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| Strand: Patterns and Algebra | | Topic: Patterns and functions |
| Foundation Level: Level statement  Students investigate patterns in their environments and are developing an awareness of ‘same’ when matching | | |
| Example learning outcomes:  Students copy a given pattern by choosing items from a limited selection.  Students backtrack actions in familiar routines. | | |
| Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically | | |
| Students know:  patterns and routines (real or representations of these) have elements (auditory, visual or physical) that are repeated  patterns and routines (real or representations of these) have elements (auditory, visual or physical) that can change  everyday language that relates to patterns and routines. | Students may:  participate in making patterns in everyday situations or routines or representations of these:   * clapping patterns related to rhymes, chants and songs * horticulture (e.g. planting using a pattern such as a pink flower followed by a white flower) * traffic lights (e.g. red, green, orange, red …) * games (e.g. turn taking — your turn, my turn) * setting the table (e.g. fork, knife, spoon) * setting a table at a picnic, at home, at school * cooking (e.g. bread, butter, range of fillings) * dancing (e.g. step, clap, step, clap; change the step to a jump — jump, clap, jump, clap)   participate in songs and rhymes where one action, item or word changes on each repetition  copy a simple repeated pattern involving either auditory, visual (2D or 3D), tactile or kinaesthetic elements by matching each element  one item at a time  sort everyday objects, such as toys, money, food, utensils, photographs or drawings, that are significant and familiar  label sorted groups of objects using categories such as ‘food’ or general function words such as ‘eat’  propose, anticipate or perform the next action or element in a familiar pattern or routine  participate in the development of a picture book that illustrates the pattern of a familiar routine  use a familiar picture book to guide actions through a routine  participate in a routine that has one element changed   * investigate routines that can be backtracked: * bus to swimming pool, swimming pool to bus * taxi to classroom, classroom to taxi * classroom to playground, playground to classroom * clothes off, clothes on during toilet and swimming routines. | |

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| Level 1: Level statement  Students identify and describe patterns in their environments. They create or continue patterns and know that some can continue indefinitely, and some radiate in a number of directions. They represent the same pattern in different ways. They describe patterns or change in terms of a simple rule and can undo a pattern or change by reversing the rule.  Students describe the number value of a group of objects as ‘equal to’, ‘different from’ or ‘the same as’. They know that the number value of a group of objects stays the same  when rearranged or represented in different combinations. | | |
| Core learning outcome: PA 1.1  Students identify, describe and create patterns and change based on simple rules. | | |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | | **Core content** |
| Students know:  patterns are based on rules  how to identify patterns  how to create patterns  patterns are described as repeating parts based on a simple rule  a consistent change can  be described using a  simple rule. | Students may:  describe similarities and differences in patterns  identify and distinguish between patterns and non-patterns  identify and describe change based on simple rules  create and describe different patterns using consistent change rules  identify the repeating part of a pattern and describe the next part using the identified rule  check for consistency of change both forwards and backwards  represent a pattern in a variety of ways (e.g. on a blank grid)  translate patterns into different representations that use the same rule (e.g. using sounds, actions or materials)  check translated patterns for consistent use of the identified rule  identify, describe and create patterns involving spatial materials  identify the ‘growing part’ of a growing pattern (e.g. for the growing pattern a b; aa bb, aaa bbb, aaaa bbbb, a and b are each growing by one letter  identify and distinguish between patterns that repeat (e.g. a,b; a,b; a,b) and patterns that grow (e.g. a b; aa bb, aaa bbb, aaaa bbbb)  create rules for patterns that grow  create patterns and continue patterns that radiate in a number of directions  reverse consistent change and explain reasoning based on identified simple rules. | **Patterns**  repeating  non-patterns  spatial  translating patterns into different representations  growing  skip counting with calculators (addition and subtraction)  Functions  rules describing consistent change of one attribute both forwards and backwards (backtracking) |
| Investigations should occur in a range of contexts. For example, students could investigate:  paving and floor designs  arrangements of windows on buildings  fencing designs  designs for wrapping paper or decorative borders involving patterns  adapting dance routines. | | |

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| Level 2: Level statement  Students use rules to create and describe number patterns based on addition and subtraction. They identify number sequences that are not patterns. They complete missing  parts of, or continue, a number pattern when given the rule. They know the inverse relationship between addition and subtraction and use this to apply and then reverse simple rules.  They display the inputs and outputs of the application of rules in table form.  Students represent addition and subtraction situations using equations. They recognise and describe the equivalence or non-equivalence of two sides of an addition or subtraction equation (number sentence) and determine an unknown using a variety of self-generated and learned strategies. | | |
| Core learning outcome: PA 2.1  Students create and explain patterns, identify and describe relationships using rules and use backtracking to reverse the effects of rules involving addition and subtraction. | | |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | | **Core content** |
| Students know:  patterns are based on rules  rules describe relationships between terms  how to identify rules  how to describe relationships using rules  how to create patterns using rules  how to explain patterns using rules  addition and subtraction are the inverse of each other  backtracking reverses the effects of rules involving addition and subtraction  how to use the inverse relationship between addition and subtraction for backtracking. | Students may:  describe rules for repeating patterns and growing patterns  identify non-patterns  explain repeating parts of a pattern  explain growing parts of a pattern  use the rule for a pattern to determine the next term  identify the term before the starting point of a pattern using the rule (e.g. identify the number that comes before 6 in the pattern 6, 8, 10 …)  record patterns in a variety of ways  identify and explain the missing parts or errors in repeating and growing patterns  use the rule to translate repeating and growing patterns into number patterns  use the rule to determine the next, other, or missing number in a pattern  identify the use of addition and/or subtraction in the rule  create other patterns using the same rule  backtrack to reverse the effects of rules  explain the backtracking process involving addition undoing subtraction and vice versa  gather or generate data involving a relationship between input and output (e.g.1 kg input → output cost $2.00; 2 kg input → output cost $4.00)  experiment with input → output using rules  record results from input → output experiments on a table  identify and describe the relationships between input → output data using rules  use a rule and the relationship between sets of data to determine the next and other values  develop data sets that are similar using relationship rules (e.g. cost per kilogram for different fruit or vegetables)  create and describe other change rules (e.g. double the number that is the input in a function machine). | **Patterns**  missing terms  non-patterns or patterns with errors  spatial  translating patterns into number patterns  number patterns   * rules based on previous term * skip counting with calculators (constant function key)   **Functions**  input → output (function machines)  backtracking (inverse)   * addition or subtraction * reversing a change   relationship rules   * given * student-generated * identified   represent input → output data in table form  rules relating two sets of data |
| At each level, investigations should occur in a range of contexts. For example, students could investigate:  how to arrange books in patterns (colour, height, thickness) on a shelf  create a patchwork quilt with materials of different shape, size or texture  counting patterns using addition and subtraction  sequences of street or seat numbers  games with function machines  scoring patterns in codes of football or cricket. | | |

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| Level 3: Level statement  Students describe relationships between sets of numbers in terms of functions or rules. They draw tables and graphs to display these relationships. They know the inverse relationship between multiplication and division and use this to reverse the effect of a rule or change.  Students represent and describe equivalence in everyday situations. They determine the missing part of an equation (number sentence) that requires either multiplication and division or addition and subtraction using a systematic guess and check strategy. | | |
| Core learning outcome: PA 3.1  Students create and continue number patterns, identify, describe and represent relationships between two quantities and use backtracking to reverse any one of the  four operations. | | |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | | **Core content** |
| Students know:  number patterns use rules  how to create and continue number patterns  how to identify and describe relationships between two quantities  relationships between two quantities can be represented using rules, tables and graphs  how to represent relationships between two quantities  backtracking reverses the effects of rules involving the four operations  mental computation strategies and computation methods for the four operations  multiplication is the reverse  of division  how to use relationships between addition and subtraction and between multiplication and division to reverse operations. | Students may:  identify and describe a pattern rule involving any one of the four operations  identify consistent rules for a variety of given number patterns using any one of the four operations  create number patterns using any one of the four operations based on a given, identified or student-generated rule  check for consistent application of a rule and identify non-patterns or patterns with errors  use rules in reverse to undo a change or to determine a missing term (e.g. to find the missing term  or the previous term in a pattern, such as …, 9, 15, 22, 30,…, 49)  explain the process identifying the inverse operation  continue a number pattern using mental computation strategies and computation methods  describe the effect of changing the operation used to create a pattern  describe the effect of changing the starting number/s of a pattern  use the inverse of operations to apply backtracking to reverse the effect of the rules  check for consistency of the use of the inverse of the rule when backtracking  identify and represent relationships between two quantities and describe as a rule or consistent change  use the rule to continue patterns forwards and backwards based on the previous term  create patterns using different rules and represent the generated or gathered data on tables  and graphs  identify the position of any term based on the application of the rule using knowledge and relationships of operations, mental computation strategies and computation methods. | **Patterns**  number (including patterns with decimals)   * rules based on previous term * calculators (whole and decimal numbers involving any operations) * missing term * non-patterns or patterns with errors * rules based on the position of terms (one operation only)   Functions  input →output (function machines)  backtracking (inverse)   * multiplication or division * reversing a change   representations of relationships  rules, tables, graphs |
| Investigations should occur in a range of contexts. For example, students could investigate:  patterns used in public transport timetables  scoring used in games  historical number patterns, such as Fibonacci sequence  relationships between two sets of data, such as the purchase price for items, the time of day and the temperature, age and growth, computer game scores  how to finish partially completed tables. | | |

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| Level 4: Level statement  Students identify and create representations of patterns and functions and use their knowledge of functions and inverses to determine unknowns within equations or any  position in a pattern. They apply combinations of the four operations, observing the order of operations and the presence of brackets.  Students manipulate and solve simple equations using strategies that maintain balance. They identify relationships between sets of data and distinguish between discrete  and continuous data represented in graphs and tables. | | |
| Core learning outcome: PA 4.1  Students identify and create representations of patterns and functions and apply backtracking to solve simple equations that involve combinations of the four operations. | | |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | | **Core content** |
| Students know:  a function is a mathematical relationship between two values  how to identify rules for patterns and functions  how to create patterns and functions using rules  how to represent functions  how and when to apply backtracking to solve simple equations involving combinations of the four operations  mental computation strategies and computation methods for operations. | Students may:  describe discrete data as data that is based on counts and have a finite number of values  identify and explain situations involving discrete data (e.g. matchstick puzzles)  describe continuous data as data that can be measured and take on any value over a given interval  (e.g. height of individuals, daily air temperatures, volume of an inflating balloon)  identify and explain situations involving continuous data (e.g. the relationship between height and foot length, time and water flow)  explain the relationship between sets of data presented in a table or graph  represent relationships as a rule or function involving combinations of operations (e.g. for a bath  containing 25 litres of water into which 5 litres flows each minute, the rule is: the amount of  water = (number of minutes x 5 + 25)L)  select and create representations of data using ordered pairs, tables, graphs to find a simple rule or function and justify the selection  use representations to determine or predict values of particular terms (e.g. How much will be in a  container after two hours?)  use representations to comment on trends  solve problems that require the reversing of a function expressed as an equation and represent on  graphs, tables or with sets of ordered pairs (e.g. When will a container be half full?)  identify the combinations of operations used in equations and apply the inverse to backtrack  identify the order of calculations and describe the effect on backtracking  use knowledge of operations, mental computation strategies and computation methods to solve problems. | **Patterns**  rules based on the position of terms (combinations of operations)  calculator number patterns  ordered pairs and graphs (with discrete data only)  **Functions**  input → output (with combinations  of operations)  rules relating two sets of data  backtracking (inverse)   * with combination of operations   representations of relationships   * ordered pairs * tables, line graphs, equations (number sentences) * trends * discrete data * continuous data   electronic, manual |
| **Investigations should occur in a range of contexts. For example, students could investigate:**  internet charges, such as connection fees, download costs, and monthly charges  contents in a container, such as filling a tank with water or a silo with produce  fees for borrowing books/DVDs, such as reservation fees and penalties for late returns  relationship between time elapsed and temperature change, such as the cooling time for hot drinks  relationship between quantities required for a project, such as the number of pavers to surround a given number of trees  results tables for sporting competitions using points for wins, draws and losses. | | |

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| Level 5: Level statement  Students identify when relationships exist between two sets of everyday data and use functions expressed in words or symbols, or represented in tables and graphs, to describe  these relationships. They identify relationships that are linear and express these using equations.  Students use algebraic reasoning and conventions, including graphical representations, to solve problems and justify their solutions. | | |
| Core learning outcome: PA 5.1\*  Students interpret and compare different representations of linear and simple non-linear functions and solve the related problems. | | |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | | **Core content** |
| Students know:  linear functions form a straight line graph  simple non-linear functions  how to distinguish between linear and simple non-linear functions  different ways of representing linear and simple non-linear functions  how to interpret representations of linear and simple non-linear functions  how to compare different representations of linear and non-linear functions  mental computation strategies and computation methods for operations  methods for solving linear and simple non-linear equations. | Students may:  classify life-related situations into those that can be modelled as linear functions (e.g. simple interest) or simple non-linear functions (e.g. area of garden beds)  identify different representations of the same situations  interpret and compare different representations of the same situation to determine the most appropriate  distinguish between the independent and dependent variables  analyse and interpret sets of ordered pairs, trends or graphs to develop a rule in words and symbols  identify and discuss situations involving either discrete or continuous data  select and create representations of discrete and continuous data manually or electronically (e.g. graphs, tables, ordered pairs and rules expressed in words or symbols)  use a graph to find the value of the dependent or independent variable given the value of the other variable  substitute into the rule to find the value of the dependent or independent variable given the other variable and use backtracking techniques. | Patterns  Functions  ordered pairs (four quadrants)  representations of variables   * words * symbols   linear models   * representations (tables, line graphs, linear equations, proportion equations) * dependent and independent variables * discrete and continuous data * trends   non-linear models   * dependent and independent variables * discrete and continuous data * representations (tables, line graphs) * trends   representations of relationships  electronic, manual |
| At each level, investigations should occur in a range of contexts. For example, students could investigate:  hiring rates for things such as clothing, equipment, sporting goods or venues  catering requirements for events  construction costs for landscaping features. | | |

**\*** This outcome may be best demonstrated in conjunction with PA 5.

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| Level 6: Level statement  Students analyse problems from realistic situations and model them with equations using algebraic symbols, graphs and tables. They select and present representations that best display  the relationships. They provide solutions or make predictions based on these models. | | |
| Core learning outcome: PA 6.1\*  Students create mathematical models of realistic situations and use interpretations of the models to draw conclusions or make decisions. | | |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | | **Core content** |
| Students know:  how to create mathematical models of realistic situations  how to interpret mathematical models of realistic situations  how to represent mathematical models algebraically  how to draw conclusions from mathematical models  how to use interpretations of models to draw conclusions  or make decisions. | Students may:  identify possible models to represent a realistic situation  apply appropriate processes to create a mathematical model by:   * determining whether the situation involves discrete or continuous data * representing the model using tables, graphs or algebraic equations * identifying and analysing the trends using dependent and independent variables to describe the rule * determining the impact of variables on the situation and solving linear equations as required   discuss and justify the reasonableness of the created or selected model  interpret mathematical models and draw conclusions  make and justify decisions for future actions or consequences based on interpretations or conclusions. | Patterns  Functions  linear models   * equations * representations (tables, graphs) * trends   non-linear models   * representations (tables, graphs) * trends   representations of relationships  electronic, manual |
| **Investigations should occur in a range of contexts. For example, students could investigate:**  travel networks with distance covered versus time taken  the length of queues against the time taken to be served  fitness levels by monitoring heart rate before, during and after exercise  changes in crowd numbers at a rock concert or sporting event as the starting time approaches. | | |

**\*** This outcome may be best demonstrated in conjunction with PA 6.2.