- changing the operations:
For example, for $13 - 8$, when students think ‘8 add what to make 13’, they could extend the strategy to work out:
 - $33 - 28$
 - $330 - 280$.

Students could apply the same strategy when working out $5.5 - 3$ by thinking ‘3 add what to make 5.5’ and extend this to work out $55.5 - 33$.

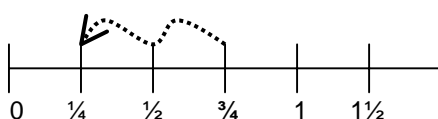
Other student-generated strategies may include:

- addition strategies — for example, students work out $246 - 75$ by thinking ‘75 add 125 to make 200, add 46 to make 246, therefore the answer is 125 add 46’
- breaking up numbers and visualising the computation on a hundred board — for example, students work out $34 - 22$ by visualising the position of 34 on the blank hundred board, go back 20, then go back 2



Blank hundred board

- making numbers manageable — for example, students work out $86 - 38$ by thinking ‘add 2 to both numbers to make the operation manageable as $88 - 40$ ’. The difference between the two sets of numbers remains the same
- visualising:
 - familiar objects — for example, students work out $\frac{3}{4} - \frac{1}{2}$ by visualising a clock face or a rectangle divided into four parts
 - positions of numbers on a number line — for example, students work out $\frac{3}{4} - \frac{1}{2}$



Multiplication

- skip counting:
For example, students may use memorised skip counting sequences such as 3, 6, 9 ... or, for fractions, $\frac{1}{2}$, $1\frac{1}{2}$, 2, $2\frac{1}{2}$... Students should be encouraged to use other strategies that are more efficient and have more flexible applications.
- by 2s:
For example, for 2×6 , when students think doubles, they can extend this strategy to larger numbers:
 - 2×60 by thinking 'double 60'
 - 2×600 by thinking 'double 600'.
- by 4s:
For example, for 4×6 , when students think 'double double 6', they can extend this strategy to larger numbers:
 - 4×60 by thinking 'double 60 to get 120, then double 120', or double double 60.
- by 5s:
For example, for 5×6 , when students think 'multiply by 10 and halve the answer', they could extend this strategy to larger numbers:
 - 5×60 by thinking 'multiply 60 by 10 and halve the answer'. This strategy is generalisable to larger numbers.
- by 6s:
For example, for 6×3 , when students think 'build up from 5×3 and add 3 more', they could extend this strategy to larger numbers:
 - 60×3 by thinking 'build up from 50×3 and add 30 more'.
- by 8s:
For example, for 8×3 , when students think 'double double double 3', they could extend this strategy to larger numbers:
 - 8×30 by thinking 'double double double 30'.
- by 9s:
For example, for 9×3 , when students think '10 times 3 and take off 3', they could extend this strategy to larger numbers:
 - 9×30 by thinking '10 times 30 and take off 30'
 - 19×3 by thinking '20 times 3 and take off 3'.
- place value:
For example, for 14×5 , when students think ' 10×5 and add 4×5 ', they could extend this strategy to larger numbers:
 - 114×5 by thinking ' 100×5 , add 10×5 , then add 4×5 '.
 The same strategy can be extended to decimal fractions. For example, students work out:
 - 6×15.5 by thinking ' 6×10 , add 6×5 , then add 6×0.5 '.

Other student-generated strategies may include:

- breaking up numbers — for example, students could work out 15×8 by thinking ' $5 \times 3 \times 2 \times 4$ rearrange to $5 \times 2 \times 3 \times 4$ ' to make the operation manageable
- doubling and halving — for example, students could work out 17×4 by thinking ' 34×2 , that is, double 34'.

Division

- related multiplication facts:
For example, if students work out $48 \div 6$ by thinking '6 x ? makes 48', they could extend this strategy to work out:
 - $480 \div 6$ by thinking '6 times how many tens makes 48 tens?'

Other student-generated strategies may include:

- breaking up numbers — for example, students could work out $64 \div 4$ by thinking ' $60 \div 4 + 4 \div 4$ '; $78 \div 3$ by thinking ' $60 \div 3 + 18 \div 3$ '
- factoring — for example, students could work out $90 \div 6$ by thinking ' $90 \div 3$ then $\div 2$ '
- visualisation — for example, students could work out $\frac{3}{4}$ of 12 by 'seeing' 12 counters in four groups of 3 and thinking ' $\frac{1}{4}$ of 12 is 3; $\frac{3}{4}$ of 12 is 3 times 3'.

Teaching mental computation

Teachers and students need to share and develop reasons why they value mental computation in a range of life situations. The power and flexibility of mental computation strategies enable individuals to function efficiently and effectively in everyday life. Opportunities for teaching mental computation can be built into teaching sequences used for mathematical investigations involving all strands of mathematics and for other key learning areas.

Students bring a range of personalised strategies to their investigations of life situations. They should be frequently asked to explain how they performed a mental computation, to justify their choice of strategy or strategies, and to evaluate the effectiveness and efficiency of the strategies used. Such discussions contribute to increased understandings and a broadening of the range of strategies available to all students.

To increase students' repertoires of mental computation strategies, teachers need to use visual and oral modes to present the mathematical ideas within an investigation. Teachers need to model a range of mental computation strategies for students and validate student-generated strategies. Teachers need to show that they value student-generated strategies and encourage the exploration of those strategies in different situations.

Students experiment with the same strategy applying it to whole numbers, fractions and percentages, and basic facts. They also experiment with extensions of strategies by applying them to larger numbers. They should be encouraged to create and use different strategies to perform computations to increase their repertoire, and to question the efficiency and mathematical validity of using a strategy in a particular situation.

Some further suggestions for teaching mental computation include:

- encouraging students to solve a problem mentally before resorting to calculators or paper and pencil
- discouraging the view that written methods are superior to mental methods
- emphasising an understanding of the underlying arithmetical processes involved, rather than teaching rules (e.g. to multiply by ten, add a zero)
- emphasising methods for adjusting numbers (other than by using the traditional rounding rules) based on the relationships between the numbers to be used in the calculation
- presenting calculations orally and in writing
- presenting calculations in a range of contexts — for example, involving fractions, money, measures
- helping students identify relationships between numbers by making connections using:
 - concrete materials, number lines (blank and numbered)
 - patterns on number boards and calculators
 - place value
- encouraging the development of 'mind pictures' to support mental computation strategies.

Computations occur in the mind, whether using mental or written techniques. Recording takes place to:

- support the short-term memory
- provide pathway/s to solution/s
- document the solution
- allow solutions to be checked.

As the majority of calculations in the daily lives of adults involve mental computation, it is imperative that students develop the knowledge, strategies and procedures that enable them to make mental computation their first choice in situations that require a calculation.

Strategies associated with mental computation should be developed explicitly throughout the schooling years, and should not be restricted to the recall of basic facts. People who are competent in mental computation tend to use a range of personal methods which are adapted to suit the particular number and situation. Therefore, students should be encouraged to develop personal mental computation strategies, to experiment with and compare strategies used by others, and to choose from amongst their available strategies to suit their own strengths and the particular context.

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