About measurement

Knowledge of measures in common use and skill in making measurements are of practical use every day. Understanding the basic principles of measurement and the confidence that can be placed in various kinds of measurements also assist people to make sensible interpretations of the results of a wide range of inquiries. The ability to make informed judgments about measures is important when actual measurement is difficult or awkward or tolerances large. It is of direct practical benefit for estimating (e.g. The burglar who ran away was about 1.8 m tall) and for judging the reasonableness of a result (e.g. Could the average height of women in Australia really be 217 cm?). For the majority of people, most of the measurements they make are likely to be based on such judgments.

Measurement involves the application of number to spatial (and other) qualities of objects and events. Therefore, spatial, numerical and measurement concepts generally develop together.

A National Statement on Mathematics for Australian Schools, 1991

The Measurement strand involves development of the knowledge, procedures and strategies associated with two topics:

- Length, mass, area and volume, which develops understandings of estimation and measurement of these attributes, associated units of measure and the relationships between them
- Time, which develops understandings of units and conventions associated with measuring and recording the passage and duration of time.

The separation into two topics takes account of the similarities between the measurement of length, mass, area and volume, and the different number systems (base 10) used as the basis for measurement of these attributes as opposed to those used for the measurement of time.

Learning experiences in measurement provide opportunities for learning in other areas of mathematics including number, patterns and algebra, chance and data, and space.

Measurement requires students to develop dexterity in the use of a range of measuring instruments, competence in the use of mathematical processes such as counting, and a sound understanding of whole number and decimal number numeration. Making sense of measurement also requires students to develop personal referents and mind pictures (visualisations) for a variety of measures to aid estimation.

Measurement terms and attributes

The concept underlying measurement is the comparison of one thing with another according to a specified attribute. Attributes can be spatial: length, area and volume; physical: mass (weight) and temperature; or have no obvious physical connection with objects: time. Some measurement attributes seem to be easier for students than others. For example, length and area appear to be easier than volume due to the complexity of spatial visualisation and/or quantifying involved. These difficulties can be reduced if explicit links are made between the one-dimensional, two-dimensional and three-dimensional aspects of these attributes.

Children come to school with early mathematical understandings developed from personal experiences with the major attributes (e.g. length and time) but very little idea of others (e.g. mass). Identification of attributes is not something that should be left to early years teachers only. Many children take years and many experiences to gain complete understanding of notions such as area, volume and mass.

Before students can compare or measure an attribute (e.g. mass), they must be aware of what the attribute is. This has to come from experiencing instances of the attribute and requires careful development of language. Teachers need to help students to perceive measurable attributes and identification of these in the environment is helpful.

As learning in this strand is about what measurement means as well as about how to measure, it is imperative that students develop an understanding of the language of measurement from an early age so they are able to explain and justify their reasoning. Different attributes are sometimes described using the same terms and this is a potential source of confusion. For example, short is used to describe length and height. High and low are used to compare heights and sounds (intangible).

In the early years of schooling, measurement activities that focus on comparisons help students understand the idea that they are measuring specific attributes of objects. Such comparisons should proceed from direct to indirect.

Direct comparisons involve directly aligning the attributes to be compared. Activities should include comparing two similar objects (e.g. two children of different heights) and different objects (e.g. the length of a pair of scissors and a pencil case). Initial activities should involve comparisons of two objects. Comparing and ordering three or more objects is difficult as students are required to identify an object as being both bigger than and smaller than other objects at the same time. Such comparisons lead to the development of transitive reasoning, which is crucial for measurement. Transitive reasoning involves thinking that follows the line that, for example, if A is longer than B, and B is longer than C, then A is longer than C.

Indirect comparison is the process of comparing two objects that cannot be directly aligned, such as the length of a desk and the height of a doorway. An intermediary device, such as a length of paper or a piece of string, is required to assist with the comparison. Subsequently, the notion of using a unit to compare measurements is introduced through non-standard units and followed by standard units.

Non-standard units include things such as body parts (e.g. hand spans, stride lengths) and classroom or kitchen materials (e.g. pencils, paper clips, cups, spoons). Non-standard units are practical, personal and familiar, and are used in real-life situations. With use, some may become handy personal referents for estimation. For example, an individual may know that their hand span is approximately 20 centimetres, that 14 of their normal paces will take them approximately 10 metres, or that a cup holds approximately 250 millilitres. The early use of non-standard units in measurement activities may strengthen links between measurement and number. This is achieved through the development of counting, understandings of the quantity of numbers, and of the position of a number relative to other numbers in the measurement context.

The use of different referents by individuals and variations arising from the use of the same referent to obtain measurements can lead students to see to the need for **standard units**. For example, where students are using paces to measure the distance between two objects, the leg length of students may result in different totals. Standard units enable more accurate and consistent measuring in different places by different people and facilitate communication that yields the same understanding about measurements.

The standardised system of measures used in Australia is metric. Unlike some European countries, Australia uses only measures that are related by 1000 (10^3). The exception is *centimetre*. Understanding that the prefix *milli* denotes one thousandth of a unit (e.g. a milligram is one thousandth of a gram), *centi* denotes one hundredth of a unit (e.g. centimetre), and *kilo* denotes 10^3 or 1000 times a given unit (e.g. a kilometre is one thousand metres) will assist students to understand the relationship between units of measure.

Some non-standard units have historical significance (e.g. feet, hands). **Historical units** of measure include those used from biblical times, such as cubit, and those used in Australia prior to the conversion from the imperial to the metric system of measurement. Familiarity with terms used for imperial measures such as miles and gallons may be helpful for students as these measurements are still used in some countries including the United States of America.

Experience with both non-standard and standard units contributes to the development of estimation skills. Thinking, reasoning and working mathematically in terms of measurement requires the ability to **estimate** in terms of units. Students should be provided with many opportunities to practise estimation of length, mass, area and volume using a variety of senses. For example, sight may be used to estimate length, area and volume, and touch (hefting) to estimate mass. Estimation helps students recognise when a measurement is reasonable and is a lifelong skill used in many professions. Students should be encouraged to estimate and make an informed judgment before they measure.

There are at least three strategies for estimation that students need to learn. These are use of referents, chunking and unitising. Examples of the use of referents include individuals using knowledge of their own height or the height of a metre to estimate the height of a window, or counting 'one cat and dog', 'two cats and dogs', three cats and dogs' … (for seconds) to estimate durations of time. Chunking involves breaking the measurable attribute into 'chunks' then adding the estimations. For example, the length of a room could be estimated using the positions of windows and furniture to divide it into chunks: chunk one being from the corner to the window; chunk two from the window to the table; and chunk three from the table to the corner. Unitising involves dividing the measurable attribute into equal parts of a familiar length. For example, the length of a swimming pool could be estimated by visualising its length divided into a number of equal parts each of a metre. When these strategies are used for estimation, students realise that estimating is a thinking exercise rather than a guessing activity.

Students in the early years may have difficulty understanding that lengths stay the same when an object is moved to a different position (e.g. vertically, horizontally), or that volume remains the same for an amount of water regardless of whether the glass is short and wide or tall and narrow. Similarly, students may have difficulty comprehending that the mass of a ball of dough remains constant when the dough is rolled out into a 'snake'. Conservation is a concept that needs to be taught and understood if students are to successfully solve measurement and space problems. Students need many opportunities to overcome the impenetrability that seems to surround understandings about conservation.

Some students do not attend to the placement of objects when directly comparing objects. For example, when objects of the same length are positioned as shown below, the second object is often identified as being longer. Students need to be directed to look at both ends when comparing length.

Mass poses particular problems for some students because they confuse it with volume, size or quantity. A common misconception is that the larger the size of an object, the greater the mass. This occurs when the judgment made is based on sight (a transfer from learning about length) rather than on feel. For example, students may think that a tennis ball is heavier than a golf ball because it is larger. In relation to quantity, students may think that three foam balls are heavier than a soccer ball because there are more of them.

Units of measure

Comparisons are sufficient when all that is required is a statement about equivalence or non-equivalence, such as 'This ribbon is longer than that ribbon'. When a more definitive judgment is required, however, measurements need to be taken using some unit of measure.

Students need to learn that there are certain conventions to be observed in the **measuring process** to ensure universal understanding of a measurement. These are:

- the whole attribute needs to be measured; there should be no gaps or overlapping
- the unit of measure needs to be appropriate for the attribute being measured. Examples include measuring the 2D attribute of area with 2D non-standard units, such as a plane shape or the standard units of square centimetres; measuring the 3D attribute of volume with non-standard units such as blocks or the standard units of cubic centimetres
- the unit of measure should be uniform (e.g. using straws only to measure length, not a mixture of straws and pens).

Students also need to understand that measurements cannot be compared if different units of measure have been used. The size of the unit and the level of care required for measuring depend on the reason for measuring. Situations in which an exact measurement is not possible because the actual measurement lies between successive units, provide opportunities for discussions about rounding and estimation of fractional parts of a unit. The purpose for measuring will influence the degree of accuracy required.

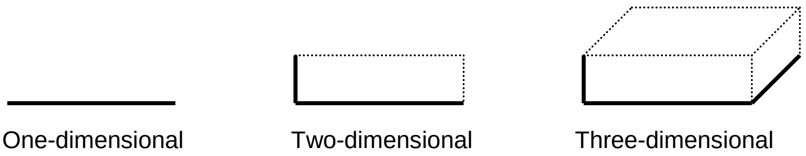
Teaching students to measure with readily available measures, standard and non-standard, provides them with a skill that will help them with practical problem solving.

Relationships

Students need to understand the inverse relationship between the size of a unit and the number of units in the measure. That is, the larger the unit, the fewer required to measure and vice versa.

Students need extensive experiences with different attributes and units of measure to develop their conceptual understandings before embarking on the use of formulas. They should be provided with opportunities to investigate areas and volumes, and to generalise about how and when various attributes are related. For example, from investigations of the areas of rectangles, students could generalise about the relationship between area, length and width.

The relationship between length, area and volume is one-dimensional, two-dimensional and three-dimensional respectively.



Investigations of the volumes of prisms will help students understand how the volume relates to the height and the area of the base of the prism.

The introduction of formulas before students have a sound understanding of the relationships between the different dimensions has been recognised as a cause of many difficulties students experience with area and volume.

Time

Time is a pervasive part of most students' school experiences and the language of time is part of their lives. Learning about time, therefore, can be embedded in daily and weekly routines rather than being taught in isolated contexts. The relative abstraction of the concept of time and the inability for direct comparisons to be made between the length of time being measured and the units being used to measure it set this measurement topic apart from the others.

Where no measuring instruments are available, awareness of the passage of time depends on body rhythms such as hunger and tiredness, periodic events in nature such as sunrise and sunset, and regularly occurring events such as breakfast time and lunchtime. Learning experiences about time should develop students' understandings of the passage and duration of time, and of point in time. Knowledge of point in time and the associated language is critical as once students understand this concept, they are able to communicate their experiences of the passage of time up to and from that point. For example, 'We are at school now. We walked to school this morning. We will walk home after school. Tomorrow is Saturday so there is no school.' Students should be provided with many opportunities to engage in discussions about past, present and future time.

Students' understandings of the passage of time are facilitated by the purposeful use of calendars (across cultures). Initially, they will create timetables and calendars that reflect their idiosyncratic view of the world and use few conventions. The early focus in the use of calendars should be on daily schedules with students sequencing events as they occur. Students can repeat this each day to build up the understanding of a week from the accumulation of daily events. They can then investigate ways of representing the week's activities so all the days are visible on one page. In doing so, the need to use conventions, such as abbreviations to create the sequence and to represent a number of events in a limited space, will become evident.

Because of the abstract nature of the concept of time and the fact that it is not a visible attribute, students begin using standard units and instruments to help them experience the passing of time. They also see the changes in instruments that are used for measuring time. Students need to relate their understanding of number concepts to their experiences with time. This means that it is easier to read a digital clock than it is to read an analogue clock because students are required to read numbers rather than interpret the position of the hands of a clock which point to different numbers that represent different units — hours, minutes and seconds. Students are able to demonstrate higher levels of accuracy reading time on digital clocks much earlier than reading analogue time.

When reading analogue time, many students will attend to the numeral that the hand has passed most recently to read the hour times as they cannot easily connect time to the next hour. For example, they will often say 40 minutes past an hour rather than 20 minutes to the next hour. They need to start to think forward in time and about the relationships that exist between past (e.g. 40 minutes past 8) and future time (e.g. 20 minutes to 9). They also need to use subtraction to come back to the point in time.

As students' understandings of the concept of time develop, they are able to work with 24-hour and 12-hour time interchangeably and explore the relationship between longitude and time zones.

Conclusion

For students to become proficient with measurement, they need to have experiences with a variety of measuring instruments, from commonly used household measures to scientific gauges. Such experiences should be in contexts that are meaningful to students and give them a sense of accomplishment and an appreciation of the usefulness of measurement.

Measurement connects topics within mathematics and with other key learning areas, and provides opportunities for teachers to integrate their curriculum in meaningful ways.

Resources

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