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| Strand: Patterns and Algebra | | Topic: Equivalence and equations |
| Foundation Level: Level statement  Students investigate patterns in their environments and are developing an awareness of ‘same’ when matching. | | |
| Example learning outcomes:  Students show an awareness of ‘same’ in relation to people, objects, places or small collections. | | |
| Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically | | |
| Students know:  matched items require elements that are the same or belong together  everyday language that relates to ‘same’. | Students may:  show an awareness of ‘same’ by matching when sorting objects or repeating an action  match same texture cards from a limited selection  create a ‘balanced’ picture (e.g. adding the missing eye to a face, matching the two pieces of familiar pictures when cut vertically or horizontally)  construct equivalent ‘buildings’ with blocks, such as one person adding a block to their building, the other person adding the same type of block to their building so the buildings are the same, or pack the same items in two lunch packs  participate in painting and folding activities that create balanced pictures  construct mobiles that have the same items on each side  participate in ‘balance’ games (e.g. balance beams, balance pans)  copy familiar actions (e.g. putting a shoe on one foot, matching the action by putting a shoe on the other foot; placing a knife with every fork because  they ‘go together’)  copy the ‘same’ action or sound  match one attribute in familiar situations involving quantity, colour, size, shape, or texture (e.g. one knife goes with one fork; two shapes for you,  two shapes for me). | |

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| Level 1: Level statement  Students identify and describe patterns in their environments. They create or continue patterns and know that some can continue indefinitely, and some radiate in a number of directions. They represent the same pattern in different ways. They describe patterns or change in terms of a simple rule and can undo a pattern or change by reversing the rule.  Students describe the number value of a group of objects as ‘equal to’, ‘different from’ or ‘the same as’. They know that the number value of a group of objects stays the same when rearranged or represented in different combinations. | | |
| Core learning outcome: PA 1.2  Students compare and describe arrangements of objects and combinations of numbers to 10 using the language of equivalence. | | |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | | **Core content** |
| Students know:  language of equivalence  equivalent collections have the same value  equivalent collections can have different arrangements  equivalent collections can have different combinations  how to compare arrangements and combinations of numbers to 10. | Students may:  decide ways to compare the value of collections in different arrangements and to use different representations  compare the value of collections of objects and describe as ‘equal to’, ‘same as’ or ‘different from’  make different combinations of the same number and describe as equivalent  compare and describe different arrangements of the same number of objects using the language of equivalence. | Equivalence   * conservation * language * equal to, same as * different from   Representations   * objects   pictures |
| Investigations should occur in a range of contexts. For example, students could investigate:  different arrangements of a given number of objects for packaging, such as arranging 10 biscuits on a tray  creating teams of 10 players for a game using different combinations of numbers, such as three wearing red and seven wearing blue, or four wearing red and six wearing blue)  different combinations of pieces of fruit (up to 10) to place on a fruit platter. | | |

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| Level 2: Level statement  Students use rules to create and describe number patterns based on addition and subtraction. They identify number sequences that are not patterns. They complete missing parts of, or continue, a number pattern when given the rule. They know the inverse relationship between addition and subtraction and use this to apply and then reverse simple rules. They display the inputs and outputs of the application of rules in table form.  Students represent addition and subtraction situations using equations. They recognise and describe the equivalence or non-equivalence of two sides of an addition or subtraction equation (number sentence) and determine an unknown using a variety of self-generated and learned strategies. | | |
| Core learning outcome: PA 2.2  Students represent and describe equivalence in equations that involve addition and subtraction. | | |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | | **Core content** |
| Students know:   * language to describe the balance represented in equations * equations are balanced if both sides have the same value * things that are equal to the same thing are equal to each other (transitive relation) * how to represent equivalence in equations that involve addition and subtraction * mental computation strategies and computation methods for addition and subtraction. | Students may:   * represent the equivalence of situations in different ways * explain the notion of balance as equivalence * represent equivalence concretely, verbally, pictorially, electronically or symbolically, or combinations of these * identify and describe situations encountered that are not equivalent * represent situations that are not equivalent * check and describe equivalence using a range of strategies * identify and describe different combinations to balance equations * identify and use a range of strategies to determine unknowns * explain transitive relations as things that are equal to the same thing as being equal to each other  (e.g. if a is taller than b and b is taller than c, then a is taller than c) * identify and represent transitive relations * use a range of strategies to identify unknowns and maintain the balance of equations. | Equivalence  conservation  balance  transitive relation  language   * equal to, same as * not equal to, different from   unknowns   * missing addend * guess and check   Representations  objects  equations (number sentences)  symbols   * equals (=) * does not equal (≠) * for unknowns (shapes, boxes, question marks, spaces, lines) |
| Investigations should occur in a range of contexts. For example, students could investigate:  combinations of coins made to the same value  the comparative weight of objects using a pan balance  equivalent quantities of ingredients for cooking activities, such as one-and-a-half cups of sugar are equal to three half cups of sugar  possible combinations to score a given number when rolling two or more dice in a game. | | |
| Level 3: Level statement  Students describe relationships between sets of numbers in terms of functions or rules. They draw tables and graphs to display these relationships. They know the inverse relationship between multiplication and division and use this to reverse the effect of a rule or change.  Students represent and describe equivalence in everyday situations. They determine the missing part of an equation (number sentence) that requires either multiplication and division or addition and subtraction using a systematic guess and check strategy. | | |
| Core learning outcome: PA 3.2  Students represent and describe equivalence in equations that involve combinations of multiplication and division or addition and subtraction. | | |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | | **Core content** |
| Students know:  language to describe the balance represented in equations  how to distinguish between equations involving multiplication and division, or addition and subtraction  how to represent equivalence in equations that involve combinations of multiplication and division, or addition and subtraction  how to balance equations  mental computation strategies and computation methods for operations. | Students may:  identify and explain equivalence and non-equivalence in equations  represent and describe equations that are balanced  use language and symbols to create equations that have unknowns (e.g. ‘I have two cans of baked beans on this side of the scales and seven on the other side. How many cans do I need to add to balance the scales? This statement can be written as: 2 + 🞏 = 7)  use guess and check or other methods to balance equations  identify and represent possible solutions to a problem involving different combinations of addition and subtraction  use knowledge of addition and subtraction operations, mental computation strategies and computation methods to solve problems represented by equations  represent and describe equivalence involving multiplication and division situations concretely, verbally, pictorially, electronically or symbolically, or using combinations of these  identify and represent possible solutions to a problem involving different combinations of multiplication and division  use knowledge of multiplication and division operations, mental computation strategies and computation methods to solve those problems. | Equivalence  conservation  balance  language   * same and different * more and less * equal, not equal * greater than, less than   unknowns   * guess and check   Representations  equations (number sentences)  symbols   * equals (=) * does not equal (≠) * greater than (>) * less than (<) * for unknowns (shapes, boxes, question marks, spaces, lines) |
| Investigations should occur in a range of contexts. For example, students could investigate:  money transactions, such as obtaining a float for a stall or holding a garage sale  situations involving time, such as viewing a movie in a number of episodes  saving plans for desired items, such as saving over an extended period of time to buy an electronic game. | | |

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| Level 4: Level statement  Students identify and create representations of patterns and functions and use their knowledge of functions and inverses to determine unknowns within equations or any position in a pattern. They apply combinations of the four operations, observing the order of operations and the presence of brackets.  Students manipulate and solve simple equations using strategies that maintain balance. They identify relationships between sets of data and distinguish between discrete and continuous data represented in graphs and tables. | | |
| Core learning outcome: PA 4.2  Students create and interpret equations, explain the effect of order of operations, and justify solutions to equations. | | |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | | **Core content** |
| Students know:  how to create equations with unknowns  how to interpret equations with unknowns  order of operations  how to explain the effects of order of operations  mental computation strategies and computation methods for solving equations  different ways to describe equivalence  how to justify solutions to equations. | Students may:  identify and describe situations involving combinations of operations  interpret equations involving combinations of operations and unknowns  create equations to represent situations involving combinations of operations and unknowns using appropriate symbols including brackets  decide on the order of operations (brackets followed by multiplication and division, left to right and then addition and subtraction, left to right)  explain the effect of the order of operations on the representation of the equation and/or solution  use knowledge of operations, mental computation strategies and computation methods to create and solve equations  describe how understandings of equivalence were used to solve equations  justify solutions to problems by using the inverse of the operations. | Equivalence  order of operations  methods for solving equations   * balance * guess and check   Representations  symbols   * equals (=) * not equals (≠) * brackets * for unknowns (shapes, boxes, question marks, spaces, lines)   arrow diagrams |
| Investigations should occur in a range of contexts. For example, students could investigate:  comparisons of discounts  possible scores in drawn games of basketball, cricket, rugby codes or Australian Rules football, such as the number of goals and the number of behinds  pavers required for landscaping. | | |

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| Level 5: Level statement  Students identify when relationships exist between two sets of everyday data and use functions expressed in words or symbols, or represented in tables and graphs to describe these relationships. They identify relationships that are linear and express these using equations.  Students use algebraic reasoning and conventions, including graphical representations, to solve problems and justify their solutions. | | |
| Core learning outcome: PA 5.2\*  Students interpret and solve linear equations related to realistic problems using algebraic and graphical methods. | | |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | | **Core content** |
| Students know:  algebraic conventions  how to algebraically and graphically represent a range of realistic problems  how to interpret different representations of linear equations  algebraic and graphical methods to solve equations. | Students may:  identify the links between realistic problems, linear functions and linear equations  represent realistic problems as linear equations  relate linear equations to the realistic problems they represent  represent equations in words, visually using graphs and arrow diagrams, or using symbols following algebraic conventions  substitute to produce a solvable equation  determine an unknown variable using a range of methods for solving equations  justify the method used to solve equations. | Equivalence  methods for solving equations   * substitution * balance * backtracking * guess and check * graphical displays * tabular data   Representations  variables   * words * letter symbols   algebraic conventions  implied multiplication (3t)  implied division ()  computer format (\*, /)  arrow diagrams  linear  proportion equations |
| Investigations should occur in a range of contexts. For example, students could investigate:  payment for delivery of advertising materials and community newspapers  wages calculated using different rates of pay such as casual, part-time and penalty rates of pay  allocation of points for sporting performance involving degrees of difficulty, such as diving and gymnastics  codes to send secret messages. | | |

\* This outcome may be best demonstrated in conjunction with PA 5.1

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| Level 6: Level statement  Students analyse problems from realistic situations and model them with equations using algebraic symbols, graphs and tables. They select and present representations that best display  the relationships. They provide solutions or make predictions based on these models. | | |
| Core learning outcome: PA 6.2\*  Students interpret and solve mathematical models of realistic situations by using algebraic, graphical and electronic methods. | | |
| **Elaborations — To support investigations that emphasise thinking, reasoning and working mathematically** | | **Core content** |
| Students know:  algebraic conventions  how to algebraically and graphically represent a range of realistic problems  how to interpret mathematical models of realistic situations  algebraic, graphical and electronic methods to solve equations. | Students may:  identify the links between realistic problems, functions and equations  represent realistic problems as linear or non-linear equations  relate equations to the realistic problems they represent  represent, interpret and record equations using words, graphs or symbols following algebraic conventions  collect like terms, simplify and expand equations as necessary to solve equations  use logical algebraic setting out when solving equations  solve mathematical equations and explain and justify reasoning. | Equivalence  methods for solving equations   * graphical methods * substitution * balance * backtracking * guess and check   simplifying  collecting like terms  expanding  Representations  linear, proportion equations  life-related non-linear models  algebraic conventions   * logical setting out   models |
| Investigations should occur in a range of contexts. For example, students could investigate:  time taken to reheat food in containers of various sizes  length of advertisement breaks during television programs depending on time of day or type of program  design of a skateboard bowl to utilise optimum speed. | | |

**\*** This outcome may be best demonstrated in conjunction with PA 6.1.