Mathematics
Queensland Comparable Assessment Tasks (QCATs) 2012
Behind the scenes
Student booklet

Given name: .................................................................
Family name: ............................................................... 
School: ........................................................................

Queensland Studies Authority
Setting the scene

Behind the scenes of a successful dance competition are a director and stage manager.

In this assessment, you will:

Follow the instructions in the director’s notes to:
- calculate areas of projected logos for painting on the backdrop
- choose suitable colours for dance competition logos
- make decisions about stage lighting
- apply your understandings to accommodate the director’s last-minute changes.

The following formulas may be useful:

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pythagoras’ theorem</td>
<td>$c^2 = a^2 + b^2$</td>
</tr>
<tr>
<td>Distance formula</td>
<td>Distance ($d$) from $A$ to $B$</td>
</tr>
<tr>
<td></td>
<td>$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$</td>
</tr>
</tbody>
</table>

Show your working

- Your teacher is looking for mathematical thinking and reasoning, not just correct answers.
- When using a calculator, show enough working so that your teacher can see the method you used.
- Ensure all answers are rounded to an appropriate number of decimal places.
- If you cannot complete a question, show what you have been able to do.
- Credit will be given if an incorrect answer is used correctly in a later question.
Projecting and painting logos on the backdrop

Director’s note #1
- Use both Logo A and Logo B for the dance competition.
- Project them onto the backdrop so they can be painted onto the surface.
- Calculate the area of each logo so you know how much paint you need.

Diagram 1: Logo A

1. Calculate the total area of Logo A.

Show all working.

Total area = .........................
2. Calculate the total area of Logo B, correct to 1 decimal place.

Show all working.

Total area = .........................
Choosing colours for the dance competition logos

Director's note #2
- Use cyan and magenta tints to mix colours for the logos.

When cyan and magenta tints are mixed into white paint, the code for the resulting colour is a pair of numbers \((c, m)\), where \(c\) is the quantity of cyan and \(m\) is the quantity of magenta tint added to each litre of paint.

Table 1: Colour chart

<table>
<thead>
<tr>
<th>Colour Name</th>
<th>blue</th>
<th>turquoise</th>
<th>purple</th>
<th>cobalt</th>
<th>slate blue</th>
<th>periwinkle</th>
</tr>
</thead>
<tbody>
<tr>
<td>((c, m))</td>
<td>(100, 100)</td>
<td>(100, 15)</td>
<td>(39, 81)</td>
<td>(60, 60)</td>
<td>(49, 56)</td>
<td>(33, 33)</td>
</tr>
</tbody>
</table>

3. Plot and label the colours in Table 1 on the cyan/magenta colour graph below.

Graph 1: Colour graph

\((c, m)\) is used in place of \((x, y)\)

Units of cyan \((c)\) per litre

Units of magenta \((m)\) per litre
4. Find the **colour difference** between slate blue and periwinkle.  
   Use Pythagoras' theorem or the distance formula (see page 3).

   
   **Show all working.**

   
   
   Distance = .................

5. List all colours from Table 1 that could match director’s note #3.  
   Justify your decisions. (You may not need to calculate the colour difference for all colours.)

   **Show all working.**
Lighting the dance floor

Diagram 3: Lighting the dance floor

Each light shines a circle of light on the floor as shown.

Remember: \( \tan \theta = \frac{\text{opposite}}{\text{adjacent}} \)

6. Find the radius \( (r) \) of the circle of light that each light shines on the floor from 3.9 m above.

Show all working.

Radius = ..........................

The rectangular dance floor can be divided into squares, each lit by one circle of light.

Diagram 4: The largest square lit by one light

Length of side of square \( s \)

Radius of circle of light on floor

Square lit by one circle of light
7. (a) Use the radius found in Question 6 and Pythagoras’ theorem to calculate the length of the side (s) of the largest square that can be lit by one circle of light (see Diagram 4).

(b) Find the length (l) and width (w) of the largest rectangular dance floor that can be completely lit by 10 lights at 3.9 m high (see Diagram 5).

Diagram 5: Rectangular dance floor seen from above

(b) Find the length (l) and width (w) of the largest rectangular dance floor that can be completely lit by 10 lights at 3.9 m high (see Diagram 5).
The director’s last minute changes

Diagram 6: Making Logo B larger

Logo B was enlarged from its original height of 5 m by moving the projector further away from the backdrop.

Director’s note #5
Sorry, a last minute change.
The logos on the backdrop need to be larger.

8. What will be the new height \( h \) of Logo B, if the projector is moved 1 metre further away from the backdrop (see Diagram 6)?

Show all working.

New height = 

\[ \text{New height} = \ldots \ldots \ldots \ldots \ldots \ldots \]
9. If Logo A is enlarged so that $a = 4.5$,
   (a) find the scale factor

   \[
   \text{Scale factor} = \dots \dots \dots \dots \dots \dots \dots
   \]

   Show all working.

   \[
   \text{Length of sides marked } b = \dots \dots \dots \dots \dots \dots \dots
   \]

   Show all working.
If the side of the small square is increased from 1 m to \((x + 1)\) m, the area of the small square = \((x + 1)^2\) square metres (see Diagram 8).

10. (a) Expand the expression \((x + 1)^2\)

\[
(x + 1)^2 =
\]

(b) Expand and simplify the expression for the total area of Enlarged Logo A.

The total area of Enlarged Logo A = \(2(3x + 3)^2 - (x + 1)^2\)

The total area of Enlarged Logo A = \(2(3x + 3)^2 - (x + 1)^2\)

= 

(c) Find the total area of Enlarged Logo A, by substituting the director’s value for \(x\) into the equation in Question 10 (b). See director’s note #6.

Director’s note #6
I’ve decided how large Logo A should be. Make \(x = 0.2\) metres

Total area = 

..............................................
11. (a) **Clearly show that** sides $a$ of the dance floor are 9 metres long (to the nearest metre).
The director thinks **6 lights** will be enough to completely light the **triangular dance floor** without being too dim.

The stage manager thinks **10 lights** are needed.

(b) Who do you agree with?

Justify your answer, including:

- the dimensions of the largest square lit by each light without being too dim
- a diagram of the **triangular dance floor**, showing the arrangement of squares lit by the lights. See Diagram 5 (page 9).
At what height \( h \) should the lights in Question 11 (b) be placed to light the **triangular dance floor** as brightly as possible?

Justify your answer, including:

- the dimensions of the square lit by each light
- diagram/s to aid your explanations.

\[ \text{Height} = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \]
Guide to making judgments — Year 9 Mathematics

**Focus:** To apply and justify strategies to solve problems, with fluent use of mathematical procedures.

<table>
<thead>
<tr>
<th>Understanding</th>
<th>Reasoning</th>
<th>Problem solving</th>
<th>Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selects and applies mathematical concepts and interprets information to:</td>
<td>Applies problem solving strategies to:</td>
<td>Calculates the areas of composite shapes (Questions 1, 2)</td>
<td>Accurately plots points on a Cartesian plane (Question 3)</td>
</tr>
<tr>
<td>• find distances on a Cartesian plane (Question 4)</td>
<td>• choose colours that meet the director’s needs (Question 5)</td>
<td>• find the dimensions of a rectangle that can be lit by 10 lights (Question 7b)</td>
<td>Expands, simplifies, manipulates and substitutes into algebraic expressions (Questions 6, 10)</td>
</tr>
<tr>
<td>• calculate unknown dimensions of right triangles and similar shapes. (Questions 6, 7a, 8, 9, 11a)</td>
<td>• determine the optimum number, arrangement and height of lights for the triangular dance floor and justifies decisions (Question 11b, c).</td>
<td>Makes use of mathematical procedures, rounding and units. (Questions 2, 4, 5, 6, 7, 10, 11)</td>
<td></td>
</tr>
</tbody>
</table>

**Understanding**
- Correctly calculates colour difference, the radius of the circle of light and the dimensions of the square lit by that circle. Finds the new height of Logo B and clearly confirms the dimensions of the triangular floor.
- Uses scale factor to find the new dimensions of Logo A. Makes significant progress in most of the following, leading to some solutions: calculating colour difference, finding the radius of a circle of light and calculating the dimensions of a square lit by that circle. (Q4, 6, 7a)
- Makes some progress in some of the following: calculating colour difference, finding the radius of a circle of light or calculating the dimensions of a square lit by that circle. (Q4, 6, 7a)

**Reasoning**
- Provides a well-reasoned justification of the strategy used to determine the number of lights for the triangular floor and their optimum height.
- Justifies choice of colours that match director’s note #3. Justifies the number of lights required, using a diagram of the triangular floor.
- Partially justifies choice of colours.

**Problem solving**
- Successfully applies a strategy to determine the required number of lights for the triangular dance floor and their optimum height.
- Calculates the area of Logo B. Uses an appropriate-sized square to determine the number of lights required to light the triangular dance floor.
- Calculates the area of Logo A, makes significant progress in calculating the area of Logo B and lists some colours that match directors note #3. Finds the dimensions of the rectangular floor and makes some progress in determining the number of lights required to light the triangular floor.
- Makes some progress in calculating the area of a composite shape and finding the dimensions of the rectangular dance floor.

**Skills**
- Correctly expands and simplifies the expression for the area of expanded Logo A in Q10b. Makes clear, accurate and efficient use of mathematical procedures. Consistently uses accurate and appropriate rounding and units.
- Correctly substitutes into and manipulates algebraic expressions in Q6, 10c. Makes systematic use of mathematical procedures and regular use of appropriate rounding and units.
- Correctly calculates colour difference, the radius of the circle of light and calculating the dimensions of a square lit by that circle. (Q4, 6, 7a)
- Makes some progress in some of the following: calculating colour difference, finding the radius of a circle of light or calculating the dimensions of a square lit by that circle. (Q4, 6, 7a)
- Successfully applies a strategy to determine the required number of lights for the triangular dance floor and their optimum height.
- Calculates the area of Logo B. Uses an appropriate-sized square to determine the number of lights required to light the triangular dance floor.
- Makes some progress in calculating the area of a composite shape and finding the dimensions of the rectangular dance floor.

**Feedback:**

- A: Correctly calculates colour difference, the radius of the circle of light and the dimensions of the square lit by that circle. Finds the new height of Logo B and clearly confirms the dimensions of the triangular floor.
- B: Provides a well-reasoned justification of the strategy used to determine the number of lights for the triangular floor and their optimum height.
- C: Justifies choice of colours that match director’s note #3. Justifies the number of lights required, using a diagram of the triangular floor.
- D: Partially justifies choice of colours.
- E: Successfully applies a strategy to determine the required number of lights for the triangular dance floor and their optimum height.