About chance

Chance deals with the concepts of randomness and the use of probability as a measure of how likely it is that particular events will occur.

A National Statement on Mathematics for Australian Schools, 1991

Elements of chance are encountered in many of the experiences students and adults have in their everyday lives. These range from predictions of the likelihood of rain, the sex of a new baby, gaming activities and the possibility of success in games and sports, to decisions about insurance and investments and announcements by business and governments. Even though students are unable to legally participate in many chance activities, such as buying lottery tickets and wagering on races or other sporting events, they will often observe these forms of gambling. Notions of probability are also encountered in many disciplines and areas of study including the physical and biological sciences, and are closely linked with the foundations, applications and interpretations of statistics.

Young people form their own, sometimes erroneous, ideas about chance. These ideas are often based on personal life experiences and preferences. Knowledge about, and an understanding of, chance enable students to make sense of their experiences, identify options, judge the likelihood of events and make informed decisions in situations of uncertainty.

The core learning outcomes of the *Years 1 to 10 Mathematics Syllabus* describe the progress of students' understandings of probability through phases. These are based on the work of Jones et al. 1997, who identified six key concepts related to probability. These are sample space, experimental probability of an event, theoretical probability of an event, probability comparisons, conditional probability and independence.

Likelihood

To develop understandings of chance, students need to experience and discuss a variety of situations that illustrate the concept of randomness. To engage with the concepts of chance, they also need to understand the language of chance (or probability), much of which is used in everyday situations. From an early age students use some of this language when they refer to occurrences or situations as being 'fair' or 'unfair', to individuals and objects as being 'lucky' or 'unlucky', and to the possibility of occurrence of events as 'always', 'sometimes' or 'never'. In the classroom, experiments that involve chance processes support the development and understanding of the everyday language of chance. Some of this language describes a continuum of values of likelihood — for example, from impossible to certain — based on the likelihood of a particular event occurring. As the learning of students progresses, they examine colloquialisms related to likelihood such as 'Buckley's chance', learn the language associated with outcomes of events, and explore theoretical, experimental and conditional probability, and single-stage and multi-stage experiments.

Familiarity with the following language is necessary to understand concepts related to chance:

- **chance experiment** — an activity for which multiple outcomes are possible but we cannot predict which outcome will occur. An experiment is designed to answer questions about a situation and involves a series of actions, observations and recordings of outcomes. For example, an experiment could be conducted to determine the chance of throwing a 6 using a six-sided die. The range of outcomes here is the numbers 1 to 6. If the experiment is repeated, the findings may be different

- **trial** — one performance within an experiment. The experiment with the die, for example, would require the die to be rolled a number of times. Each roll of the die is a trial. For example, the die may be rolled 100 times; the number of trials in this one experiment therefore would be 100

- **outcome** — the result of a trial. For example, when a six-sided die is rolled, the outcome could be any number from 1 to 6 (see sample space)

- **event** — a selected outcome or subset of all possible outcomes. In the experiment described earlier, the event would be the rolling of a 6. In another experiment with a die, the event might be rolling an odd number (1, 3 or 5)

- **probability** — the measure of how likely an event is. For example, 'What is the probability of rolling a 6?'.
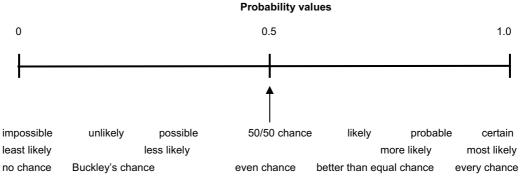
Sample space is another important term used in relation to chance activities. It is used to describe all the outcomes that could occur within an activity or experiment. For example, when rolling a six-sided die, the sample space includes 1, 2, 3, 4, 5 and 6 — all the numbers on the die. If the experiment is rolling two dice and adding the two numbers, the outcomes would be all the whole numbers from 2 to 12. Initially, students may resist, or experience difficulty in, identifying all possible outcomes because of the distractions of favoured outcomes. They need to be supported in their efforts to identify all outcomes within a sample space. The ability to do this accurately is important when students are making judgments about the likelihood of specific outcomes and it is a prerequisite for determining the probability theoretically.
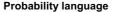
The term **probability values** is used to describe the continuum of probabilities for outcomes in any sample space. Initially, students use words to describe the limits of the continuum — from impossible to certain — and anything in between — possible.

As students' understandings of chance develop, they progress to the use of numbers to describe the probability values on the continuum of probabilities. This continuum has limits of 0 — impossible, will not occur — and 1 — certain, absolutely sure to occur. These limits can also be represented using percentages (0% and 100%), decimal fractions or common fractions.

For example, an event that is likely to occur can have its probability estimated at 80%, 0.8 or $^4/_5$ and a very unlikely event could have its probability estimated at 5%, 0.05 or $^1/_{20}$. An event that is just as likely to occur as not could be estimated as having a likelihood of about 50%, 0.5, ½ or be described as 50-50.

Choosing the appropriate representation for describing the probability will assist with interpretation. For example, it is easier to say that the probability of rolling a 4 on a standard die is $^1/_6$ (one-sixth) rather than a decimal fraction (0.166666….) or a percentage (16.666666….%) because recurring decimals are involved.

Probability values

0	0.5	1.0

impossible	unlikely	possible	50/50 chance	likely	probable	certain
least likely		less likely			more likely	most likely
no chance	Buckley's chance		even chance	better than equal chance		every chance

Probability language

Experimental probability is another common term used in relation to chance. It is the actual likelihood of the occurrence of a particular outcome obtained from the results of an experiment by dividing the number of successful trials by the total number of trials. To find the experimental probability of being dealt an ace in a card game, it is necessary to deal many different hands (under the same conditions) and to record the number of trials and the number of times an ace is dealt. For example,

Number of trials = 30

Number of times an ace is dealt = 10

Experimental probability = 10/30 = α = 33.3%

Experimental probability can focus on those sample spaces where the outcomes are equally likely or not equally likely. Experiments involving equally likely outcomes that could be conducted include tossing a coin, spinning a spinner with coloured sections of equal size, and drawing a counter from a bag containing counters of different colours and recording the outcomes of each trial. Experiments where outcomes are not equally likely could include rolling two dice and recording the sum in a table or graph, dropping a drawing pin or matchbox and recording the outcomes. As students' understandings develop, they use a variety of numerical values to describe the estimated probability.

Relative frequency is the term used to compare how often an event occurs with the number of trials performed. It is the ratio of the number of observations of an outcome to the total number of observations. For example, in an experiment to investigate the chance of landing on red when using a spinner with four equal sectors, 20 trials were conducted and the event (landing on red) occurred six times. The relative frequency is 6:20. Relative frequency is another description of experimental probability.

Ratio is also used to describe 'odds'. In the experiment with the spinner, six of the 20 trials were successful and 14 were failures. The 'odds against' landing on red in this experiment are therefore described as 14:6 (failures:successes). The 'odds in favour' of landing on red would be described as 6:14.

The term 'success fraction' is another way of describing experimental probability. For the experiment with the spinner described above, the success fraction is 6/20.

As the number of trials increases, the experimental probability gets closer and closer to the theoretical probability.

Theoretical probability describes the chances of events occurring by calculating results that would occur under ideal conditions. It is the chance that something should happen in theory and can be worked out by a mathematical calculation. The theoretical probability of a toss of a coin resulting in a 'head' is ½ or 50% because there are two sides to a coin — two possible outcomes.

When all the outcomes are equally likely, the theoretical probability of an outcome is determined by dividing one by the number of outcomes.

Using the example of a spinner, theoretical probability of any arrangement of sectors can also be worked out.

- If a spinner is divided into four equal sectors (90° each), each section has a theoretical probability of ¼, 25% or 0.25 of being spun.
- If another spinner has eight equal sectors (45° each), the theoretical probability of each sector being spun is ⅛, 12.5% or 0.125.

Similarly, the faces on a cube (such as a die) have equal chances of being rolled and the theoretical probability of each outcome can be calculated. As the faces of a die are equally likely to be rolled, the theoretical probability of any number on the face of a six-sided die is $\frac{1}{6}$.

In contrast, when rolling two dice and recording the sum, the chances of rolling numbers between 2 and 12 are not equally likely. This is illustrated in the table below.

1st die

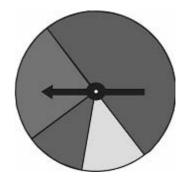| 2nd die	+	1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Sample space of sum of numbers when rolling two dice

When using spinners, different combinations of colours can be used to create situations where different theoretical probabilities are involved:

- red can occupy half of the circle (theoretical probability ½)
- blue can occupy one-quarter (theoretical probability ¼)
- green can occupy one-eighth (theoretical probability ⅛)
- yellow can occupy the remaining one-eighth (theoretical probability ⅛)

Equal sections – equal probabilities   **Unequal sections – unequal probabilities**

To be able to distinguish between situations where the outcomes are equally likely to occur and those where they are not, students need to examine the conditions that might affect likelihood of outcomes, such as the shape and/or size of an object. Students should be provided with opportunities to engage in activities and conduct experiments to gather data where both situations can be examined and compared.

The term conditional probability is used to describe probability under certain conditions. It is the probability of an event occurring given that one or more other events have taken place. In experiments where no replacement is made after a specific outcome has occurred, the probabilities of the remaining outcomes change because the number of possible outcomes is fewer.

For example, in an experiment or card-drawing game using a pack of 52 cards, the probability of drawing a heart is 13 in 52 or 1 in 4. After the first card has been drawn, the probability changes depending on the conditions applied to the experiment. If the first card drawn is replaced, the probability remains 1 in 4.

Where there is non-replacement of drawn cards, the probability of drawing a heart changes depending on whether the first card drawn was a successful trial; that is, if it was a heart.

The nature of an activity usually determines whether or not replacement occurs. For example, when tossing coins, rolling dice and using spinners, replacement always occurs because of the nature of the materials and the next toss, roll or spin is independent of previous events.

While the notion of replacement (or not) may be part of the conditions of activities and experiments in early chance experiences, conditional probability is formalised and calculated only when students have consolidated their understandings of probability.

During practical activities and experiments, students experience **simple events**, that is, events that are produced by a single random generator. Examples include an experiment involving tossing a coin and recording 'head' as the outcome of a trial, or recording '5' as the outcome of a trial in an experiment involving rolling one die.
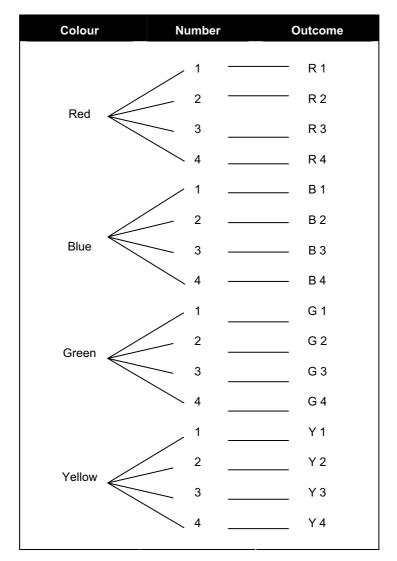
These should be compared with **compound events**. Some compound events occur with single-stage experiments. For example, in drawing a single card from a deck, getting a card that is both black and a colour is a compound event with a single-stage experiment. Another compound event with a single-stage experiment is getting a card that is black or a colour. Compound events also occur with multistage experiments. For example, in rolling two dice, the event of getting a 4 on one die and a 6 on the other is a compound event. A more complex compound event involves getting an even number on each die.

During practical activities and experiments designed to gather data about frequency of occurrence and likelihood, a condition might exist where several outcomes are grouped together and recorded as a result under that group. For example, the outcomes of rolling two dice might be recorded in two groups — group one is any sum of the faces that is less than 7, and group two are those sums of 7 and above. This is a **multi-outcome event**. Group one would contain the outcomes 2, 3, 4, 5 and 6. Another example of a multi-outcome event is rolling an odd number when using a six-sided die. The possible multi-outcomes are 1, 3 and 5.

As students' understandings develop, they use the data collected from experiments to represent and model probabilities using lists, tables or tree diagrams as appropriate. The data and the models could be used to support comments about fairness and bias. The following models show the sample space of equally likely outcomes of a two-stage experiment involving spinning two spinners — one with four equally likely numbers and one with four equally likely colours.

List	Table					
Red and 1				**Number**		
Red and 2						
Red and 3						
Red and 4			1	2	3	4
Blue and 1		Red	R 1	R 2	R 3	R 4
Blue and 2		Blue	B 1	B 2	B 3	B 4
Blue and 3						
Blue and 4	**Colour**	Green	G 1	G 2	G 3	G 4
Green and 1		Yellow	Y 1	Y 2	Y 3	Y 4
Green and 2						
Green and 3						
Green and 4						
Yellow and 1						
Yellow and 2						
Yellow and 3						
Yellow and 4						

Tree diagram (Two-stage experiment)

Colour	Number	Outcome

Red
1 — R 1
2 — R 2
3 — R 3
4 — R 4

Blue
1 — B 1
2 — B 2
3 — B 3
4 — B 4

Green
1 — G 1
2 — G 2
3 — G 3
4 — G 4

Yellow
1 — Y 1
2 — Y 2
3 — Y 3
4 — Y 4

These representations may assist students to calculate theoretical probability.

Using a table to represent an experiment with more than two stages is difficult; however, expanding a tree diagram for more than two stages is an easier way to represent the data. For example, when looking at the probability of having two girls and one boy in a family of three children, a tree diagram is an easy way to represent the sample space and find the event whose probability is required.

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              B  <  B   BBB     1/8   3 boys
                     G   BBG
   B  <                                3/8   2 boys, 1 girl
              G  <  B   BGB
                     G   BGG
                     B   GBB
   G  <       B  <                    3/8   1 boy, 2 girls
                     G   GBG
              G  <  B   GGB
                     G   GGG     1/8   3 girls
```

(8 possible outcomes)

Family of three children

Judgments

Subjective judgments

A range of factors, including personal experiences and ideas, and the opinions of others such as parents/carers, older siblings and peer groups, shape a student's views about familiar events. Irrespective of available data, young students' statements and opinions are often clouded by subjectivity. They are also often distracted by irrelevant factors. For example, they may believe that the next firework to explode or coloured bead to be selected will be red because red is their favourite colour.

Subjectivity may persist for some students over several years, despite their engagement with experimental data. This needs to be regularly challenged through practical activities and experiments.

Numerical judgments

Students require assistance with the transition from subjective thinking to naïve quantitative thinking. Young students are often reluctant (or unable) to list all the possible outcomes of experiments (Jones et al. 1997). The design of practical activities and experiments is critical in the development of students' learning, and the collection of numerical data is a key aspect of this.

Learning experiences should be designed around practical activities and experiments where data are gathered and organised, then analysed and discussed. Teachers can ask students for their predictions about the likelihood of the outcomes prior to the collection of data and discuss the reasoning that leads them to make their predictions. Students then use their collected data to make judgments or statements about the relative likelihood of outcomes and compare the experimental data with their predictions. The collected data should form the basis of convincing arguments, be used to challenge misconceptions and wrong assumptions, and predict the results of further experimentation and/or events.

The language associated with numerical judgments develops from comparing relative frequencies (e.g. this event occurred twice as often as that one), and describing the likelihood of individual events (e.g. this event occurs about once every four throws), to the identification or estimation of the probability of any event (e.g. this event has a 20%, $^1/_5$ or 0.2 probability of occurring). These notions are involved in the development of students' abilities to recognise when the probability of an event is dependent on, or independent of, the occurrence of other events, especially previous events.

Students can use extrapolations from simplified explorations for future activities, such as developing strategies to use when playing games of chance or to create rules when devising new games.

Dependence, independence and randomness

When students begin exploring aspects of chance, they will not be aware of the implications of the notions of dependence and independence of events but may make statements about the perceived fairness or unfairness of certain procedures. Comments such as 'it's not fair' may be implying that the randomness of selections for things such as favourite tasks or teams has not been guaranteed or, if someone's name is drawn a second time before everyone has had a turn, such a comment is recognising that a non-replacement condition should be applied.

When discrete materials such as cards and counters are involved, a choice about replacement (or not) of a drawn card or counter must be made. If occurrences are not replaced, the probabilities of future events are dependent on those that have already occurred. That is, they are **dependent events** — the outcome of one event depends on the outcome of another. For example, when a red marble is drawn from, and not returned to, a container holding equal numbers of red, green and blue marbles, the probability of drawing another red marble is reduced.

A further example exists in an experiment to find the probability of drawing a heart from a pack of cards with non-replacement, when the first card drawn is a heart, the probability of drawing another heart changes to 12 out of 51 (12 hearts remain in the pack of 51 cards). If the first card drawn is not a heart, the probability of drawing a heart is 13 out of 51 (13 hearts still remain in the pack which now contains 51 cards). If the second card drawn is also a heart, the probability changes to 11 out of 50. If the second card is not a heart, then the probability is 12 out of 50. Each event is dependent on the previous event.

Events that have no influence on each other are said to be **independent events** — they are unconnected with one another and have no bearing on each other. For example, the toss of a coin is not linked to other tosses of the same coin or tosses of other coins. If a student has tossed a coin nine times and has nine tails as outcomes, the student may think that the next toss will result in a head — however, the chance of that occurring is still 50-50 as coins have no memory. Each trial is independent.

The lack of a predictable order and pattern of events is described as **randomness** and is seen in many everyday events and natural phenomena. For activities and experiments, certain conditions must be present to ensure that outcomes occur in random order. For example, when counters are used, they should be all of the same size and texture, and the selection should be a 'blind' process. A deck of cards should be well shuffled and none of the cards should 'stick' together. The 'randomness' notion is closely tied with the notion of 'fairness' and all outcomes having an equal chance of occurring.

The teaching of probabilistic thinking presents a challenge for teachers as it presents concepts that may bring into question students' intuitive beliefs. Real-life mathematical investigations can provide students with opportunities 'to understand issues of probability and chance, and the consequences of intuitive thinking in their everyday lives' (Owens & Perry 2000, p. 112).

Chance has strong connections with other topics in the Mathematics key learning area and provides opportunities for teachers to integrate mathematics with other key learning areas.

Resources

Australian Education Council and Curriculum Corporation 1991, *A National Statement on Mathematics for Australian Schools*, Curriculum Corporation, Victoria, Australia.

Booker, G., Bond, D., Briggs, J., & Davey, G. 1997, *Teaching Primary Mathematics*, 2nd edn, Longman, Australia.

Jones, G. (ed.) 2005, *Exploring Probability in School: Challenges for Teaching and Learning*, Springer, New York.

Jones, G., Langrall, C., Thornton, C., & Mogill, A. 1997, 'A framework for assessing students' probabilistic thinking in instruction', *Educational Studies in Mathematics*, 32, 101–125.

Nisbet, S. 2000, 'The development of students' thinking in chance & data', in *Mathematics — Making Waves*, eds. M. Goos & T. Spencer, Proceedings of the 19th Biennial Conference of the Australian Association of Mathematics Teachers Inc., Brisbane, 13–17 January 2003, Australian Association of Mathematics Teachers Inc., Adelaide.

Nisbet, S., Jones, G., Thornton, C., & Langrall, C. 2000, 'A dicey strategy to get your M and Ms', *Australian Primary Mathematics Classroom*, 5 (3), 19–22.

Owens, K., & Perry, B., 2000, *Mathematics K–10 Literature Review*, Board of Studies, New South Wales.

Watson, J., Collis, K., & Moritz, J. 1995, 'Children's understanding of luck', *Mathematics Education Research Group of Australia Conference Proceedings*, Darwin.

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