A future world record

Image: *Womens 100m final*, jimmyharris, Creative Commons Attribution2.0, www.flickr.com/photos/jimmyharris/23519766

Analyse data for world record times to predict a record time forty years from now.

You will:

- analyse and interpret data to determine how world records have changed over time and to predict a future world record
- evaluate the reasonableness of you prediction and the limitations of your strategy
- present your findings as a mathematical report.

World record times for 1500 metres

World record times for the 1500 metres athletics event have been continually dropping, as shown in tables 1 and 2.

.Table 1: World record times: men's 1500 metres

Date	Athlete	Time (mm:ss.0)	Date	Athlete	Time (mm:ss.0)
8/06/1912	Abel Kiviat	03:55.8	6/09/1955	Gunnar Nielsen	03:40.8
5/08/1917	John Zander	03:54.7	3/08/1956	István Rózsavölgyi	03:40.6
19/06/1924	Paavo Nurmi	03:52.6	11/07/1957	Olavi Salsola	03:40.2
11/09/1926	Otto Peltzer	03:51.0	11/07/1957	Olavi Salonen	03:40.2
5/10/1930	Jules Ladoumegue	03:49.2	12/07/1957	Stanislav Jungwirth	03:38.1
9/09/1933	Luigi Beccali	03:49.2	28/08/1958	Herb Elliott	03:36.0
17/10/1933	Luigi Beccali	03:49.0	6/09/1960	Herb Elliott	03:35.6
30/06/1934	Bill Bonthron	03:48.8	8/07/1967	Jim Ryun	03:33.1
6/08/1936	Jack Lovelock	03:47.8	2/02/1974	Filbert Bayi	03:32.2
10/08/1941	Gunder Hägg	03:47.6	15/08/1979	Sebastian Coe	03:32.1
17/07/1942	Gunder Hägg	03:45.8	15/07/1980	Steve Ovett	03:32.1
17/08/1943	Arne Andersson	03:45.0	27/08/1980	Steve Ovett	03:31.4
7/07/1944	Gunder Hägg	03:43.0	28/08/1983	Sydney Maree	03:31.2
15/07/1947	Lennart Strand	03:43.0	4/09/1983	Steve Ovett	03:30.8
29/06/1952	Werner Lueg	03:43.0	16/07/1985	Steve Cram	03:29.7
4/06/1954	Wes Santee	03:42.8	23/08/1985	Saïd Aouita	03:29.5
21/06/1954	John Landy	03:41.8	6/09/1992	Noureddine Morceli	03:28.9
28/07/1955	Sándor Iharos	03:40.8	12/07/1995	Noureddine Morceli	03:27.4
6/09/1955	László Tábori	03:40.8	14/07/1998*	Hicham El Guerrouj	03:26.0

*Current record as of Oct 2013

Source: International Association of Athletics Federations (IAAF) 2013, www.iaaf.org/records/toplists/middlelong/1500-metres/outdoor/men/senior

Table 2: World record times: women's 1500 metres					
Date	**Athlete**	**Time** (mm:ss.0)	**Date**	**Athlete**	**Time** (mm:ss.0)
19/08/1927	Anna Mushkina	5:18.2	03/06/1967	Anne Rosemary Smith	4:17.3
16/09/1934	Anna Mushkina	5:07.0	24/10/1967	Maria Gommers	4:15.6
13/07/1936	Lydia Freiberg	5:02.0	02/07/1969	Paola Pigni	4:12.4
30/07/1936	Yevdokiya Vasilyeva	4:47.2	20/09/1969	Jaroslava Jehličková	4:10.7
13/09/1937	Yevdokiya Vasilyeva	4:45.2	15/08/1971	Karin Burneleit	4:09.6
10/06/1940	Anna Zaytseva-Bosenko	4:41.8	18/07/1972	Ludmila Bragina	4:06.9
17/08/1944	Yevdokiya Vasilyeva	4:38.0	04/09/1972	Ludmila Bragina	4:06.5
15/09/1946	Olga Ovsyannikova	4:37.8	07/09/1972	Ludmila Bragina	4:05.1
30/08/1952	Nina Pletnyova	4:37.0	09/09/1972	Ludmila Bragina	4:01.4
17/05/1956	Phyllis Perkins	4:35.4	28/06/1976	Tatyana Kazankina	3:56.0
16/05/1957	Diane Leather	4:30.0	06/07/1980	Tatyana Kazankina	3:55.0
19/07/1957	Diane Leather	4:29.7	03/08/1980	Tatyana Kazankina	3:52.47
08/12/1962	Marise Chamberlain	4:19.0	11/09/1993*	Qu Yunxia	3:50.46

***Current record as of Oct 2013**

Source: International Association of Athletics Federations (IAAF) 2013, www.iaaf.org/records/toplists/middlelong/1500-metres/outdoor/women/senior

Future world record times for 1500 metres

Using the data in table 1 or 2, write a mathematical report to predict the men's or women's 1500 metres world record time forty years from now.

Use the checklist below to make sure your report is complete.

Checklist for your mathematical report
☐ Introduction A brief introduction that describes the goals of the report and outlines what will be presented.
☐ Strategy Clearly show the mathematical reasoning used. You should: • state the variables • convert non-decimal variables to decimal form • generate a scatter plot and a line of best fit to represent the relationship between your variables • describe the trend displayed in your scatter plot • write an equation of a linear function that closely matches the set of points, and support your answer by showing calculations • predict the 1500 metres men's or women's world record time forty years from now, supporting your answer by showing the mathematical processes used and any assumptions you make.
☐ Discussion of results Refer to mathematical reasoning. You may wish to use other sources of information to support your discussion. Discuss: • Is your prediction reasonable? • What are the limitations of your strategy? • Should you reconsider your prediction because of the time elapsed since the world record was last broken? • Is your prediction consistent with the idea of a time in the future when the record could never again be broken? • Other ...?
☐ Conclusion A summary of your findings.
☐ References Include references to any sources of information used in your report.

A future world record

Name .....

Purpose of assessment: To use world record data to predict a record forty years from now and to evaluate a mathematical model.

Understanding and Skills			
Understanding & Fluency		**Problem solving & Reasoning**	
Display of world record data as a scatter plot and representation of the relationship as a linear function.	Use of appropriate language, conventions and symbols when graphing data, developing equations and providing explanations.	Development and evaluation of a mathematical model to determine how world records have changed over time and to predict a future world record.	Use of a mathematical argument to support a prediction. Evaluation of the suitability of the strategy used to make a prediction.
Accurate use of technologies to present data as a scatter plot. Accurate use of procedures to effectively represent the relationship as a linear function.	Consistent use of appropriate conventions and symbols when graphing data and developing equations. Consistent use of appropriate mathematical language in explanations.	Evaluation and interpretation of a scatter plot and a linear function to accurately describe a trend in world records and make a valid prediction.	Clear explanation of a mathematical argument to support the prediction, including a logical justification of the strategies used.	**A**
				B
Use of technologies and procedures to present data as a scatter plot.	Use of conventions and symbols when graphing data. Use of mathematical language in explanations.	Refers to a scatter plot to describe the trend in world records to make a rough prediction.	Description of the mathematical thinking behind the prediction and strategies used.	**C**
	Use of aspects of mathematical conventions, symbols and language.			**D**
Some use of technologies and procedures to present data.		Statements about world record trends to make a prediction.		
	Use of everyday language.		Isolated statements about strategies.	**E**

A future world record

Name .....

Purpose of assessment: To use world record data to predict a record forty years from now and to evaluate the strategy used.

			A	B	C	D	E
Understanding and Skills dimensions	Understanding & Fluency	**Recall and use of facts, definitions, technologies and procedures** Display of world record data as a scatter plot and representation of the relationship as a linear function.	Accurate use of technologies to present data as a scatter plot. Accurate use of procedures to effectively represent the relationship as a linear function.	Accurate use of technologies and procedures to present data as a scatter plot and to represent the relationship as a linear function.	Use of technologies and procedures to present data as a scatter plot.	Some use of technologies and procedures to present data.	Attempted use of technologies and procedures to present data.
		Use of mathematical language, conventions and symbols Use of appropriate language, conventions and symbols when graphing data, developing equations and providing explanations.	Consistent use of appropriate conventions and symbols when graphing data and developing equations. Consistent use of appropriate mathematical language in explanations.	Use of appropriate conventions and symbols when graphing data and developing equations. Use of appropriate mathematical language in explanations.	Use of conventions and symbols when graphing data. Use of mathematical language in explanations.	Use of aspects of mathematical conventions, symbols and language.	Use of everyday language.
	Problem solving & Reasoning	**Development of models and representations** Development and evaluation of a mathematical model to determine how world records have changed over time and to predict a future world record.	Evaluation and interpretation of a scatter plot and a linear function to accurately describe a trend in world records and make a valid prediction.	Development of a scatter plot and a linear function to describe the trend in world records and to make a prediction.	Refers to a scatter plot to describe the trend in world records to make a rough prediction.	Statements about world record trends to make a prediction.	Isolated statements about world record trends or predictions.
		Discussion of mathematical thinking, choices and strategies Use of mathematical argument to support a prediction. Evaluation of the suitability of the strategy used to make a prediction.	Clear explanation of a mathematical argument to support the prediction, including a logical justification of the strategies used.	Explanation of a mathematical argument to support the prediction, including a justification of the strategies used.	Description of the mathematical thinking behind the prediction and strategies used.	Statements about strategies used.	Isolated statements about strategies.

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<table>
<tr><th>Assessment description</th><th>Category</th></tr>
<tr><td rowspan="2">Students analyse data to determine how world records have changed over time and predict a future world record. They evaluate the reasonableness of their prediction and the limitations of the strategy.</td><td>Written</td></tr>
<tr><td>Technique
Supervised assessment</td></tr>
<tr><td>Context for assessment</td><td>Alignment</td></tr>
<tr><td rowspan="3">The assessment will involve the students writing a report to show how well they understand the mathematical ideas and concepts covered.

Students will use technology applications (including spreadsheets and graphing software) to analyse and present real-life data. They will generate a scatter plot to investigate and comment on relationships between sets of data.

Students will require access to appropriate technology and will need internet access if they wish to gather further data to assist with their evaluation of the strategy.</td><td>• Australian Curriculum 5.1, Year 10 Mathematics Australian Curriculum content and achievement standard ACARA — Australian Curriculum, Assessment and Reporting Authority, www.australiancurriculum.edu.au
• Year 10 Mathematics standard elaborations, www.qsa.qld.edu.au/yr10-maths-resources.html</td></tr>
<tr><td>Connections
This assessment can be used with the QSA Australian Curriculum resource titled Year 10 unit overview – Mathematics exemplar (Mathematics and sport), available at: www.qsa.qld.edu.au/yr10-maths-resources.html</td></tr>
<tr><td>Definitions
See the Australian Curriculum glossary for technical terms used in this assessment: www.qsa.qld.edu.au/yr10-maths-resources.html</td></tr>
<tr><td>In this assessment</td><td>Assessment materials</td></tr>
<tr><td>Student booklet
Task-specific standards — continua
Task-specific standards — matrix
Assessment resource: Indicative A response
Teacher guidelines</td><td>Computer access for all students
Word processing and spreadsheet software
Internet access if further data is required to support discussion of results</td></tr>
</table>

Teacher guidelines

Identify curriculum

Content descriptions to be taught
Statistics and Probability
Data representation and interpretation Use scatter plots to investigate and comment on relationships between two numerical variables (ACMSP251) Investigate and describe bivariate numerical data where the independent variable is time (ACMSP252) **10A** Use information technologies to investigate bivariate numerical data sets. Where appropriate use a straight line to describe the relationship allowing for variation (ACMSP279)
General capabilities (GCs) and cross-curriculum priorities (CCPs) This assessment may provide opportunities to engage with the following GCs and CCPs. Refer also to the Resources tab on the Mathematics curriculum hub: www.qsa.qld.edu.au/yr10-maths-resources.html
Literacy — **ICT capability** **Numeracy** — **Critical and creative thinking**
Achievement standard This assessment provides opportunities for students to demonstrate the following highlighted aspects.
By the end of Year 10, students recognise the connection between simple and compound interest. They solve problems involving linear equations and inequalities. They make the connections between algebraic and graphical representations of relations. Students solve surface area and volume problems relating to composite solids. They recognise the relationships between parallel and perpendicular lines. Students apply deductive reasoning to proofs and numerical exercises involving plane shapes. They compare data sets by referring to the shapes of the various data displays. They describe bivariate data where the independent variable is time. Students describe statistical relationships between two continuous variables. They evaluate statistical reports. Students expand binomial expressions and factorise monic quadratic expressions. They find unknown values after substitution into formulas. They perform the four operations with simple algebraic fractions. Students solve simple quadratic equations and pairs of simultaneous equations. They use triangle and angle properties to prove congruence and similarity. Students use trigonometry to calculate unknown angles in right-angled triangles. Students list outcomes for multi-step chance experiments and assign probabilities for these experiments. They calculate quartiles and inter-quartile ranges.
Source: ACARA, The Australian Curriculum v5.1, www.australiancurriculum.edu.au

Sequence learning

Suggested learning experiences

This assessment leads on from the learning experiences outlined in the QSA's Year 10 Mathematics unit overview. The knowledge, understanding and skills developed in the exemplar unit will prepare students to engage in this assessment.

See *Year 10 plan - Mathematics exemplar* (Exemplar unit: Mathematics and sport) www.qsa.qld.edu.au/yr10-maths-resources.html.

Adjustments for needs of learners

To make adjustments, teachers refer to learning area content aligned to the student's chronological age, personalise learning by emphasising alternate levels of content, general capabilities or cross-curriculum priorities in relation to the chronological age learning area content. The emphasis placed on each area is informed by the student's current level of learning and their strengths, goals and interests. Advice on the process of curriculum adjustment for all students and in particular for those with disability, gifted and talented or for whom English is an additional language or dialect are addressed in *Australian Curriculum — Student Diversity* materials.

For information to support students with diverse learning needs, see:

- Queensland Studies Authority materials for supporting students with diverse learning needs www.qsa.qld.edu.au/10188.html
- Australian Curriculum Student Diversity www.australiancurriculum.edu.au/StudentDiversity/Overview
- The *Melbourne Declaration on Educational Goals for Young Australians* www.mceecdya.edu.au/mceecdya/melbourne_declaration,25979.html
- *Disability Standards for Education* http://deewr.gov.au/disability-standards-education.

Develop assessment

Preparing for the assessment

Learning experiences in preparation for the assessment could include:

- Use scatter plots to determine if a relationship is linear, i.e. by adding a line of best fit.
- Write and solve linear equations using graph paper and technology (e.g. graphing calculators, spreadsheets).
- Substitute values into formulas.
- Use graphics calculators and spreadsheets to generate scatter plots, add trend lines, make predictions, etc.
- Review rounding to two decimal places and three significant figures.
- Convert time (mm:ss.00) into decimal notation or into seconds.
- Examine exemplars of how to communicate mathematically (including mathematical language conventions and the use of equation writers in word processing software to write algebraic and statistical notation and symbols).

Implementing

- Allow sufficient time for students to complete assessment — additional time may be required if students wish to gather further data to assist with their discussion of the results.
- Discuss requirements of task with the class.
- Supervise students and assist with unpacking requirements of specific parts of the task as necessary.
- Remind students to make use of appropriate conventions, symbols and mathematical language when writing their report.

Making judgments

When making judgments about the evidence in student responses to this assessment, teachers are advised to use the task-specific standards provided. The development of these task-specific standards has been informed by the Queensland Year 10 standard elaborations. See www.qsa.qld.edu.au/yr10-maths-resources.html.

The Queensland standard elaborations for Mathematics

The Queensland Year 10 Standard elaborations for Mathematics is a resource to assist teachers to make consistent and comparable evidence-based A to E (or equivalent) judgments. It should be used in conjunction with the Australian Curriculum achievement standard and content descriptions for the relevant year level.

The Queensland Year 10 Standard elaborations provide a basis for judging *how well* students have demonstrated what they know, understand and can do using the Australian Curriculum achievement standard.

The Australian Curriculum achievement standards dimensions of Understanding and Skills are used to organise the Queensland Mathematics standard elaborations. Understanding and Skills in Mathematics are organised as Understanding, Fluency, Problem solving and Reasoning

The valued features of Mathematics are drawn from the achievement standard and the content descriptions for Understanding and Skills are organised as:

- Mathematical understandings
- Recall and use of facts, definitions, technologies and procedures
- Use of mathematical language, conventions and symbols
- Use of problem solving strategies
- Modelling and representation
- Results and conclusions of investigations and inquiries
- Communication of mathematical thinking, choices and strategies.

Task-specific standards

Task-specific standards give teachers:

- a tool for directly matching the evidence of learning in the student response to the standards
- a focal point for discussing student responses
- a tool to help provide feedback to students.

Task-specific standards are not a checklist; rather they are a guide that:

- highlights the valued features that are being targeted in the assessment and the qualities that will inform the overall judgment
- specifies particular *targeted aspects* of the curriculum content and achievement standard
- aligns the valued feature, task-specific descriptor and assessment
- allows teachers to make consistent and comparable on-balance judgments about student work by matching the qualities of student responses with the descriptors
- clarifies the curriculum expectations for learning at each of the five grades (A–E or the early years equivalent)
- shows the connections between what students are expected to know and do, and how their responses will be judged and the qualities that will inform the overall judgment

- supports evidence-based discussions to help students gain a better understanding of how they can critique their own responses and achievements, and identify the qualities needed to improve
- encourages and provides the basis for conversations among teachers, students and parents/carers about the quality of student work and curriculum expectations and related standards.

Task-specific valued features

Task-specific valued features are the discrete aspects of the valued features of Mathematics targeted in a particular assessment and incorporated into the task-specific standards for that assessment. They are selected from the Queensland Mathematics standard elaborations valued features drawn from the Australian Curriculum achievement standard and content descriptions.

Task-specific valued features for this assessment			
The following identifies the valued features for this assessment and makes explicit the understandings and skills that students will have the opportunity to demonstrate. This ensures that the alignment between what is taught, what is assessed and what is reported is clear.			
Australian Curriculum achievement standard dimensions	**Australian Curriculum proficiency strands**	**Queensland standard elaborations valued features**	**Task-specific valued features**
Understanding and Skills	Understanding & Fluency	Recall and use of facts, definitions, technologies and procedures	Display of world record data as a scatter plot and representation of the relationship as a linear function.
		Use of mathematical language, conventions and symbols	Use of appropriate language, conventions and symbols when graphing data, developing equations and providing explanations
	Problem solving & Reasoning	Development of models and representations	Development and evaluation of a mathematical model to determine how world records have changed over time and to predict a future world record.
		Discussion of mathematical thinking, choices and strategies	Use of mathematical argument to support a prediction. Evaluation of the suitability of the strategy used to make a prediction.

The task-specific standards for this assessment are provided in two models using the same task-specific valued features:

- a matrix
- a continua.

Matrix and Continua

Task-specific standards can be prepared as a matrix or continua. Both the continua and the matrix:

- use the Queensland standard elaborations to develop task-specific descriptors to convey expected qualities in student work — A to E or equivalent
- highlight the same valued features from the Queensland standard elaborations that are being targeted in the assessment and the qualities that will inform the overall judgment
- incorporate the same task-specific valued features i.e. make explicit the particular understanding / skills students have the opportunity to demonstrate for each selected valued feature
- provide a tool for directly matching the evidence of learning in the student response to the standards to make an on-balance judgment about student achievement
- assist teachers to make consistent and comparable evidence-based A to E or equivalent judgments.

Continua

The continua model of task-specific standards uses the dimensions of the Australian Curriculum achievement standard to organise task-specific valued features and standards as a number of reference points represented progressively along an A-E continuum. The task-specific valued features at each point are described holistically. The task-specific descriptors of the standard use the relevant degrees of quality described in the Queensland standard elaborations.

Teachers determine a position along each continuum that best matches the evidence in the students' responses to make an on-balance judgment about student achievement on the task.

The continua model is a tool for making an overall on-balance judgment about the assessment and for providing feedback on task specific valued features.

Matrix

The matrix model of task-specific standards uses the structure of the Queensland standard elaborations to organise the task-specific valued features and standards A to E. The task-specific descriptors of the standard described in the matrix model use the same degrees of quality described in the Queensland standard elaborations.

Teachers make a judgment about the task-specific descriptor in the A to E (or equivalent) cell of the matrix that best matches the evidence in the students' responses in order to make an on-balance judgment about how well the pattern of evidence meets the standard.

The matrix is a tool for making both overall on-balance judgments and analytic judgments about the assessment. Achievement in each valued feature of the Queensland standard elaboration targeted in the assessment can be recorded and feedback can be provided on the task-specific valued features.

Use feedback

Feedback to students	This assessment provides opportunities to give feedback to students on how well they: • generate a scatter plot and plotting a line of best fit: – converting times into decimal notation and rounding them, before using them for graphing – selecting the correct axes and scale for the independent and dependent variables, when using graph paper – manipulating scatter plots in spreadsheet software to best display the data – inputting work into spreadsheets/calculators • formulate a linear equation (10A): – using the graph to find the gradient and y-axis intercept to substitute into the equation of a straight line – inputting work into spreadsheets/calculators • check for reasonableness: – verifying the reasonableness of the solution by checking that the answer fits the question, e.g. by applying existing strategies, using a variety of techniques, estimation, working backward • discuss the results: – making links between results of the mathematical strategy, the assumptions made, the limitations of the strategy, and the plausibility of predictions back to the original question and life-related situations.
Resources	For guidance on providing feedback, see the professional development packages titled: • *About feedback* www.qsa.qld.edu.au/downloads/p_10/as_feedback_about.doc • *Seeking and providing feedback* www.qsa.qld.edu.au/downloads/p_10/as_feedback_provide.doc.