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| Strand: Patterns and Algebra | Topic: Patterns and functions |
| Foundation Level: Level statementStudents investigate patterns in their environments and are developing an awareness of ‘same’ when matching |
| Example learning outcomes:Students copy a given pattern by choosing items from a limited selection.Students backtrack actions in familiar routines. |
| Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically |
| Students know: patterns and routines (real or representations of these) have elements (auditory, visual or physical) that are repeated patterns and routines (real or representations of these) have elements (auditory, visual or physical) that can changeeveryday language that relates to patterns and routines. | Students may:participate in making patterns in everyday situations or routines or representations of these: * clapping patterns related to rhymes, chants and songs
* horticulture (e.g. planting using a pattern such as a pink flower followed by a white flower)
* traffic lights (e.g. red, green, orange, red …)
* games (e.g. turn taking — your turn, my turn)
* setting the table (e.g. fork, knife, spoon)
* setting a table at a picnic, at home, at school
* cooking (e.g. bread, butter, range of fillings)
* dancing (e.g. step, clap, step, clap; change the step to a jump — jump, clap, jump, clap)

participate in songs and rhymes where one action, item or word changes on each repetitioncopy a simple repeated pattern involving either auditory, visual (2D or 3D), tactile or kinaesthetic elements by matching each element one item at a time sort everyday objects, such as toys, money, food, utensils, photographs or drawings, that are significant and familiar label sorted groups of objects using categories such as ‘food’ or general function words such as ‘eat’propose, anticipate or perform the next action or element in a familiar pattern or routine participate in the development of a picture book that illustrates the pattern of a familiar routine use a familiar picture book to guide actions through a routine participate in a routine that has one element changed * investigate routines that can be backtracked:
* bus to swimming pool, swimming pool to bus
* taxi to classroom, classroom to taxi
* classroom to playground, playground to classroom
* clothes off, clothes on during toilet and swimming routines.
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| Level 1: Level statementStudents identify and describe patterns in their environments. They create or continue patterns and know that some can continue indefinitely, and some radiate in a number of directions. They represent the same pattern in different ways. They describe patterns or change in terms of a simple rule and can undo a pattern or change by reversing the rule.Students describe the number value of a group of objects as ‘equal to’, ‘different from’ or ‘the same as’. They know that the number value of a group of objects stays the same when rearranged or represented in different combinations. |
| Core learning outcome: PA 1.1Students identify, describe and create patterns and change based on simple rules. |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | **Core content** |
| Students know:patterns are based on rules how to identify patterns how to create patterns patterns are described as repeating parts based on a simple rulea consistent change can be described using a simple rule. | Students may:describe similarities and differences in patternsidentify and distinguish between patterns and non-patternsidentify and describe change based on simple rulescreate and describe different patterns using consistent change rulesidentify the repeating part of a pattern and describe the next part using the identified rulecheck for consistency of change both forwards and backwardsrepresent a pattern in a variety of ways (e.g. on a blank grid)translate patterns into different representations that use the same rule (e.g. using sounds, actions or materials)check translated patterns for consistent use of the identified ruleidentify, describe and create patterns involving spatial materialsidentify the ‘growing part’ of a growing pattern (e.g. for the growing pattern a b; aa bb, aaa bbb, aaaa bbbb, a and b are each growing by one letteridentify and distinguish between patterns that repeat (e.g. a,b; a,b; a,b) and patterns that grow (e.g. a b; aa bb, aaa bbb, aaaa bbbb)create rules for patterns that growcreate patterns and continue patterns that radiate in a number of directionsreverse consistent change and explain reasoning based on identified simple rules. | **Patterns**repeatingnon-patternsspatialtranslating patterns into different representationsgrowingskip counting with calculators (addition and subtraction)Functionsrules describing consistent change of one attribute both forwards and backwards (backtracking) |
| Investigations should occur in a range of contexts. For example, students could investigate:paving and floor designsarrangements of windows on buildingsfencing designsdesigns for wrapping paper or decorative borders involving patternsadapting dance routines. |

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| Level 2: Level statementStudents use rules to create and describe number patterns based on addition and subtraction. They identify number sequences that are not patterns. They complete missing parts of, or continue, a number pattern when given the rule. They know the inverse relationship between addition and subtraction and use this to apply and then reverse simple rules. They display the inputs and outputs of the application of rules in table form. Students represent addition and subtraction situations using equations. They recognise and describe the equivalence or non-equivalence of two sides of an addition or subtraction equation (number sentence) and determine an unknown using a variety of self-generated and learned strategies. |
| Core learning outcome: PA 2.1Students create and explain patterns, identify and describe relationships using rules and use backtracking to reverse the effects of rules involving addition and subtraction. |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | **Core content** |
| Students know:patterns are based on rulesrules describe relationships between termshow to identify ruleshow to describe relationships using rules how to create patterns using ruleshow to explain patterns using rulesaddition and subtraction are the inverse of each otherbacktracking reverses the effects of rules involving addition and subtractionhow to use the inverse relationship between addition and subtraction for backtracking. | Students may:describe rules for repeating patterns and growing patternsidentify non-patternsexplain repeating parts of a patternexplain growing parts of a patternuse the rule for a pattern to determine the next termidentify the term before the starting point of a pattern using the rule (e.g. identify the number that comes before 6 in the pattern 6, 8, 10 …)record patterns in a variety of waysidentify and explain the missing parts or errors in repeating and growing patternsuse the rule to translate repeating and growing patterns into number patternsuse the rule to determine the next, other, or missing number in a patternidentify the use of addition and/or subtraction in the rulecreate other patterns using the same rulebacktrack to reverse the effects of rulesexplain the backtracking process involving addition undoing subtraction and vice versagather or generate data involving a relationship between input and output (e.g.1 kg input → output cost $2.00; 2 kg input → output cost $4.00)experiment with input → output using rulesrecord results from input → output experiments on a tableidentify and describe the relationships between input → output data using rulesuse a rule and the relationship between sets of data to determine the next and other valuesdevelop data sets that are similar using relationship rules (e.g. cost per kilogram for different fruit or vegetables)create and describe other change rules (e.g. double the number that is the input in a function machine). | **Patterns**missing termsnon-patterns or patterns with errorsspatialtranslating patterns into number patternsnumber patterns* rules based on previous term
* skip counting with calculators (constant function key)

**Functions**input → output (function machines)backtracking (inverse)* addition or subtraction
* reversing a change

relationship rules* given
* student-generated
* identified

represent input → output data in table formrules relating two sets of data |
| At each level, investigations should occur in a range of contexts. For example, students could investigate:how to arrange books in patterns (colour, height, thickness) on a shelfcreate a patchwork quilt with materials of different shape, size or texturecounting patterns using addition and subtractionsequences of street or seat numbersgames with function machines scoring patterns in codes of football or cricket. |

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| Level 3: Level statementStudents describe relationships between sets of numbers in terms of functions or rules. They draw tables and graphs to display these relationships. They know the inverse relationship between multiplication and division and use this to reverse the effect of a rule or change. Students represent and describe equivalence in everyday situations. They determine the missing part of an equation (number sentence) that requires either multiplication and division or addition and subtraction using a systematic guess and check strategy. |
| Core learning outcome: PA 3.1Students create and continue number patterns, identify, describe and represent relationships between two quantities and use backtracking to reverse any one of the four operations. |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | **Core content** |
| Students know:number patterns use ruleshow to create and continue number patternshow to identify and describe relationships between two quantitiesrelationships between two quantities can be represented using rules, tables and graphshow to represent relationships between two quantities backtracking reverses the effects of rules involving the four operationsmental computation strategies and computation methods for the four operationsmultiplication is the reverse of divisionhow to use relationships between addition and subtraction and between multiplication and division to reverse operations. | Students may:identify and describe a pattern rule involving any one of the four operationsidentify consistent rules for a variety of given number patterns using any one of the four operationscreate number patterns using any one of the four operations based on a given, identified or student-generated rulecheck for consistent application of a rule and identify non-patterns or patterns with errorsuse rules in reverse to undo a change or to determine a missing term (e.g. to find the missing term or the previous term in a pattern, such as …, 9, 15, 22, 30,…, 49) explain the process identifying the inverse operationcontinue a number pattern using mental computation strategies and computation methodsdescribe the effect of changing the operation used to create a patterndescribe the effect of changing the starting number/s of a patternuse the inverse of operations to apply backtracking to reverse the effect of the rulescheck for consistency of the use of the inverse of the rule when backtrackingidentify and represent relationships between two quantities and describe as a rule or consistent changeuse the rule to continue patterns forwards and backwards based on the previous termcreate patterns using different rules and represent the generated or gathered data on tables and graphsidentify the position of any term based on the application of the rule using knowledge and relationships of operations, mental computation strategies and computation methods. | **Patterns**number (including patterns with decimals)* rules based on previous term
* calculators (whole and decimal numbers involving any operations)
* missing term
* non-patterns or patterns with errors
* rules based on the position of terms (one operation only)

Functionsinput →output (function machines)backtracking (inverse)* multiplication or division
* reversing a change

representations of relationshipsrules, tables, graphs |
| Investigations should occur in a range of contexts. For example, students could investigate:patterns used in public transport timetablesscoring used in gameshistorical number patterns, such as Fibonacci sequencerelationships between two sets of data, such as the purchase price for items, the time of day and the temperature, age and growth, computer game scoreshow to finish partially completed tables.  |

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| Level 4: Level statementStudents identify and create representations of patterns and functions and use their knowledge of functions and inverses to determine unknowns within equations or any position in a pattern. They apply combinations of the four operations, observing the order of operations and the presence of brackets. Students manipulate and solve simple equations using strategies that maintain balance. They identify relationships between sets of data and distinguish between discrete and continuous data represented in graphs and tables. |
| Core learning outcome: PA 4.1Students identify and create representations of patterns and functions and apply backtracking to solve simple equations that involve combinations of the four operations. |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | **Core content** |
| Students know:a function is a mathematical relationship between two valueshow to identify rules for patterns and functionshow to create patterns and functions using ruleshow to represent functionshow and when to apply backtracking to solve simple equations involving combinations of the four operations mental computation strategies and computation methods for operations. | Students may:describe discrete data as data that is based on counts and have a finite number of valuesidentify and explain situations involving discrete data (e.g. matchstick puzzles)describe continuous data as data that can be measured and take on any value over a given interval (e.g. height of individuals, daily air temperatures, volume of an inflating balloon)identify and explain situations involving continuous data (e.g. the relationship between height and foot length, time and water flow)explain the relationship between sets of data presented in a table or graphrepresent relationships as a rule or function involving combinations of operations (e.g. for a bath containing 25 litres of water into which 5 litres flows each minute, the rule is: the amount of water = (number of minutes x 5 + 25)L)select and create representations of data using ordered pairs, tables, graphs to find a simple rule or function and justify the selection use representations to determine or predict values of particular terms (e.g. How much will be in a container after two hours?)use representations to comment on trendssolve problems that require the reversing of a function expressed as an equation and represent on graphs, tables or with sets of ordered pairs (e.g. When will a container be half full?)identify the combinations of operations used in equations and apply the inverse to backtrack identify the order of calculations and describe the effect on backtracking use knowledge of operations, mental computation strategies and computation methods to solve problems. | **Patterns**rules based on the position of terms (combinations of operations) calculator number patternsordered pairs and graphs (with discrete data only)**Functions**input → output (with combinations of operations)rules relating two sets of databacktracking (inverse)* with combination of operations

representations of relationships* ordered pairs
* tables, line graphs, equations (number sentences)
* trends
* discrete data
* continuous data

electronic, manual |
| **Investigations should occur in a range of contexts. For example, students could investigate:**internet charges, such as connection fees, download costs, and monthly chargescontents in a container, such as filling a tank with water or a silo with producefees for borrowing books/DVDs, such as reservation fees and penalties for late returnsrelationship between time elapsed and temperature change, such as the cooling time for hot drinksrelationship between quantities required for a project, such as the number of pavers to surround a given number of treesresults tables for sporting competitions using points for wins, draws and losses.  |

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| Level 5: Level statementStudents identify when relationships exist between two sets of everyday data and use functions expressed in words or symbols, or represented in tables and graphs, to describe these relationships. They identify relationships that are linear and express these using equations.Students use algebraic reasoning and conventions, including graphical representations, to solve problems and justify their solutions. |
| Core learning outcome: PA 5.1\* Students interpret and compare different representations of linear and simple non-linear functions and solve the related problems. |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | **Core content** |
| Students know:linear functions form a straight line graph simple non-linear functionshow to distinguish between linear and simple non-linear functionsdifferent ways of representing linear and simple non-linear functionshow to interpret representations of linear and simple non-linear functionshow to compare different representations of linear and non-linear functionsmental computation strategies and computation methods for operationsmethods for solving linear and simple non-linear equations. | Students may:classify life-related situations into those that can be modelled as linear functions (e.g. simple interest) or simple non-linear functions (e.g. area of garden beds) identify different representations of the same situations interpret and compare different representations of the same situation to determine the most appropriate distinguish between the independent and dependent variables analyse and interpret sets of ordered pairs, trends or graphs to develop a rule in words and symbols identify and discuss situations involving either discrete or continuous data select and create representations of discrete and continuous data manually or electronically (e.g. graphs, tables, ordered pairs and rules expressed in words or symbols)use a graph to find the value of the dependent or independent variable given the value of the other variablesubstitute into the rule to find the value of the dependent or independent variable given the other variable and use backtracking techniques. | PatternsFunctionsordered pairs (four quadrants)representations of variables * words
* symbols

linear models* representations (tables, line graphs, linear equations, proportion equations)
* dependent and independent variables
* discrete and continuous data
* trends

non-linear models* dependent and independent variables
* discrete and continuous data
* representations (tables, line graphs)
* trends

representations of relationshipselectronic, manual |
| At each level, investigations should occur in a range of contexts. For example, students could investigate:hiring rates for things such as clothing, equipment, sporting goods or venuescatering requirements for eventsconstruction costs for landscaping features. |

**\*** This outcome may be best demonstrated in conjunction with PA 5.

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| Level 6: Level statementStudents analyse problems from realistic situations and model them with equations using algebraic symbols, graphs and tables. They select and present representations that best display the relationships. They provide solutions or make predictions based on these models. |
| Core learning outcome: PA 6.1\*Students create mathematical models of realistic situations and use interpretations of the models to draw conclusions or make decisions. |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | **Core content** |
| Students know:how to create mathematical models of realistic situations how to interpret mathematical models of realistic situations how to represent mathematical models algebraicallyhow to draw conclusions from mathematical modelshow to use interpretations of models to draw conclusions or make decisions. | Students may:identify possible models to represent a realistic situationapply appropriate processes to create a mathematical model by:* determining whether the situation involves discrete or continuous data
* representing the model using tables, graphs or algebraic equations
* identifying and analysing the trends using dependent and independent variables to describe the rule
* determining the impact of variables on the situation and solving linear equations as required

discuss and justify the reasonableness of the created or selected model interpret mathematical models and draw conclusionsmake and justify decisions for future actions or consequences based on interpretations or conclusions. | PatternsFunctionslinear models* equations
* representations (tables, graphs)
* trends

non-linear models* representations (tables, graphs)
* trends

representations of relationshipselectronic, manual  |
| **Investigations should occur in a range of contexts. For example, students could investigate:**travel networks with distance covered versus time takenthe length of queues against the time taken to be servedfitness levels by monitoring heart rate before, during and after exercise changes in crowd numbers at a rock concert or sporting event as the starting time approaches. |

**\*** This outcome may be best demonstrated in conjunction with PA 6.2.