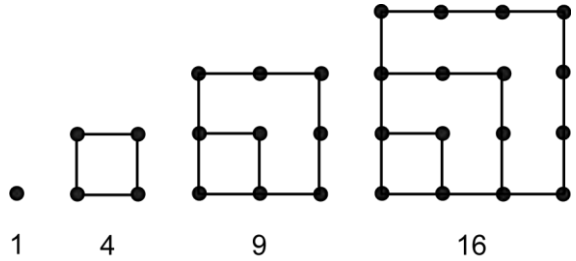
Square numbers

The Year 6 Mathematics Queensland Comparable Assessment Task (QCAT) investigates specific properties of square numbers. This document provides background information and advice for teachers to understand the relationships between **Number**, **Square number** and the **Difference between consecutive pairs** of square numbers, that students will be investigating in Question 2 (page 5, Student booklet).

Identifying square numbers

Square numbers are generated by multiplying a number by itself.

Square numbers are those that can be represented by a square array of dots.



For example:

1 is a square number because 1 × 1 = 1 and this can be represented as a square array of dots.

4 is a square number because 2 × 2 = 4 and this can be represented as a square array of dots.

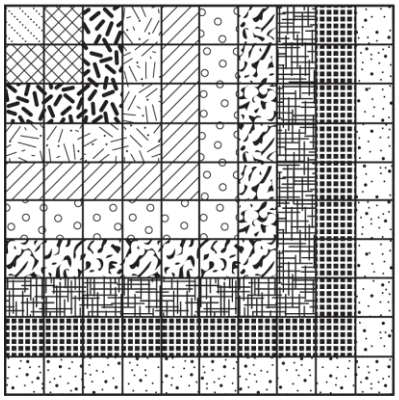
Position on a multiplication facts table

Square numbers occupy the spaces from top left to bottom right diagonal in the multiplication table shown below.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

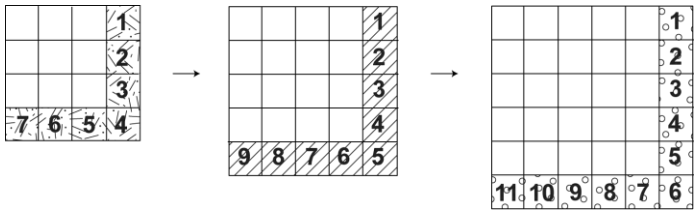
Square number patterns

Using different colours to shade adjacent squares highlights the amount of extra squares required to build the next square number. The amount of **Extra squares shaded** represents the **Difference between consecutive pairs**.



Number	Square number	Extra squares shaded	Generating the square number
1	1	-	1
2	4	3	$1^2 + 3 = 2^2$
3	9	5	$2^2 + 5 = 3^2$
4	16	7	$3^2 + 7 = 4^2$
5	25	9	$4^2 + 9 = 5^2$
6	36	11	$5^2 + 11 = 6^2$
7	49	13	$6^2 + 13 = 7^2$
8	64	15	$7^2 + 15 = 8^2$
9	81	17	$8^2 + 17 = 9^2$
10	100	19	$9^2 + 19 = 10^2$

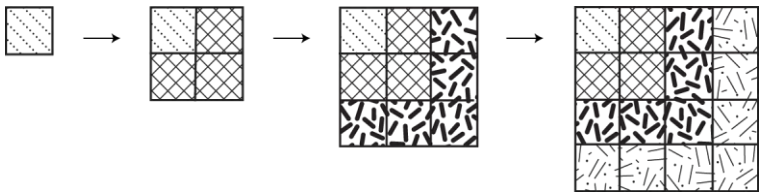
Another way to represent this diagrammatically is:



Students can describe the pattern created by calculating the **Difference between consecutive pairs** as the sequence of odd numbers.

Number	Difference between consecutive pairs	Generating the square number
1	1	1
2	3	$1 + 3 = 2^2$
3	5	$(1 + 3) + 5 = 3^2$
4	7	$(1 + 3 + 5) + 7 = 4^2$
5	9	$(1 + 3 + 5 + 7) + 9 = 5^2$
6	11	$(1 + 3 + 5 + 7 + 9) + 11 = 6^2$
7	13	$(1 + 3 + 5 + 7 + 9 + 11) + 13 = 7^2$
8	15	$(1 + 3 + 5 + 7 + 9 + 11 + 13) + 15 = 8^2$
9	17	$(1 + 3 + 5 + 7 + 9 + 11 + 13 + 15) + 17 = 9^2$
10	19	$(1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17) + 19 = 10^2$

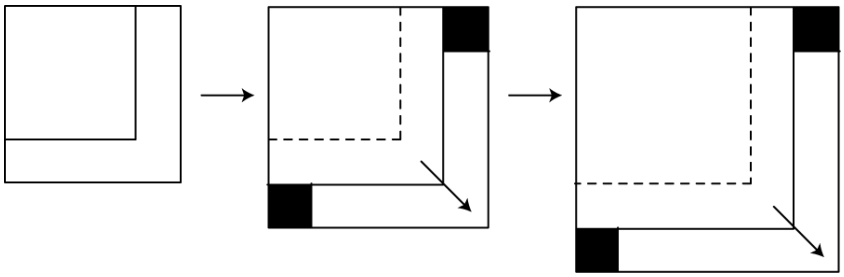
A way to represent this diagrammatically is:



Students can also describe the pattern created by calculating the **Difference between consecutive pairs** as increasing by two.

Number	Difference between consecutive pairs	Generating the square number
1	1	1
2	3 = 1 + 2	$1^2 + (1 + 2) = 2^2$
3	5 = 3 + 2	$2^2 + (3 + 2) = 3^2$
4	7 = 5 + 2	$3^2 + (5 + 2) = 4^2$
5	9 = 7 + 2	$4^2 + (7 + 2) = 5^2$
6	11 = 9 + 2	$5^2 + (9 + 2) = 6^2$
7	13 = 11 + 2	$6^2 + (11 + 2) = 7^2$
8	15 = 13 + 2	$7^2 + (13 + 2) = 8^2$
9	17 = 15 + 2	$8^2 + (15 + 2) = 9^2$
10	19 = 17 + 2	$9^2 + (17 + 2) = 10^2$

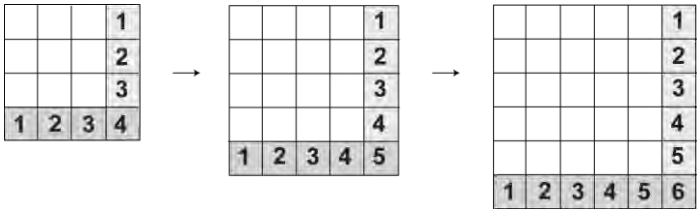
This pattern can be illustrated visually using the reverse L-shape as shown below. The next square number is generated by adding the same number of squares from the previous reverse L-shape and then two more at the ends, shown here in black.



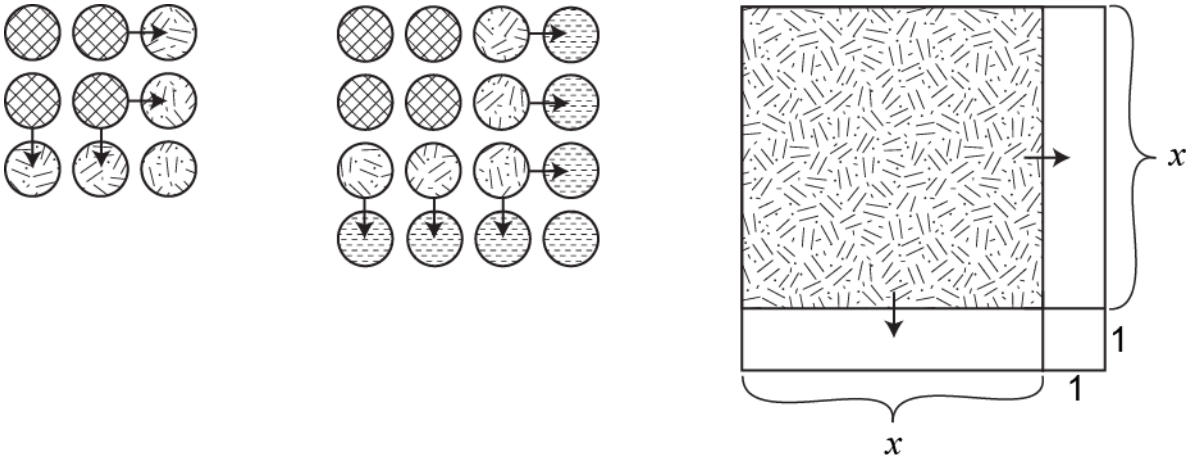
Students can describe the relationship between **Number** and **Difference between consecutive pairs** as the sum of consecutive numbers.

Number	Difference between consecutive pairs	Generating the square number
1	1	1
2	3 = 1 + 2	$1^2 + (1 + 2) = 2^2$
3	5 = 2 + 3	$2^2 + (2 + 3) = 3^2$
4	7 = 3 + 4	$3^2 + (3 + 4) = 4^2$
5	9 = 4 + 5	$4^2 + (4 + 5) = 5^2$
6	11 = 5 + 6	$5^2 + (5 + 6) = 6^2$
7	13 = 6 + 7	$6^2 + (6 + 7) = 7^2$
8	15 = 7 + 8	$7^2 + (7 + 8) = 8^2$
9	17 = 8 + 9	$8^2 + (8 + 9) = 9^2$
10	19 = 9 + 10	$9^2 + (9 + 10) = 10^2$

A way to represent this diagrammatically is:



Students can also describe the relationship between **Number** and **Difference between consecutive pairs** as the algebraic relationship, $2x + 1$.



Number	Difference between consecutive pairs	Generating the square number
1	1	1
2	$3 = 2 \times 1 + 1$	$1^2 + (2 \times 1 + 1) = 2^2$
3	$5 = 2 \times 2 + 1$	$2^2 + (2 \times 2 + 1) = 3^2$
4	$7 = 2 \times 3 + 1$	$3^2 + (2 \times 3 + 1) = 4^2$
5	$9 = 2 \times 4 + 1$	$4^2 + (2 \times 4 + 1) = 5^2$
6	$11 = 2 \times 5 + 1$	$5^2 + (2 \times 5 + 1) = 6^2$
7	$13 = 2 \times 6 + 1$	$6^2 + (2 \times 6 + 1) = 7^2$
8	$15 = 2 \times 7 + 1$	$7^2 + (2 \times 7 + 1) = 8^2$
9	$17 = 2 \times 8 + 1$	$8^2 + (2 \times 8 + 1) = 9^2$
10	$19 = 2 \times 9 + 1$	$9^2 + (2 \times 9 + 1) = 10^2$