Mathematics

Queensland Comparable Assessment Tasks (QCATs) 2012

Behind the scenes

Student booklet

9

Given name: ....

Family name: ....

School: ....

Queensland Government

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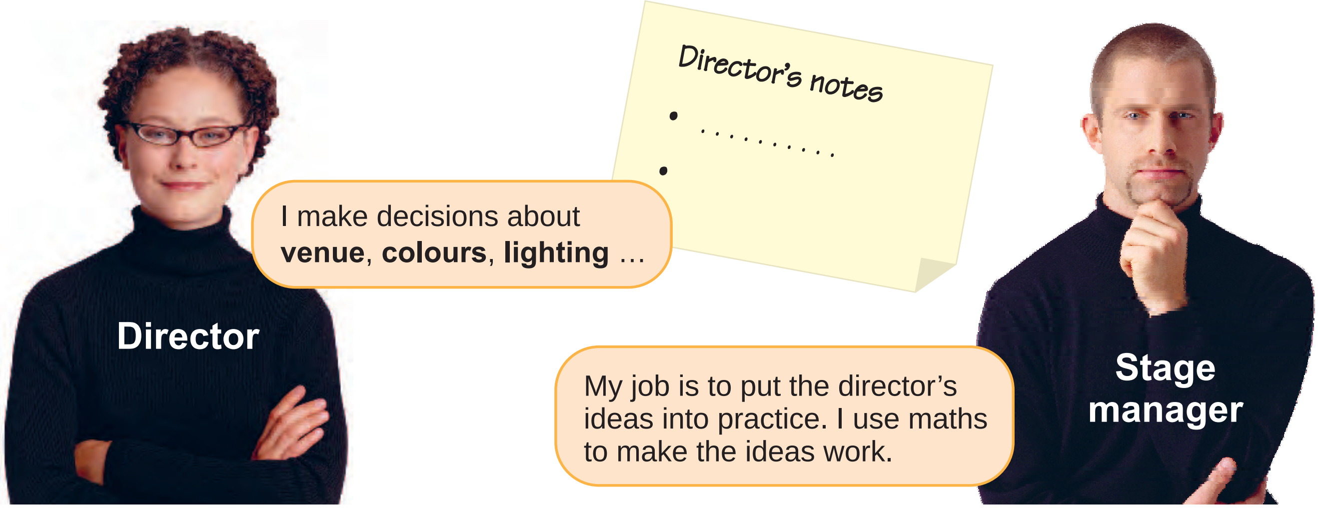
Images p. 1 Dancers: <www.123rf.com/photo_7887714_hip-hop-men-performing-and-act-over-an-urban-background.html>; p. 2 Dance montage: <www.123rf.com/photo_10448221_three-dancers>, <www.123rf.com/photo_8343118_portrait-of-an-attractive-young-bboy-dancer-showing-moves-against-grey-background.html>; p. 3 Director: PhotoDisc_OS29005; Stage manager: PhotoDisc_OS35077; p. 4, pp. 11–12 Silhouettes of dancers: <www.sxc.hu/photo/1206163>, <www.sxc.hu/photo/1206163>. All other images © QSA.

Setting the scene

Behind the scenes of a successful dance competition are a director and stage manager.

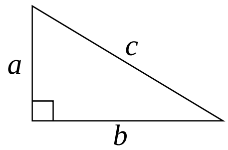
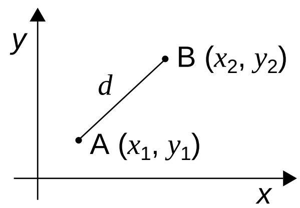


In this assessment, you will:

Follow the instructions in the director's notes to:

- calculate areas of projected logos for painting on the backdrop
- choose suitable colours for dance competition logos
- make decisions about stage lighting
- apply your understandings to accommodate the director's last-minute changes.

The following formulas may be useful:

Pythagoras' theorem	$c^2 = a^2 + b^2$	
Distance formula	Distance (d) from A to B $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	

Show your working

- Your teacher is looking for mathematical thinking and reasoning, not just correct answers.
- When using a calculator, show enough working so that your teacher can see the method you used.
- Ensure all answers are rounded to an appropriate number of decimal places.
- If you cannot complete a question, show what you have been able to do.
- Credit will be given if an incorrect answer is used correctly in a later question.

Projecting and painting logos on the backdrop

Director's note #1
- Use both Logo A and Logo B for the dance competition.
- Project them onto the backdrop so they can be painted onto the surface.
- Calculate the area of each logo so you know how much paint you need.

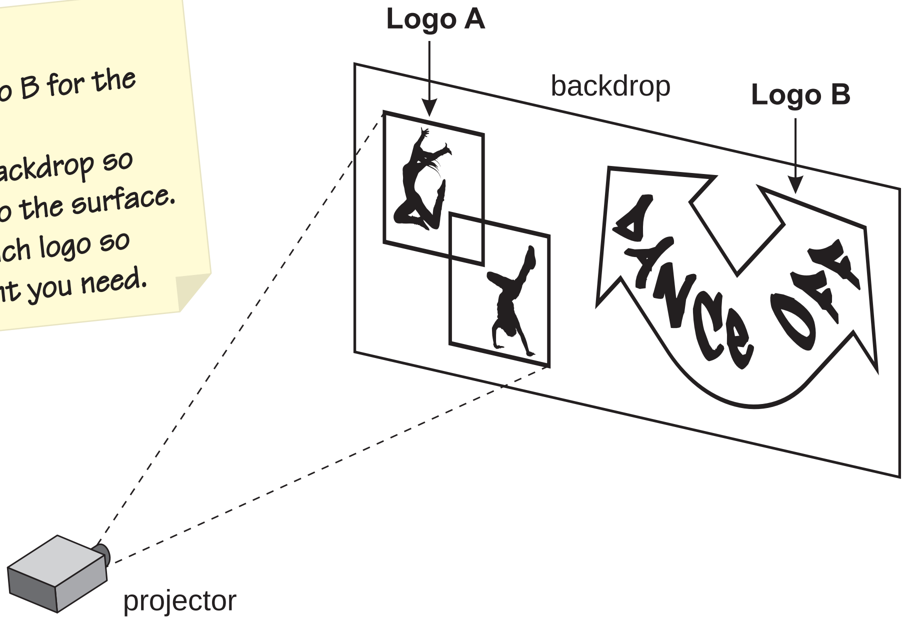
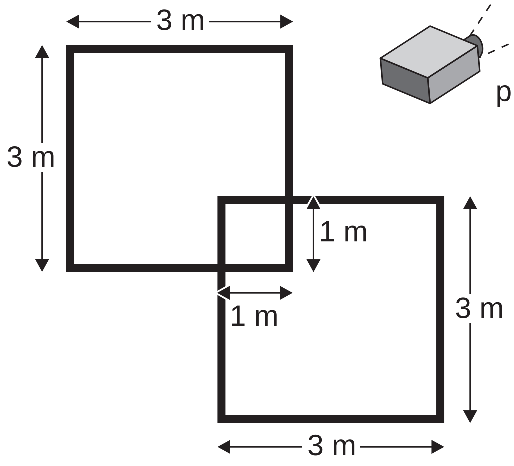


Diagram 1: Logo A

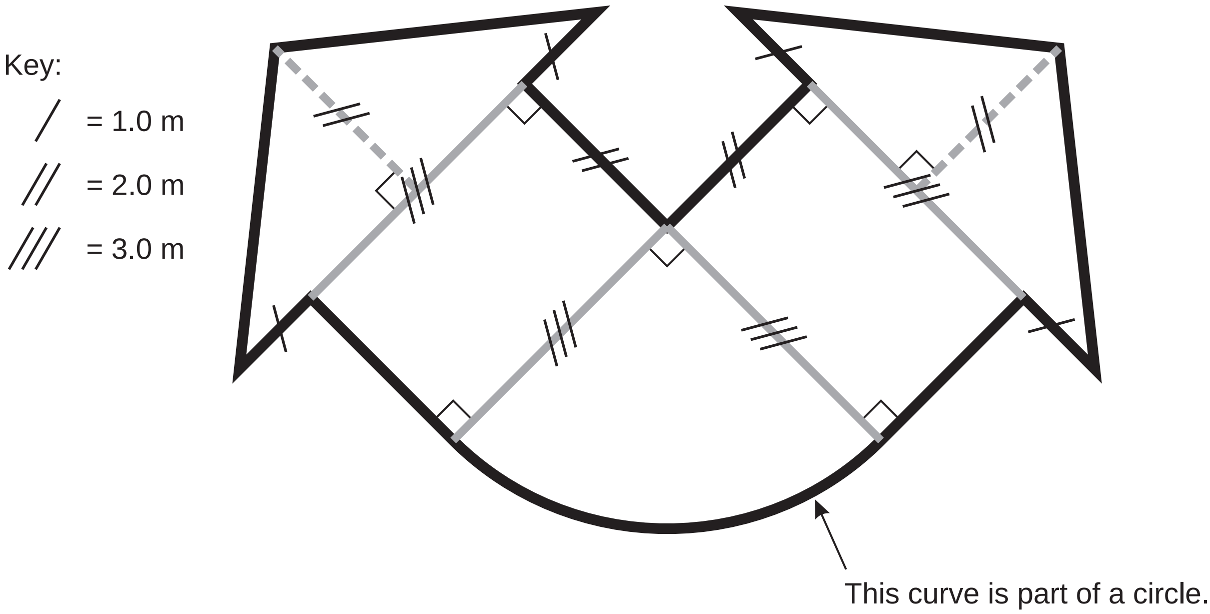


1. Calculate the total area of **Logo A**.

Show all working.

Total area = ...

Diagram 2: Logo B



2. Calculate the total area of **Logo B**, correct to 1 decimal place.

Show all working.

Total area = ...

Choosing colours for the dance competition logos

Director's note #2
- Use cyan and magenta tints to mix colours for the logos.

When **cyan** and **magenta** tints are mixed into white paint, the code for the resulting colour is a pair of numbers **(c, m)**, where **c** is the quantity of **cyan** and **m** is the quantity of **magenta** tint added to each litre of paint.

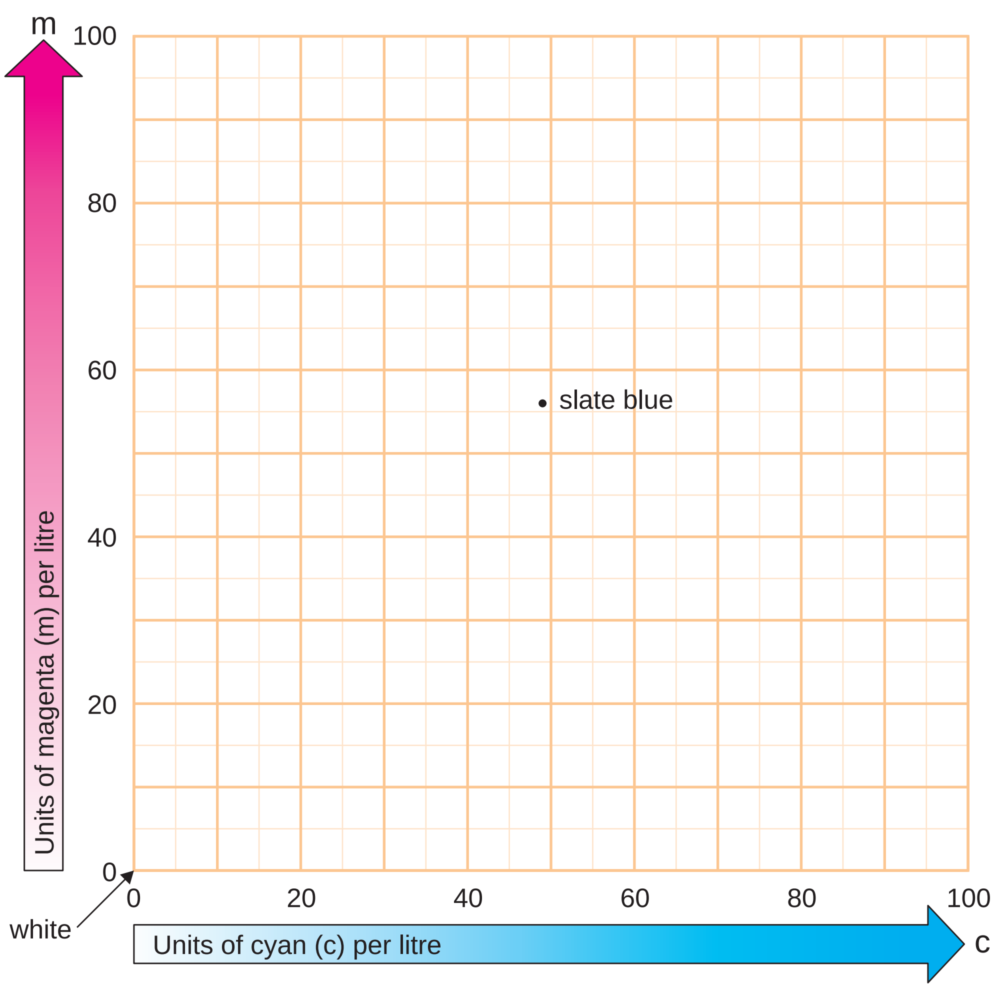
Table 1: Colour chart

Colour						
Name	blue	turquoise	purple	cobalt	slate blue	periwinkle
(c, m)	(100, 100)	(100, 15)	(39, 81)	(60, 60)	(49, 56)	(33, 33)

3. Plot and label the colours in Table 1 on the cyan/magenta colour graph below.

Graph 1: Colour graph

(c, m) is used in place of (x, y)



Colour difference is the distance between two colours when plotted on a colour graph. Colours used for logos should have a high contrast (a large colour difference).

4. Find the **colour difference** between slate blue and periwinkle.

Use Pythagoras' theorem **or** the distance formula (see page 3).

Show all working.

Distance = ...

Director's note #3

Use two colours:

- slate blue
- another colour from Table 1 with a colour difference of more than 25 units from slate blue.

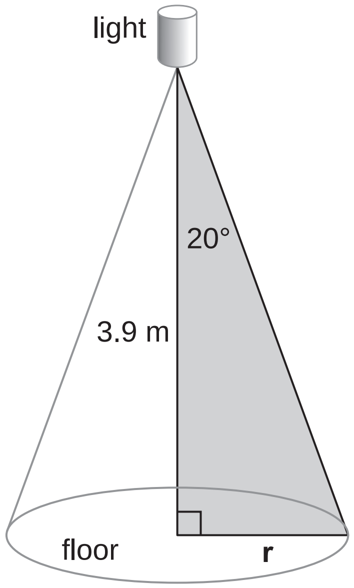
5. List all colours from Table 1 that could match director's note #3.

Justify your decisions. (You may not need to calculate the colour difference for all colours.)

Show all working.

Lighting the dance floor

Diagram 3: Lighting the dance floor



> *Director's note #4*
> - *As a special effect I want the rectangular dance floor lit from above.*
> - *Use the 10 lights from the storeroom.*

Each light shines a circle of light on the floor as shown.

Remember: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

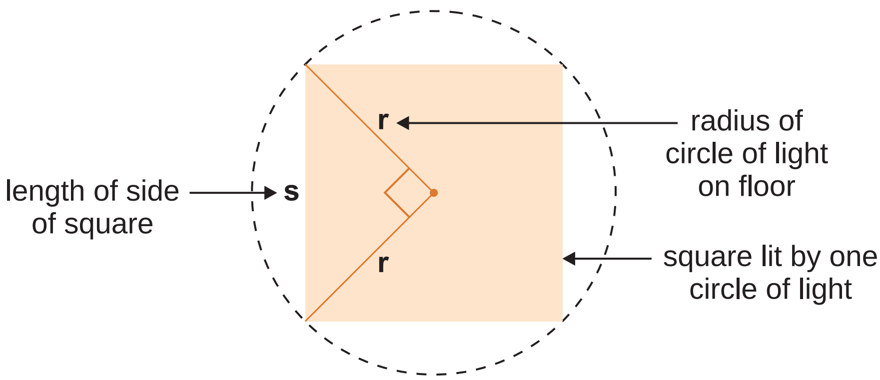
6. Find the radius (**r**) of the circle of light that each light shines on the floor from 3.9 m above.

Show all working.

Radius = ...

The rectangular dance floor can be divided into squares, each lit by one circle of light.

Diagram 4: The largest square lit by one light

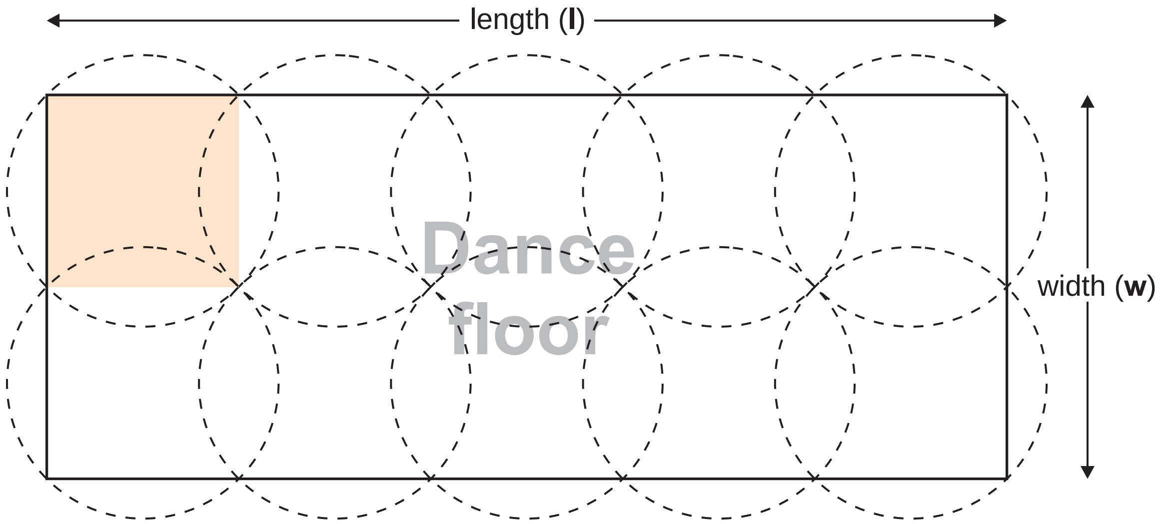


7. (a) Use the radius found in Question 6 and Pythagoras' theorem to calculate the length of the side (**s**) of the largest square that can be lit by one circle of light (see Diagram 4).

Show all working.

Length of side of square = ...

Diagram 5: Rectangular dance floor seen from above



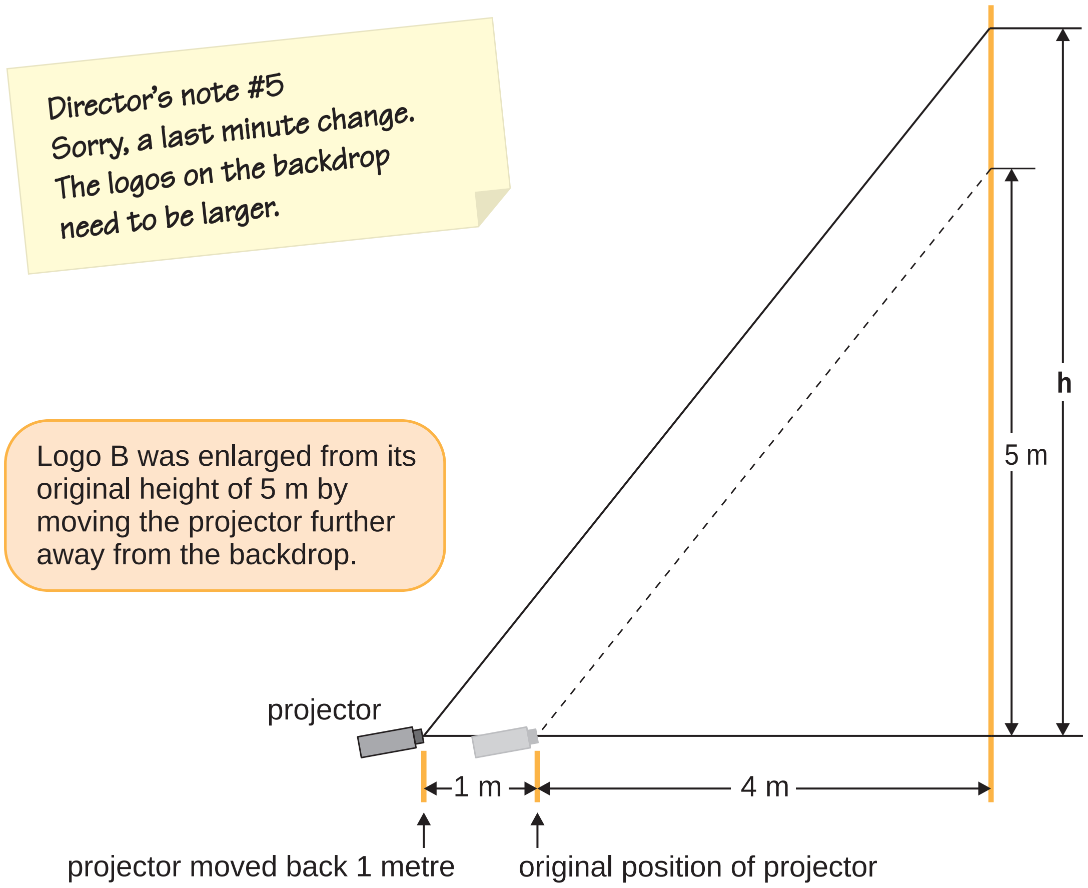
(b) Find the length (**l**) and width (**w**) of the largest rectangular dance floor that can be completely lit by 10 lights at 3.9 m high (see Diagram 5).

Show all working.

Length (**l**) = ...   Width (**w**) = ...

The director's last minute changes

Diagram 6: Making Logo B larger



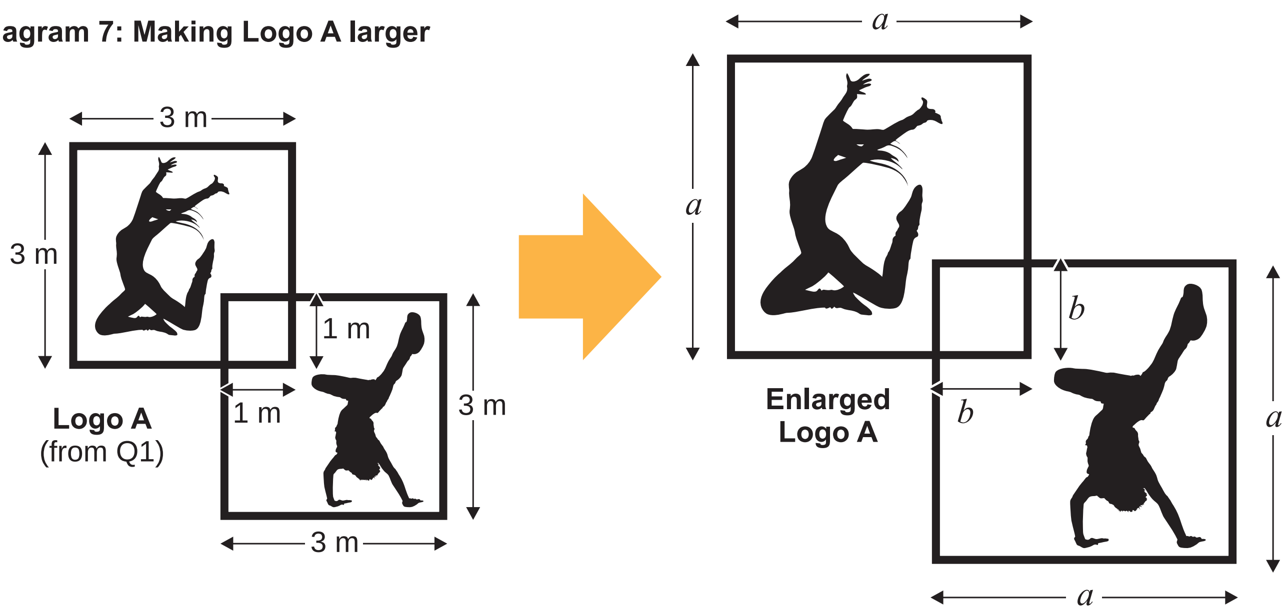
8. What will be the new height (**h**) of Logo B, if the projector is moved 1 metre further away from the backdrop (see Diagram 6)?

Show all working.

New height = ...

Making Logo A larger
The director is not sure how large to make Logo A.

Diagram 7: Making Logo A larger



The original and enlarged logos are similar figures.

9. If Logo A is enlarged so that $a = 4.5$,

(a) find the scale factor

Show all working.

Scale factor = ...

(b) find the lengths of the sides marked b in Diagram 7.

Show all working.

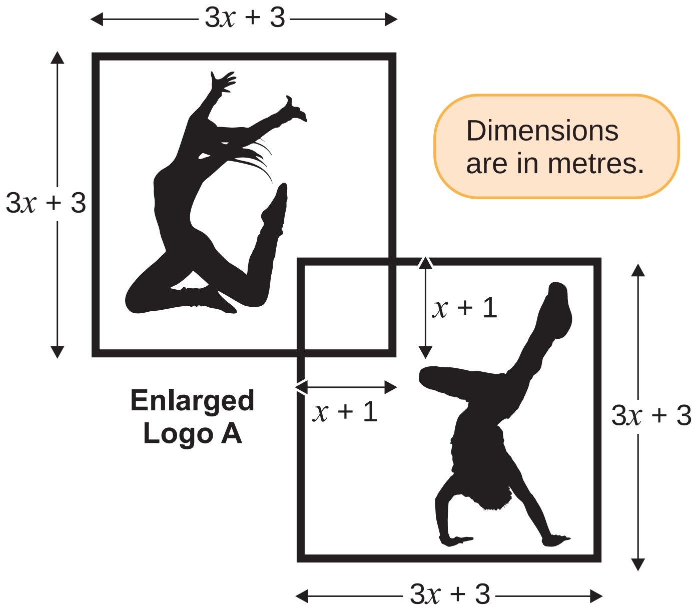
Length of sides marked b = ...

If the side of the small square is increased from 1 m to $(x + 1)$ m, the area of the small square = $(x + 1)^2$ square metres (see Diagram 8).

10. (a) Expand the expression $(x + 1)^2$

$(x + 1)^2 =$

Diagram 8: Enlarged Logo A



The total area of Enlarged Logo A = $2(3x + 3)^2 - (x + 1)^2$

(b) Expand and simplify the expression for the total area of **Enlarged Logo A**.

The total area of Enlarged Logo A = $2(3x + 3)^2 - (x + 1)^2$

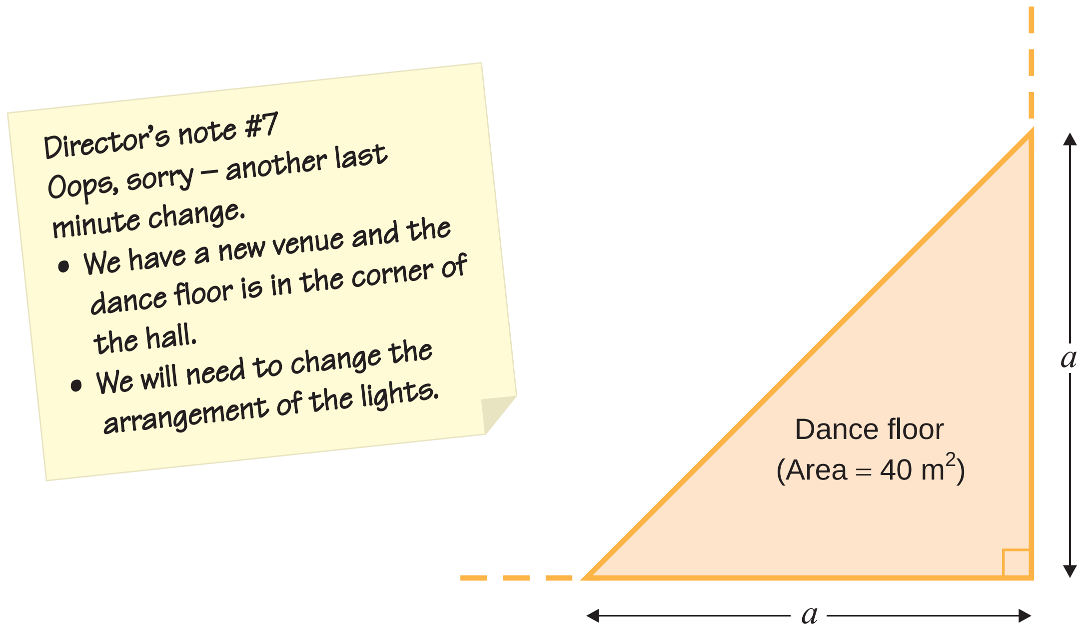
=

(c) Find the total area of **Enlarged Logo A**, by substituting the director's value for x into the equation in Question 10 (b). See director's note #6.

Director's note #6
I've decided how large Logo A should be.
Make $x = 0.2$ metres

Total area = ...

Diagram 9: Triangular dance floor seen from above

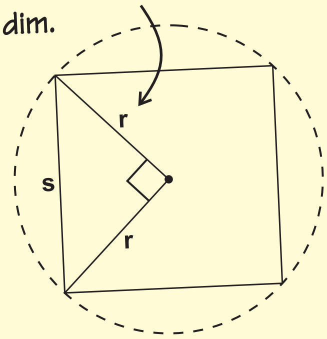


11. (a) **Clearly show that** sides a of the dance floor are 9 metres long (to the nearest metre).

Show all working.

Director's note #8

- We can adjust the height of the lights.
- The higher the lights are, the dimmer the light on the floor.
- Make sure the radius (r) of each circle of light is <u>no more than 1.8 metres</u> or the light will be too dim.
- 6 lights should be enough.



The director thinks **6 lights** will be enough to completely light the **triangular dance floor** without being too dim.

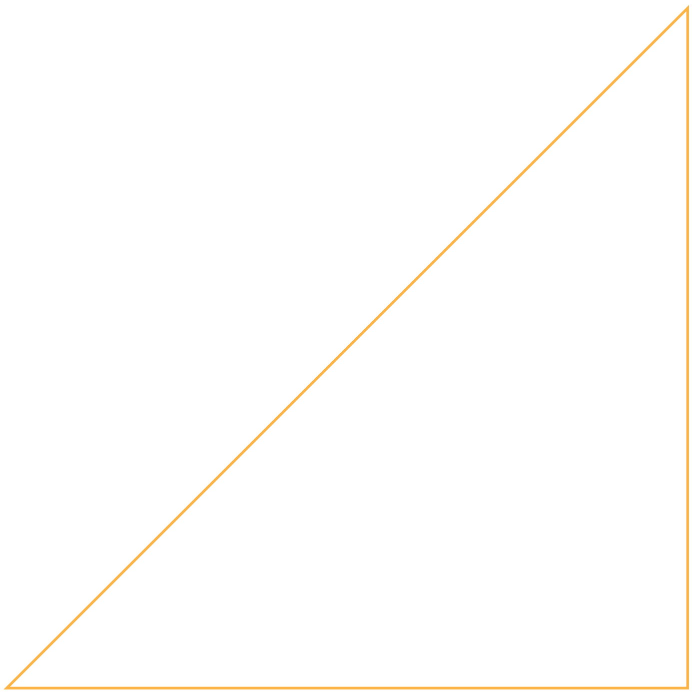
The stage manager thinks **10 lights** are needed.

(b) Who do you agree with?

Justify your answer, including:

- the dimensions of the largest square lit by each light without being too dim
- a diagram of the **triangular dance floor**, showing the arrangement of squares lit by the lights. See Diagram 5 (page 9).

Show all working.



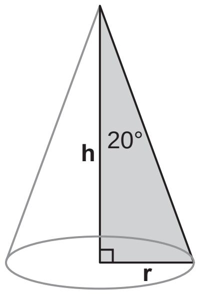
I agree with . . . . , that . . . . lights are needed.

To make the stage brighter, the stage manager can adjust the height of the lights.

(c) At what height (**h**) should the lights in Question 11 (b) be placed to light the **triangular dance floor** as brightly as possible?

Justify your answer, including:

- the dimensions of the square lit by each light
- diagram/s to aid your explanations.



Show all working.

Height = ...

Guide to making judgments — Year 9 Mathematics

Student name . . . .

Focus: To apply and justify strategies to solve problems, with fluent use of mathematical procedures.

	Understanding		Skills	
	Understanding	**Reasoning**	**Problem solving**	**Fluency**
	Selects and applies mathematical concepts and interprets information to: • find distances on a Cartesian plane (Question 4) • calculate unknown dimensions of right triangles and similar shapes. (Questions 6, 7a, 8, 9, 11a)	Applies problem solving strategies to: • calculate the areas of composite shapes (Questions1, 2) • choose colours that meet the director's needs (Question 5) • find the dimensions of a rectangle that can be lit by 10 lights (Question 7b) • determine the optimum number, arrangement and height of lights for the triangular dance floor and justifies decisions (Question 11b, c).		Accurately plots points on a Cartesian plane (Question 3) Expands, simplifies, manipulates and substitutes into algebraic expressions (Questions 6, 10) Makes use of mathematical procedures, rounding and units. (Questions 2, 4, 5, 6 ,7, 10, 11)
A	Correctly calculates colour difference, the radius of the circle of light and the dimensions of the square lit by that circle. (Q4, 6, 7a) Finds the new height of Logo B and clearly confirms the dimensions of the triangular floor.	Provides a well-reasoned justification of the strategy used to determine the number of lights for the triangular floor and their optimum height.	Successfully applies a strategy to determine the required number of lights for the triangular dance floor and their optimum height.	Correctly expands and simplifies the expression for the area of expanded Logo A in Q10b. Makes clear, accurate and efficient use of mathematical procedures. Consistently uses accurate and appropriate rounding and units.
B		Justifies choice of colours that match director's note #3. Justifies the number of lights required, using a diagram of the triangular floor.	Calculates the area of Logo B. Uses an appropriate-sized square to determine the number of lights required to light the triangular dance floor.	Correctly substitutes into and manipulates algebraic expressions in Q6, 10c. Makes systematic use of mathematical procedures and regular use of appropriate rounding and units.
C	Uses scale factor to find the new dimensions of Logo A. Makes significant progress in most of the following, leading to some solutions: calculating colour difference, finding the radius of a circle of light and calculating the dimensions of a square lit by that circle. (Q4, 6, 7a)	Partially justifies choice of colours.	Calculates the area of Logo A, makes significant progress in calculating the area of Logo B and lists some colours that match directors note #3. Finds the dimensions of the rectangular floor and makes some progress in determining the number of lights required to light the triangular floor.	Accurately locates all points on a colour graph and correctly expands a simple binomial expression in Q10a. Recalls and substitutes into relevant formulas for the calculation of most areas.
D	Makes some progress in some of the following: calculating colour difference, finding the radius of a circle of light or calculating the dimensions of a square lit by that circle. (Q4, 6, 7a)		Makes some progress in calculating the area of a composite shape and finding the dimensions of the rectangular dance floor.	Plots some points on a colour graph, recalls some formulas and makes use of simple mathematical procedures.
E				

Feedback: . . . .